8.

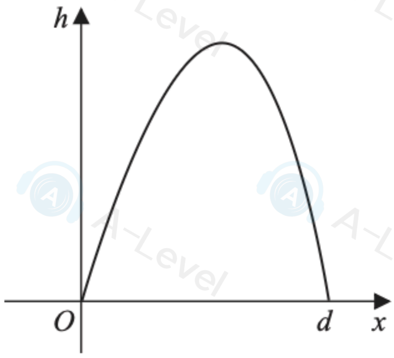

Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,e^{0.02x} \qquad 0 \leqslant x \leqslant d$$

Use the model to answer parts (a), (b) and (c).

(a) Find the value of d, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

(b) Show that the maximum value of h occurs when

$$x = 50\ln\left(\frac{150}{x+50}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = 50\ln\left(\frac{150}{x_n+50}\right) \qquad \text{with } x_1 = 30$$

(c) (i) find the value of x_2 to 2 decimal places,

(ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

(3)

2.

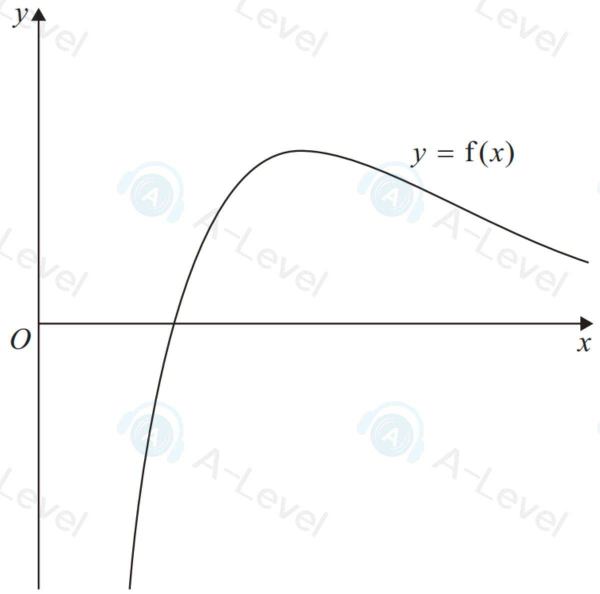

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x^2 + 3x - 4}{\mathrm{e}^x} - \frac{1}{x^2} \qquad x \in \mathbb{R} \quad x \neq 0$$

(a) Show that $\mathrm{f}(x) = 0$ has a root α in the interval $[1, 2]$

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \sqrt[3]{\frac{\mathrm{e}^x + 4x^2}{2x + 3}}$$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[3]{\frac{\mathrm{e}^{x_n} + 4x_n^{\ 2}}{2x_n + 3}} \qquad \text{with } x_1 = 1$$

find, to 4 decimal places,

(c) (i) the value of x_3

(ii) the value of α

(3)

1. $$g(x) = x^6 + 2x - 1000$$

(a) Show that $g(x) = 0$ has a root α in the interval $[3, 4]$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \qquad \text{with } x_1 = 3$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α.
Give your answer to 4 decimal places.

(3)

1. $$f(x) = 2\sec x + 6x - 3 \qquad 0 < x < \frac{\pi}{2}$$

The equation $f(x) = 0$ has a single root α

(a) Show that $0.1 < \alpha < 0.2$

(2)

(b) Show that α is a solution of

$$x = \frac{1}{2} - \frac{1}{3\cos x}$$

(1)

The iterative formula

$$x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$$

is used to find α

(c) Starting with $x_1 = 0.15$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

2: $$f(x) = 4x^3 + 2x^2 - 12$$

The equation $f(x) = 0$ has a single root α.

(a) Show that α lies in the interval [1, 2]

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{k}{2x+1}}$$

where k is a constant to be found.

(2)

(c) Using the iteration formula

$$x_{n+1} = \sqrt{\frac{k}{2x_n + 1}}$$

with $x_1 = 1$ and the value of k found in part (b), find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α.

(3)

DO NOT WRITE IN THIS AREA

2. A curve has equation $y = f(x)$ where

$$f(x) = x^4 - 5x^2 + 4x - 7 \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [2, 3]

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$$

(1)

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{5x_n^2 - 4x_n + 7}{x_n}}$$

is used to find α

(c) Starting with $x_1 = 2$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

DO NOT WRITE IN THIS AREA

1. A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[1, 2]$

(2)

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root α.

(b) (i) Find the value of x_2 to 4 decimal places.

(ii) Find, by repeated iteration, the value of α, giving your answer to 4 decimal places.

(3)

7: A continuous curve has equation

$$y = e^{-x^2}\sin 3x \qquad 0 \leqslant x \leqslant \frac{\pi}{3}$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3}\arctan\left(\frac{3}{2x}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = \frac{1}{3}\arctan\left(\frac{3}{2x_n}\right) \qquad x_1 = 0.4$$

(b) find the value of

(i) x_2

(ii) x_4

giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places.

(2)