

4. The number of bacteria on a surface is being monitored.

The number of bacteria, N , on the surface, t hours after monitoring began is modelled by the equation

$$\log_{10} N = 0.35t + 2$$

Use the equation of the model to answer parts (a) to (c).

- (a) Find the initial number of bacteria on the surface.

(1)

- (b) Show that the equation of the model can be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of b to 2 decimal places.

(3)

- (c) Hence find the rate of growth of bacteria on the surface exactly 5 hours after monitoring began.

(2)

5. A hot piece of metal is cooled by dropping it into water. The temperature, $H^\circ\text{C}$, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \quad t \geq 0$$

Use the equation of the model to answer parts (a) to (d).

- (a) Find the initial temperature of the piece of metal.

(1)

- (b) On Diagram 1, sketch the graph of H against t . On your sketch, state the equation of the asymptote to the curve.

(2)

- (c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(3)

- (d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)

- 9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D e^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

- (a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

- (b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

- (c) Find the value of T , giving your answer to 2 decimal places.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

4. The function f is defined by

$$f(x) = 2x^2 - 5 \quad x \geq 0 \quad x \in \mathbb{R}$$

- (a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

- (b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

Using algebra and showing your working,

- (c) find the exact x coordinate of P

(3)

9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\frac{3 \sin \theta \cos \theta}{\cos \theta + \sin \theta} = (2 + \sec 2\theta)(\cos \theta - \sin \theta)$$

can be written in the form

$$3 \sin 2\theta - 4 \cos 2\theta = 2 \quad (3)$$

(b) Hence solve for $\pi < x < \frac{3\pi}{2}$

$$\frac{3 \sin x \cos x}{\cos x + \sin x} = (2 + \sec 2x)(\cos x - \sin x)$$

giving the answer to 3 significant figures.

(5)

5. The functions f and g are defined by

$$f(x) = 2 + 5 \ln x \quad x > 0$$

$$g(x) = \frac{6x - 2}{2x + 1} \quad x > \frac{1}{3}$$

(a) Find $f^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

10:

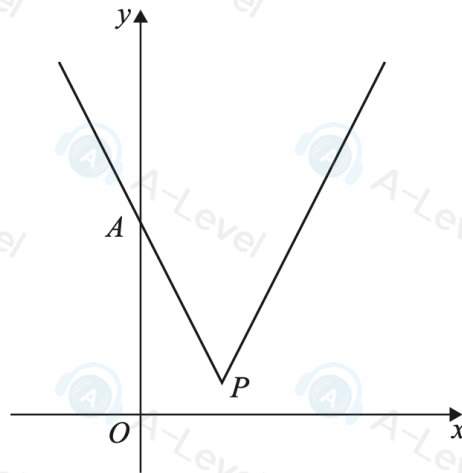


Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \quad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y -axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k ,

- the y coordinate of A
- the coordinates of P

(3)

(b) Find, in terms of k , the range of values of x which satisfy

$$|kx - 10| + k \geq 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = f(x)$ at 2 distinct points,

(c) find the range of values of k .

(4)

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