3. The share price, £V, of a company is being monitored.

A graph is drawn of $\log_{10} V$ against t, where t is the number of years after monitoring began.

The graph is a straight line passing through the points $(0, 2)$ and $(5, 2.25)$

Using this information,

(a) find an equation for the line in the form

$$\log_{10} V = mt + c$$

where m and c are constants.

(2)

(b) Write the answer to part (a) in the form

$$V = ab^t$$

where a and b are constants to be found.

Give the exact value of a and the value of b to 3 significant figures.

(3)

When $t = T$, the rate of increase in the share price of the company was £50 per year.

(c) Find the value of T, giving your answer to the nearest integer.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

3. (i) The variables x and y are connected by the equation

$$y = \frac{10^6}{x^3} \qquad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

(3)

(ii)

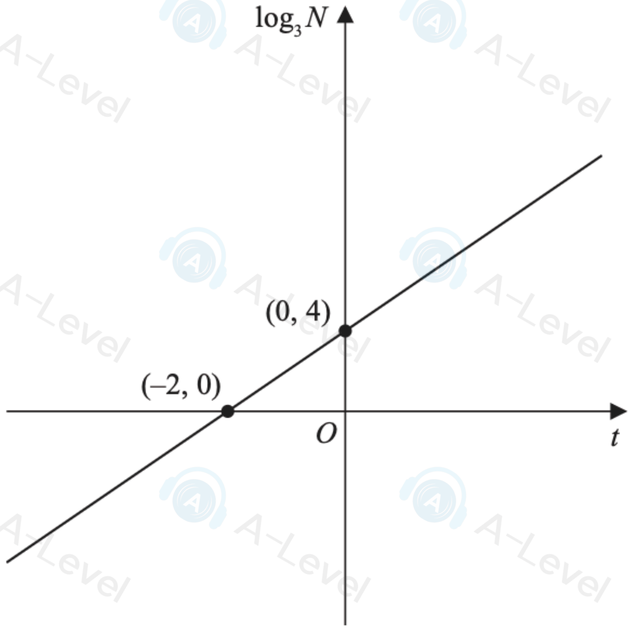

Figure 2

Figure 2 shows the linear relationship between $\log_3 N$ and t.

Show that $N = ab^t$ where a and b are constants to be found.

(3)

9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D\,\mathrm{e}^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

(a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

(b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

(c) Find the value of T, giving your answer to 2 decimal places.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

DO NOT WRITE IN THIS AREA

7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = A\mathrm{e}^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000\mathrm{e}^{-0.6t} \qquad t \geqslant 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T, giving your answer to 3 significant figures.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

DO NOT WRITE IN THIS AREA

6. An area of sea floor is being monitored.

The area of the sea floor, S km^2, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where p and q are constants and t is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

(a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$ **(2)**

(b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of p and the value of q each to 3 significant figures. **(3)**

(c) With reference to the model, interpret the value of the constant q **(1)**

3:

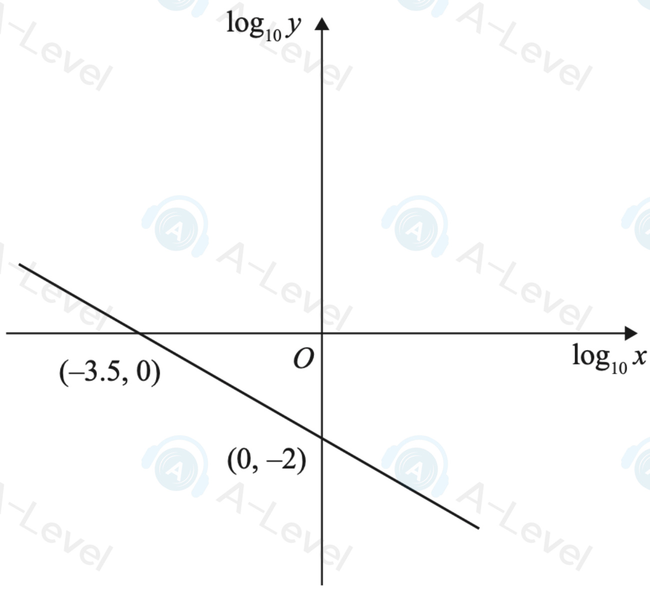

Figure 1

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points (–3.5, 0) and (0, –2) as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$ **(2)**

(b) Hence, or otherwise, express y in the form px^q where p and q are rational constants. **(3)**

5. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The number of squirrels in a forest is being studied.

The number of squirrels, N, in the forest, t years after the start of the study, is modelled by the equation

$$N = \frac{4000e^{0.1t}}{19 + e^{0.2t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the number of squirrels in the forest at the start of the study.
(1)

(b) Find $\dfrac{dN}{dt}$
(2)

The number of squirrels in the forest is at a maximum when $t = T$

Using the answer to part (b),

(c) show that $e^{0.2T} = A$, where A is a constant to be found.
(2)

(d) Hence find the maximum number of squirrels in the forest.

Show your working and give your answer to the nearest whole number.
(3)

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of A.
(1)

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of T. Give your answer to 2 decimal places.
(3)

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(Solutions relying entirely on calculator technology are not acceptable.)
(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.
(1)

3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, £D, is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

(a) Write this equation in the form

$$D = ab^t$$

where a and b are constants to be found. Give each value to 4 significant figures.

(3)

When $t = T$, the total amount of money raised is £45 000

According to the model,

(b) find the value of T, giving your answer to 3 significant figures.

(2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

(c) determine whether or not the charity will achieve its aim.

(2)

3.

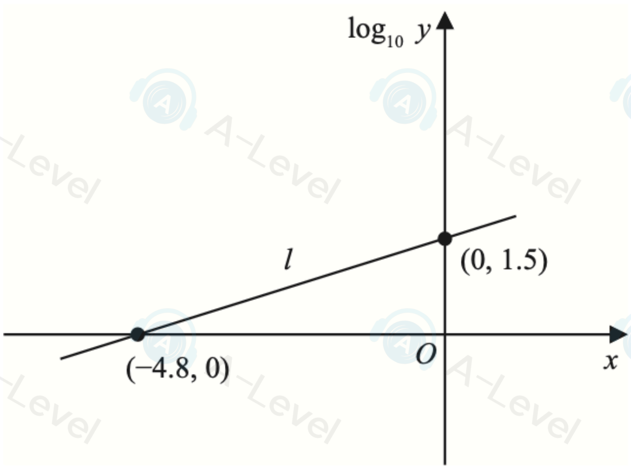

Figure 1

The line l in Figure 1 shows a linear relationship between $\log_{10} y$ and x.

The line passes through the points (0, 1.5) and (−4.8, 0) as shown.

(a) Write down an equation for l.

(2)

(b) Hence, or otherwise, express y in the form kb^x, giving the values of the constants k and b to 3 significant figures.

(3)

2. The weed on the surface of a pond is being monitored.

The surface area of the pond covered by the weed, $A\,\text{m}^2$, is modelled by the equation

$$\log_{10} A = 1 + 0.03t$$

where t is the number of weeks after monitoring began.

Use the equation of the model to answer parts (a) and (b).

(a) Find the surface area of the pond initially covered by the weed.

(1)

After T weeks, $25\,\text{m}^2$ of the pond is covered by the weed.

(b) Find the value of T, giving your answer to 2 decimal places.

(2)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, T °C, of the air in a room t minutes after a heat source is switched off, is modelled by the equation

$$T = 10 + Ae^{-Bt}$$

where A and B are constants.

Given that the temperature of the air in the room at the instant the heat source was switched off was 18 °C,

(a) find the value of A

(1)

Given also that, exactly 45 minutes after the heat source was switched off, the temperature of the air in the room was 16 °C,

(b) find the value of B to 3 significant figures.

(3)

Using the values for A and B,

(c) find, according to the model, the rate of change of the temperature of the air in the room exactly two minutes after the heat source was switched off.
Give your answer in $°\text{C}\,\text{min}^{-1}$ to 3 significant figures.

(2)

(d) Explain why, according to the model, the temperature of the air in the room cannot fall to 5 °C

(1)

5. A hot piece of metal is cooled by dropping it into water. The temperature, H°C, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280\mathrm{e}^{-0.05t} + 24 \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a) to (d).

(a) Find the initial temperature of the piece of metal.

(1)

(b) On Diagram 1, sketch the graph of H against t. On your sketch, state the equation of the asymptote to the curve.

(2)

(c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(3)

(d) Show by differentiation that

$$\frac{\mathrm{d}H}{\mathrm{d}t} = a + bH$$

where a and b are constants to be found.

(3)

2.

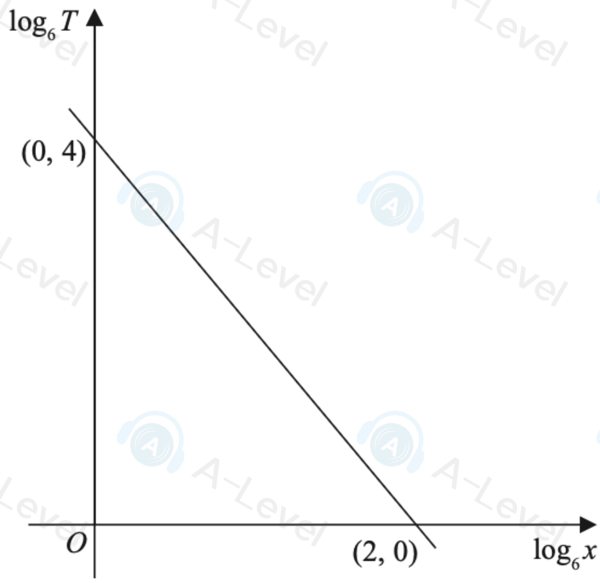

Figure 1

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$

The line passes through the points (0, 4) and (2, 0) as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of T when $x = 216$

(3)

(b) Find an equation, not involving logs, linking T with x

(3)

8.

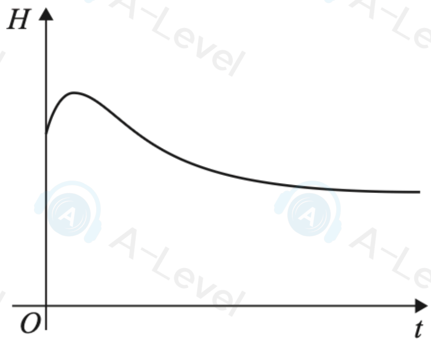

Figure 4

The heart rate of a horse is being monitored.
The heart rate H, measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t.

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L.

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T, giving your answer to 3 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{8}{1 + 4e^{-0.9t_n}}\right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A.

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C,

(b) find the value of k, giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

4. The number of bacteria on a surface is being monitored.

The number of bacteria, N, on the surface, t hours after monitoring began is modelled by the equation

$$\log_{10} N = 0.35t + 2$$

Use the equation of the model to answer parts (a) to (c).

(a) Find the initial number of bacteria on the surface.

(1)

(b) Show that the equation of the model can be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of b to 2 decimal places.

(3)

(c) Hence find the rate of growth of bacteria on the surface exactly 5 hours after monitoring began.

(2)