4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + q\ln 2$, where p and q are integers.

(5)

2.

$$\mathrm{f}(x) = \cos x + 2\sin x$$

(a) Express f(x) in the form $R\cos(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$\mathrm{g}(x) = 3 - 7\mathrm{f}(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of $\mathrm{g}(x)$,

(ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places.

(3)

2.

$$\mathrm{g}(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $\mathrm{g}(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where A, B and C are integers to be found.

(3)

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x)\,dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

9.

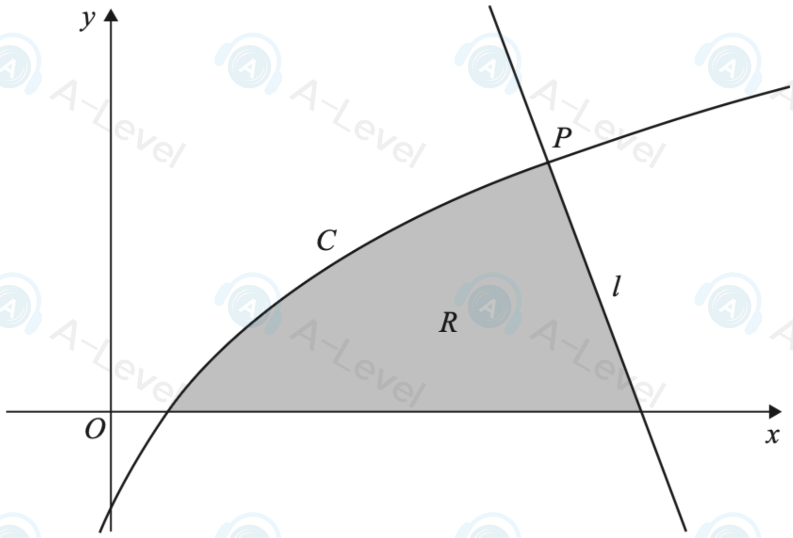

Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{6x^2 + 4x - 2}{2x + 1} \qquad x > -\frac{1}{2}$$

(a) Find $f'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

(b) Show that an equation for l is

$$16y + 5x = 106$$

(3)

(c) Write $f(x)$ in the form $Ax + B + \dfrac{D}{2x + 1}$ where A, B and D are constants.

(3)

The region R, shown shaded in Figure 5, is bounded by C, l and the x-axis.

(d) Use algebraic integration to find the exact area of R, giving your answer in the form $P + Q\ln 3$, where P and Q are rational constants.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

3. (a) Using the identity for $\cos(A+B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

(2)

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,\mathrm{d}x$$

(4)

7. The curve C has equation

$$x = 3\tan\left(y - \frac{\pi}{6}\right) \qquad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C.

Given that the tangent to C at P crosses the x-axis at the point Q.

(b) find, in simplest form, the exact x coordinate of Q.

(5)

6.

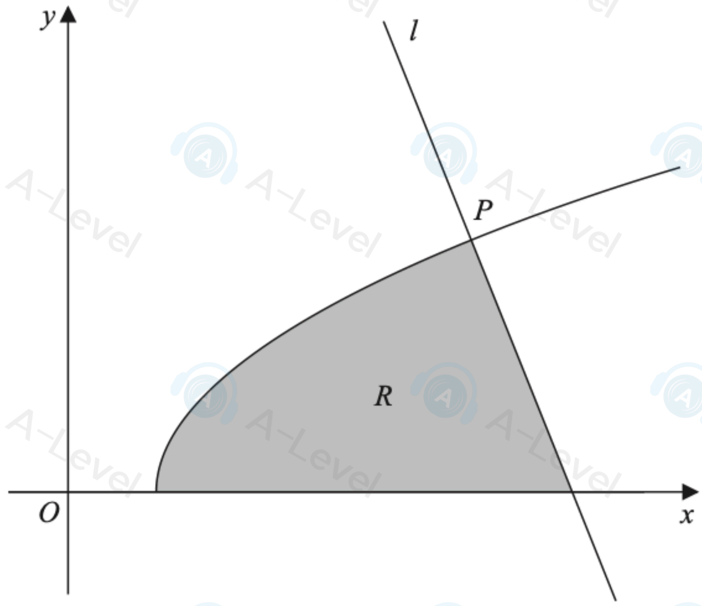

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

5: Find

(i) $\int \sin^2 3x \, dx$

(2)

(ii) $\int x(x^2 + 4)^{\frac{3}{2}} \, dx$

(2)

3. (i) Find $\dfrac{d}{dx}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form. **(2)**

(ii) (a) Find $\dfrac{d}{dx}\left(3x^2 - 4\right)^6$ **(2)**

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2 - 4\right)^5 \, dx = R$$

where R is an integer to be found.

(Solutions relying on calculator technology are not acceptable.) **(3)**