3.

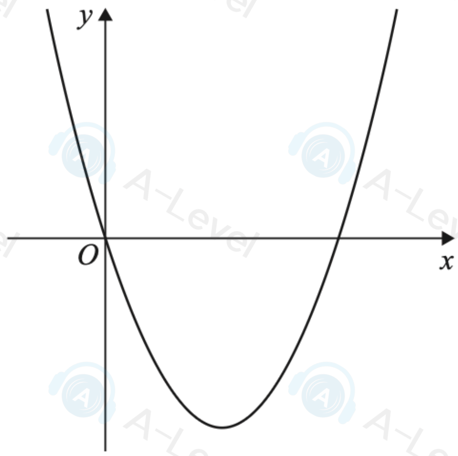

Figure 2

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x^2 - 10x \qquad x \in \mathbb{R}$$

(a) Solve the equation

$$\mathrm{f}(|x|) = 48$$

(3)

(b) Find the set of values of x for which

$$|\mathrm{f}(x)| \geqslant \frac{5}{2}x$$

(4)

4. The function f is defined by

$$\mathrm{f}(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

(b) On Diagram 1, sketch the curve with equation $y = \mathrm{f}^{-1}(x)$.

(2)

The curve with equation $y = \mathrm{f}(x)$ meets the curve with equation $y = \mathrm{f}^{-1}(x)$ at the point P

Using algebra and showing your working,

(c) find the exact x coordinate of P

(3)

1.

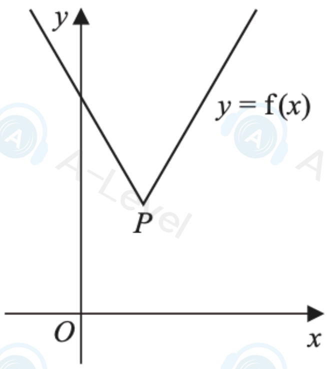

Figure 1

Figure 1 shows a sketch of the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2|x - 5| + 10$$

The point P, shown in Figure 1, is the vertex of the graph.

(a) State the coordinates of P

(2)

(b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(*Solutions relying on calculator technology are not acceptable.*)

(2)

(c) Find the point to which P is mapped, when the graph with equation $y = \mathrm{f}(x)$ is transformed to the graph with equation $y = 3\mathrm{f}(x - 2)$

(2)

5. The functions f and g are defined by

$$\mathrm{f}(x) = 2 + 5\ln x \qquad x > 0$$

$$\mathrm{g}(x) = \frac{6x - 2}{2x + 1} \qquad x > \frac{1}{3}$$

(a) Find $\mathrm{f}^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

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DO NOT WRITE IN THIS AREA

8.

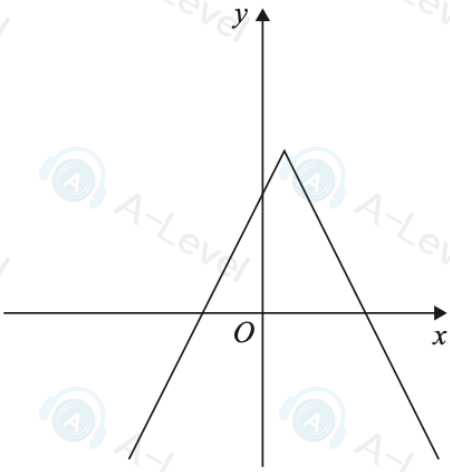

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b,

(i) the coordinates of the maximum point of the graph,

(ii) the coordinates of the point of intersection of the graph with the y-axis,

(iii) the coordinates of the points of intersection of the graph with the x-axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)

6.

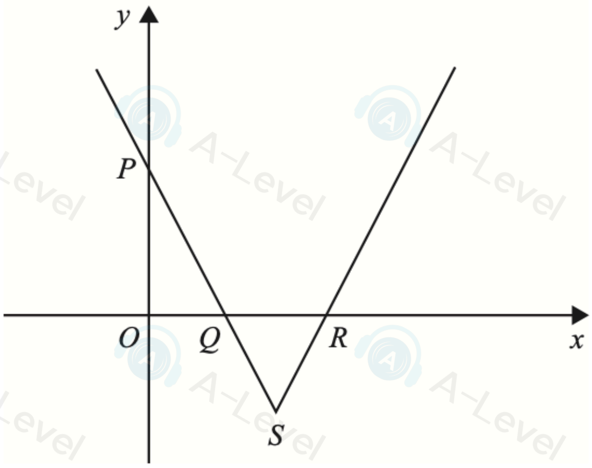

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y-axis at the point P
- cuts the x-axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a, the coordinates of

(i) point P

(ii) points Q and R

(iii) point S

(4)

(b) Find, in simplest form in terms of a, the values of x for which

$$|3x - 5a| - 2a = |x - 2a|$$

(4)

7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y-axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

giving any solution for x in terms of a and b.

(2)

4. The function f is defined by

$$f(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \qquad x \in \mathbb{R} \quad x > 2$$

(a) Show that $f(x) = \dfrac{2x}{3x-5}$

(3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear.

(3)

The function g is defined by

$$g(x) = 3 + 2\ln x \qquad x \geqslant 1$$

(c) Find g^{-1}

(3)

(d) Find the exact value of a for which

$$gf(a) = 5$$

(4)

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

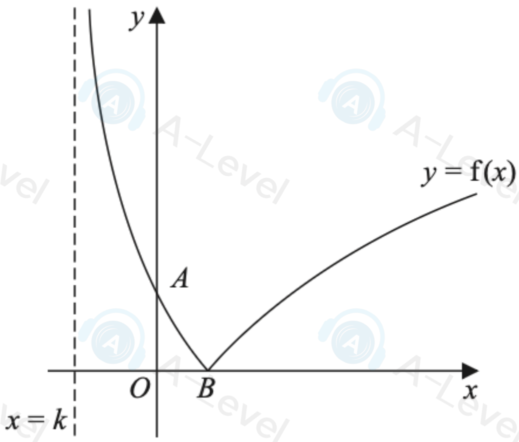

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x+1)| \qquad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y-axis at point A
- meets the x-axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x+1)| > 3$$

(5)

1: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \frac{2x}{3x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = 4 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Find the value of $gf(1)$

(2)

(b) Find the range of f

(2)

(c) Find $f^{-1}(x)$

(2)

(d) Solve $f^{-1}(x) = f(x)$

(2)

8: The functions f and g are defined by

$$f(x) = \ln(2x - 3) \qquad x \in \mathbb{R} \quad x > 2$$

$$g(x) = \frac{3x - 2a}{x - a} \qquad x \in \mathbb{R} \quad x \neq a$$

where a is a positive constant.

(a) Find the exact value of $fg(2a)$

(2)

(b) Find f^{-1}

(3)

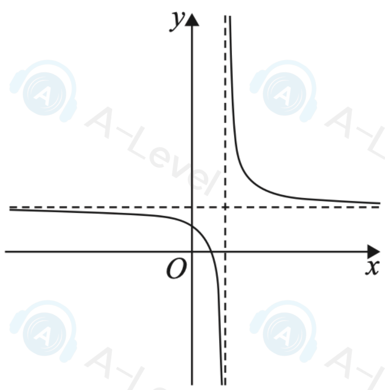

Figure 3

Figure 3 shows a sketch of the curve with equation $y = g(x)$

(c) Sketch a graph of the curve with equation $y = |g(x)|$

Indicate clearly on your sketch the equation of the **horizontal** asymptote.

(3)

The equation $|g(x)| = 10$ has two solutions.

Given that the larger solution is 8

(d) (i) find the value of a,

(ii) find the value of the smaller solution.

(4)

6.

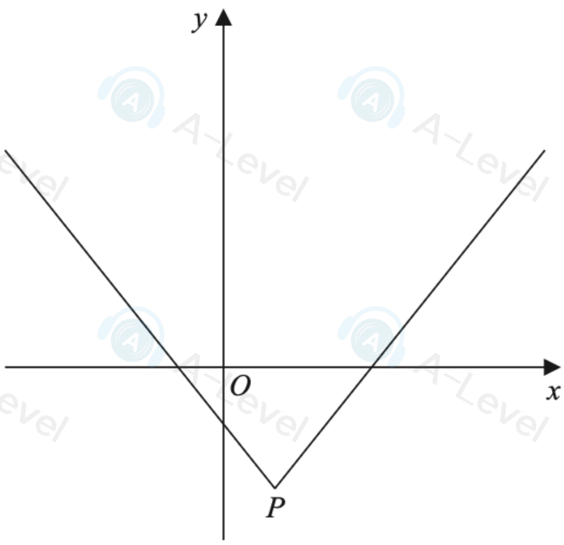

Figure 2

Figure 2 shows a sketch of the graph $y = f(x)$, where

$$f(x) = 3|x-2| - 10$$

The vertex of the graph is at point P, shown in Figure 2.

(a) Find the coordinates of P

(2)

(b) Find $ff(0)$

(2)

(c) Solve the inequality

$$3|x-2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

$$f(|x|) = 0$$

(3)

1. The point $P(-4, -3)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(2x)$

(1)

(b) $y = 3f(x - 1)$

(2)

(c) $y = |f(x)|$

(1)

10:

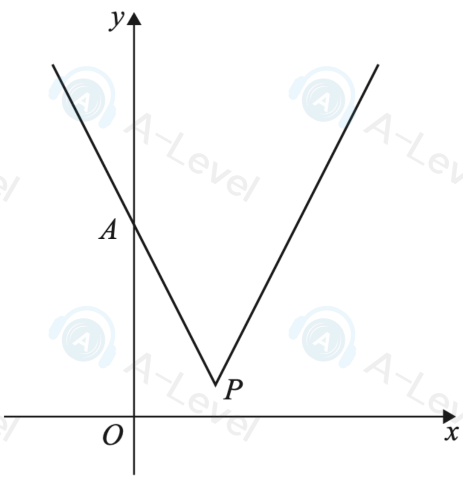

Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y-axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k,

(i) the y coordinate of A

(ii) the coordinates of P

(3)

(b) Find, in terms of k, the range of values of x which satisfy

$$|kx - 10| + k \geqslant 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = \mathrm{f}(x)$ at 2 distinct points,

(c) find the range of values of k.

(4)

1: The point $P(6, -2)$ lies on the continuous curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

Find the point to which P is mapped when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = 2\mathrm{f}(3x)$

(2)

(b) $y = \mathrm{f}(x - 2) + 8$

(2)

(c) $y = \mathrm{f}^{-1}(x)$

(1)

6. The function f is defined by

$$\mathrm{f}(x) = \frac{4x + 3}{x - 2} \qquad x \neq 2$$

(a) Find f^{-1}

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{ax + b}{cx + d}$$

where a, b, c and d are integers to be found.

(3)

The point $P(3, 15)$ lies on the curve with equation $y = \mathrm{f}(x)$.

(c) Find the point to which P is mapped when $y = \mathrm{f}(x)$ is transformed to the curve with equation $y = 2\mathrm{f}(3x) + 8$

(2)

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \ln(x^2 + 3) \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3 + 5x}{x + 2} \qquad x \in \mathbb{R} \quad x > -2$$

(a) State the range of f **(1)**

(b) Find g^{-1} **(3)**

(c) Find fg(0) **(2)**

(d) Find the exact value of a for which

$$g\left(e^{2a}\right) = f\left(\sqrt{e^4 - 3}\right)$$

(3)

2. The function f is defined by

$$f(x) = \frac{x + 3}{x - 4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find ff(6) **(2)**

(b) Find f^{-1} **(3)**

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

1. The functions f and g are defined by

$$f(x) = 9 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = \frac{3}{2x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Write down the range of f **(1)**

(b) Find the value of fg(1.5) **(2)**

(c) Find g^{-1} **(3)**

8.

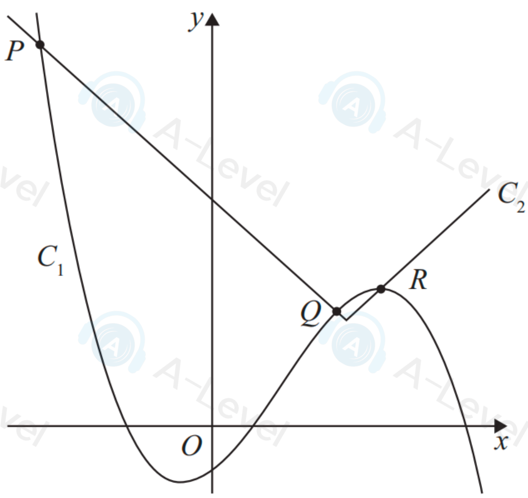

Figure 2

Figure 2 shows a sketch of the graph C_1 with equation

$$y = -2x^3 + 5x^2 + 4x - 3$$

and a sketch of the graph C_2 with equation

$$y = a + |5x + b|$$

where a and b are constants.

Given that P has coordinates $(-2, 25)$

(a) show that

$$a = 15 + b$$

(2)

Given also that R has coordinates $(2, 9)$

(b) find the value of a and the value of b

(3)

Using the answer to part (b),

(c) state the coordinates of the vertex of C_2

(2)

(d) Find, using algebra, the coordinates of Q. Show each stage of your working.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

6. The functions f and g are defined by

$$f(x) = 6 - \frac{21}{2x + 3} \qquad x \geqslant 0$$

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}$$

(a) Find $gf(2)$

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$gg(x) = 126$$

(3)