

9.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Express

$$6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$$

in terms of $\sin 2\theta$ and $\cos 2\theta$ only.

(3)

(b) Hence show that the equation

$$3 \cot 2\theta - 14 = 6 \sin^2 \theta \cot 2\theta + 4 \sin \theta \cos \theta$$

can be written in the form

$$5 \sin^2 2\theta + 14 \sin 2\theta - 3 = 0$$

(3)

(c) Hence solve, for $0 < x < 90^\circ$, the equation

$$3 \cot 2x - 14 = 6 \sin^2 x \cot 2x + 4 \sin x \cos x$$

giving your answers to one decimal place.

(3)

9.

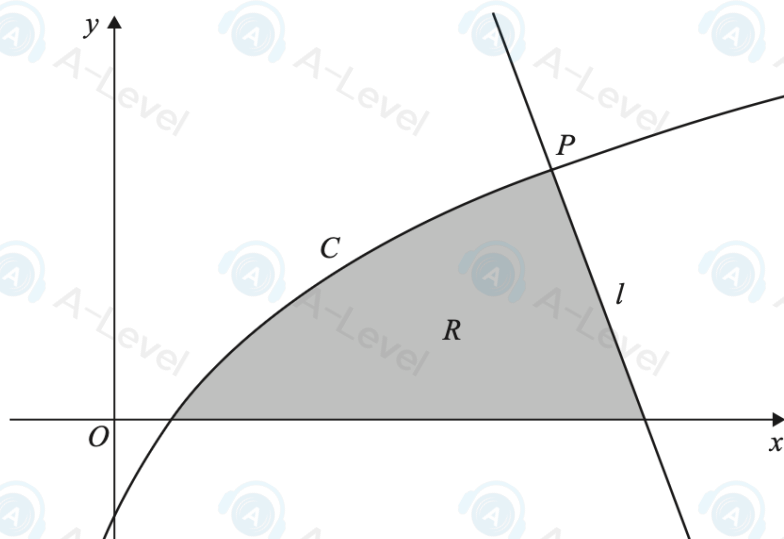


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{6x^2 + 4x - 2}{2x + 1} \quad x > -\frac{1}{2}$$

(a) Find $f'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

(b) Show that an equation for l is

$$16y + 5x = 106 \quad (3)$$

(c) Write $f(x)$ in the form $Ax + B + \frac{D}{2x+1}$ where A , B and D are constants.

(3)

The region R , shown shaded in Figure 5, is bounded by C , l and the x -axis.

(d) Use algebraic integration to find the exact area of R , giving your answer in the form $P + Q \ln 3$, where P and Q are rational constants.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\frac{3 \sin \theta \cos \theta}{\cos \theta + \sin \theta} = (2 + \sec 2\theta)(\cos \theta - \sin \theta)$$

can be written in the form

$$3 \sin 2\theta - 4 \cos 2\theta = 2 \quad (3)$$

(b) Hence solve for $\pi < x < \frac{3\pi}{2}$

$$\frac{3 \sin x \cos x}{\cos x + \sin x} = (2 + \sec 2x)(\cos x - \sin x)$$

giving the answer to 3 significant figures.

(5)

9.

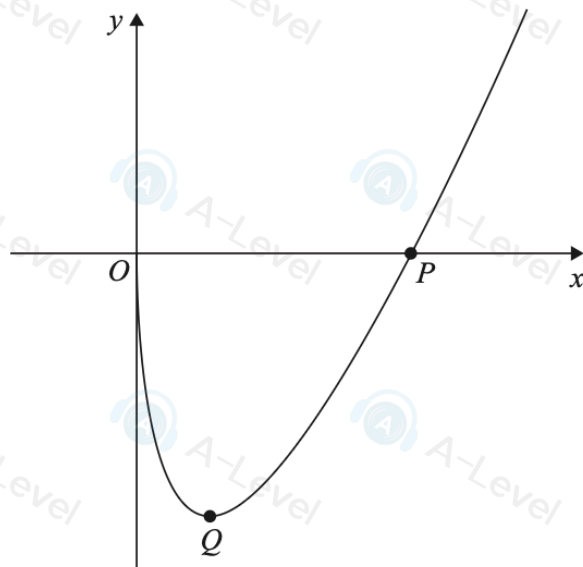
**Figure 1**

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = 6\sqrt{x} \ln(4x) \quad x > 0$$

The curve cuts the x -axis at point P

(a) State the x coordinate of P

(1)

The point Q , shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $g(x)$ where

$$g(x) = -2f(x)$$

(2)

3. (i) Find $\frac{d}{dx} \ln(\sin^2 3x)$ writing your answer in simplest form.

(2)

(ii)(a) Find $\frac{d}{dx} (3x^2 - 4)^6$

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x(3x^2 - 4)^5 dx = R$$

where R is an integer to be found.

(Solutions relying on calculator technology are not acceptable.)

(3)

3:

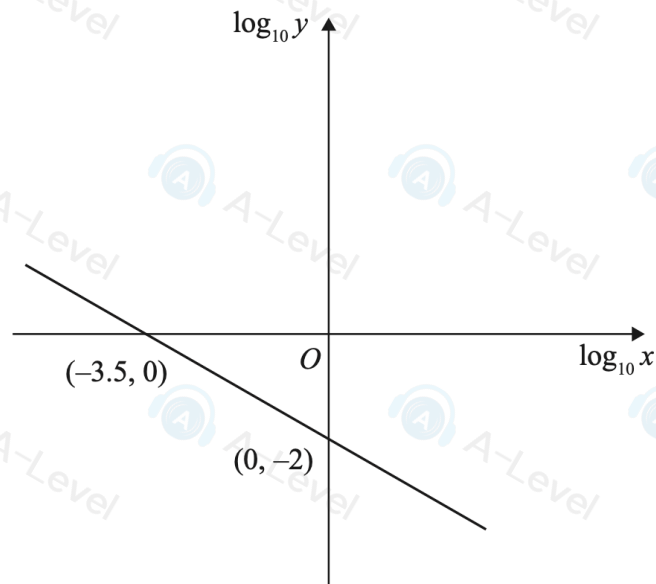
**Figure 1**

Figure 1 shows a linear relationship between $\log_{10} y$ and $\log_{10} x$

The line passes through the points $(-3.5, 0)$ and $(0, -2)$ as shown.

(a) Find an equation linking $\log_{10} y$ with $\log_{10} x$

(2)

(b) Hence, or otherwise, express y in the form px^q where p and q are rational constants.

(3)

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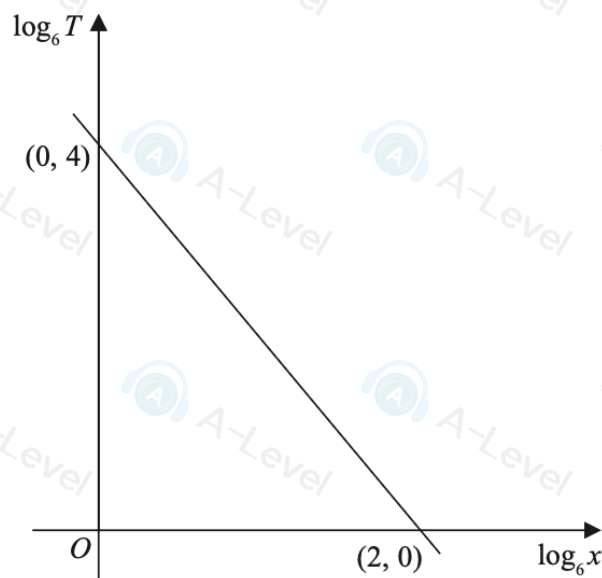


Figure 1

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$.

The line passes through the points $(0, 4)$ and $(2, 0)$ as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of T when $x = 216$

(3)

(b) Find an equation, not involving logs, linking T with x

(3)

- 9: The amount of an antibiotic, x milligrams, in the bloodstream of a horse, t hours after the antibiotic had been administered, is given by the formula

$$x = D e^{-0.2t}$$

where D milligrams is the dose of the antibiotic given to the horse.

A dose of 30 mg of the antibiotic is given to the horse.

- (a) Find the amount of the antibiotic in the bloodstream of the horse 8 hours after the dose is given. Give your answer in mg to 2 decimal places.

(2)

A second dose of 20 mg is given to the horse 8 hours after the first dose.

- (b) Show that the amount of the antibiotic in the bloodstream of the horse, 2 hours after the second dose is given, is 17.5 mg to one decimal place.

(1)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the amount of the antibiotic in the bloodstream is 10 mg.

- (c) Find the value of T , giving your answer to 2 decimal places.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

4. The function f is defined by

$$f(x) = 2x^2 - 5 \quad x \geq 0 \quad x \in \mathbb{R}$$

- (a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

- (b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

Using algebra and showing your working,

- (c) find the exact x coordinate of P

(3)

4. $f(x) = 8 \sin x \cos x + 4 \cos^2 x - 3$

- (a) Write $f(x)$ in the form

$$a \sin 2x + b \cos 2x + c$$

where a , b and c are integers to be found.

(3)

- (b) Use the answer to part (a) to write $f(x)$ in the form

$$R \sin(2x + \alpha) + c$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 significant figures.

(3)

- (c) Hence, or otherwise,

- (i) state the maximum value of $f(x)$
 (ii) find the **second** smallest positive value of x at which a maximum value of $f(x)$ occurs. Give your answer to 3 significant figures.

(3)

5.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**

- (i) Solve, for
- $0 < x < \pi$

$$(x - 2)(\sqrt{3} \sec x + 2) = 0 \quad (3)$$

- (ii) Solve, for
- $0 < \theta < 360^\circ$

$$10 \sin \theta = 3 \cos 2\theta \quad (4)$$

8. (a) Prove that

$$2 \operatorname{cosec}^2 2\theta (1 - \cos 2\theta) \equiv 1 + \tan^2 \theta \quad (4)$$

- (b) Hence solve for
- $0 < x < 360^\circ$
- , where
- $x \neq (90n)^\circ$
- ,
- $n \in \mathbb{N}$
- , the equation

$$2 \operatorname{cosec}^2 2x (1 - \cos 2x) = 4 + 3 \sec x$$

giving your answers to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.) (4)

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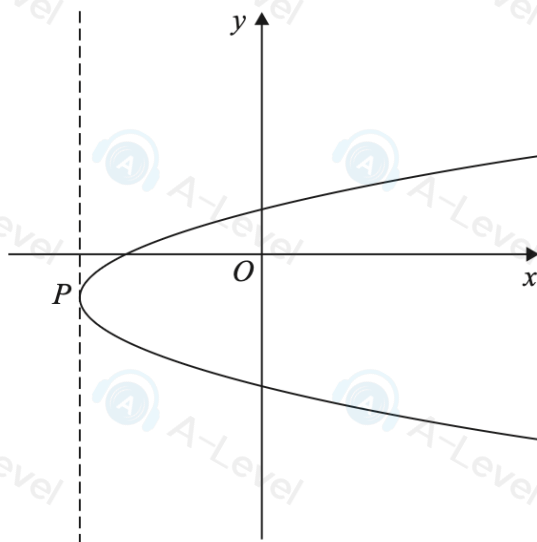


Figure 1

Figure 1 shows a sketch of the curve with equation

$$x = 2y^2 + 5y - 6$$

- (a) Find $\frac{dy}{dx}$ in terms of y .

(2)

The point P lies on the curve and is shown in Figure 1.

Given that the tangent to the curve at P is parallel to the y -axis,

- (b) find the coordinates of P .

(3)

7: A continuous curve has equation

$$y = e^{-x^2} \sin 3x \quad 0 \leq x \leq \frac{\pi}{3}$$

The curve has a stationary point at the point P .

- (a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$$

(4)

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Using the iteration formula

$$x_{n+1} = \frac{1}{3} \arctan\left(\frac{3}{2x_n}\right) \quad x_1 = 0.4$$

(b) find the value of

(i) x_2

(ii) x_4

giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places.

(2)

9.

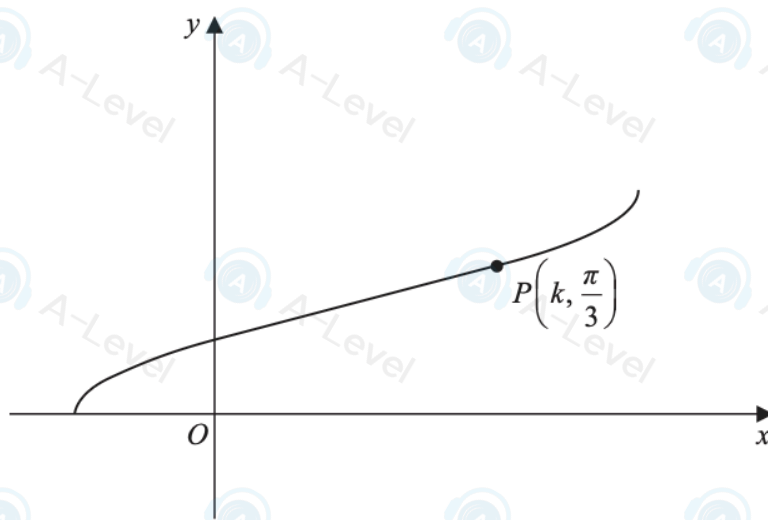


Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \quad 0 \leq y \leq \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x -axis at the point N .

(c) Find the exact area of triangle OPN , where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

5. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The temperature, $T^{\circ}\text{C}$, of the air in a room t minutes after a heat source is switched off, is modelled by the equation

$$T = 10 + Ae^{-Bt}$$

where A and B are constants.

Given that the temperature of the air in the room at the instant the heat source was switched off was 18°C ,

- (a) find the value of A (1)

Given also that, exactly 45 minutes after the heat source was switched off, the temperature of the air in the room was 16°C ,

- (b) find the value of B to 3 significant figures. (3)

Using the values for A and B ,

- (c) find, according to the model, the rate of change of the temperature of the air in the room exactly two minutes after the heat source was switched off.
Give your answer in $^{\circ}\text{C min}^{-1}$ to 3 significant figures. (2)

- (d) Explain why, according to the model, the temperature of the air in the room cannot fall to 5°C (1)

7. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (a) Given that

$$\sqrt{2} \sin(x + 45^{\circ}) = \cos(x - 60^{\circ})$$

show that

$$\tan x = -2 - \sqrt{3} \quad (4)$$

- (b) Hence or otherwise, solve, for $0 \leq \theta < 180^{\circ}$

$$\sqrt{2} \sin(2\theta) = \cos(2\theta - 105^{\circ}) \quad (4)$$

1: The point $P(6, -2)$ lies on the continuous curve with equation $y = f(x)$, $x \in \mathbb{R}$.

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = 2f(3x)$ (2)

(b) $y = f(x - 2) + 8$ (2)

(c) $y = f^{-1}(x)$ (1)

2.

$$f(x) = \cos x + 2 \sin x$$

(a) Express $f(x)$ in the form $R \cos(x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α , in radians, to 3 decimal places. (3)

$$g(x) = 3 - 7f(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of $g(x)$,

(ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places. (3)

6:

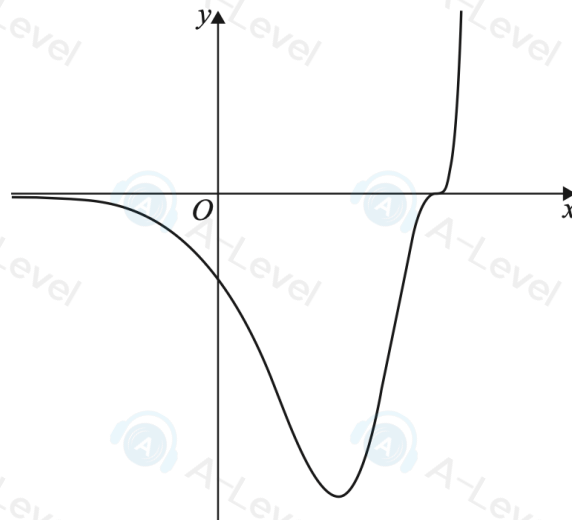


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x - 2}$$

- (a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P , Q and n are constants to be found.

(4)

- (b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,

(ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \quad x \leq \frac{3}{2}$$

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

3.

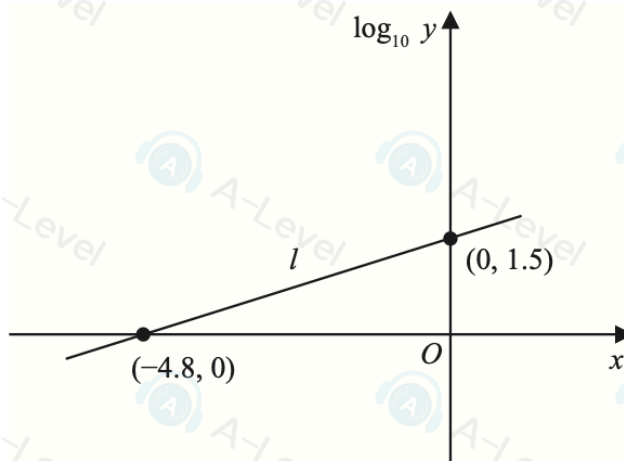


Figure 1

The line l in Figure 1 shows a linear relationship between $\log_{10} y$ and x .

The line passes through the points $(0, 1.5)$ and $(-4.8, 0)$ as shown.

- (a) Write down an equation for l .

(2)

- (b) Hence, or otherwise, express y in the form kb^x , giving the values of the constants k and b to 3 significant figures.

(3)

3. (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2 \cos^2 A - 1 \quad (2)$$

- (b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4 \cos^2 3x) dx \quad (4)$$

5. A hot piece of metal is cooled by dropping it into water. The temperature, $H^\circ\text{C}$, of the metal, t minutes after it is dropped into the water, is modelled by the equation

$$H = 280e^{-0.05t} + 24 \quad t \geq 0$$

Use the equation of the model to answer parts (a) to (d).

- (a) Find the initial temperature of the piece of metal. (1)

- (b) On Diagram 1, sketch the graph of H against t . On your sketch, state the equation of the asymptote to the curve. (2)

- (c) Find the value of t for which $H = 144$, giving your answer to 2 decimal places.

(Solutions based entirely on calculator technology are not acceptable.) (3)

- (d) Show by differentiation that

$$\frac{dH}{dt} = a + bH$$

where a and b are constants to be found.

(3)

6.

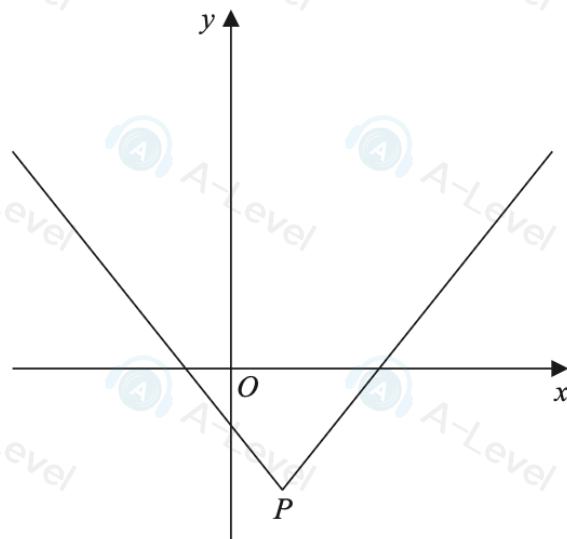


Figure 2

Figure 2 shows a sketch of the graph $y = f(x)$, where

$$f(x) = 3|x - 2| - 10$$

The vertex of the graph is at point P , shown in Figure 2.

(a) Find the coordinates of P

(2)

(b) Find $ff(0)$

(2)

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

$$f(|x|) = 0$$

(3)

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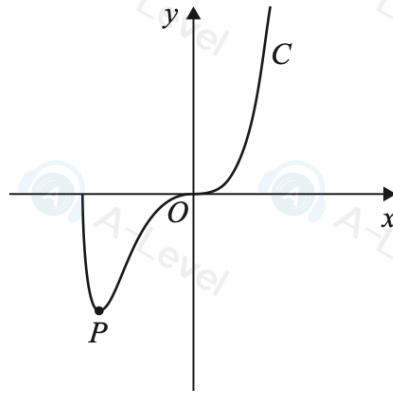


Figure 3

The curve C has equation $y = f(x)$, where

$$f(x) = x^3 \sqrt{4x + 7} \quad x \geq -\frac{7}{4}$$

(a) Show that

$$f'(x) = \frac{kx^2(2x + 3)}{\sqrt{4x + 7}}$$

where k is a constant to be found.

(4)

The point P , shown in Figure 3, is the minimum turning point on C .

(b) Find the coordinates of P .

(2)

(c) Hence find the range of the function g defined by

$$g(x) = -4f(x) \quad x \geq -\frac{7}{4}$$

(2)

The point Q with coordinates $\left(\frac{1}{2}, \frac{3}{8}\right)$ lies on C .

(d) Find the coordinates of the point to which Q is mapped when C is transformed to the curve with equation

$$y = 40f\left(x - \frac{3}{2}\right) - 8$$

(2)

2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \quad x \in \mathbb{R}, x \neq 4$$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = x^2 + 5 \quad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

4. The number of bacteria on a surface is being monitored.

The number of bacteria, N , on the surface, t hours after monitoring began is modelled by the equation

$$\log_{10} N = 0.35t + 2$$

Use the equation of the model to answer parts (a) to (c).

(a) Find the initial number of bacteria on the surface.

(1)

(b) Show that the equation of the model can be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of b to 2 decimal places.

(3)

(c) Hence find the rate of growth of bacteria on the surface exactly 5 hours after monitoring began.

(2)

6.

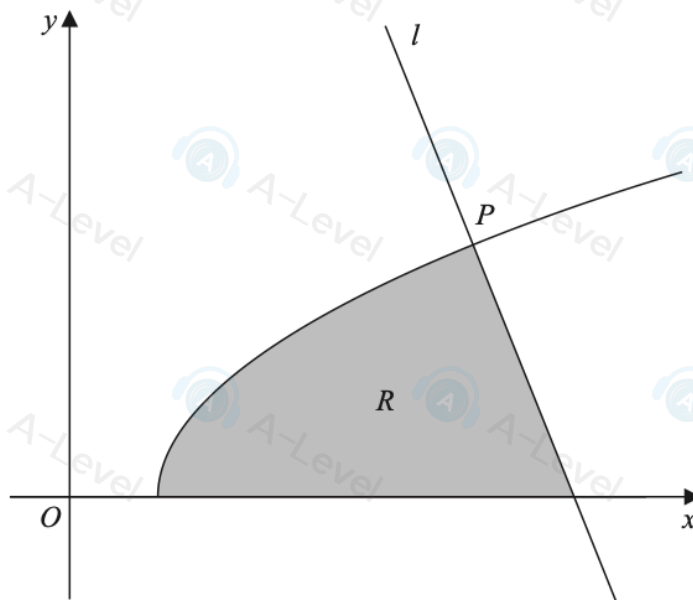


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l , shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0 \quad (5)$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and l .

(b) Use algebraic integration to find the exact area of R .

(4)

7.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 2xe^{x^2 + (3k-2)x}$$

where k is a constant.

Given that C has two distinct turning points, find the range of possible values of k .

(7)

4: The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \quad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x - 3} \quad x > 3 \quad (3)$$

(b) Hence find $f'(x)$ giving your answer in simplest form.

(2)

3.

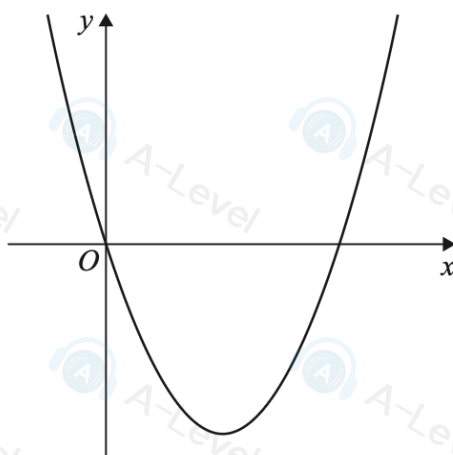


Figure 2

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2x^2 - 10x \quad x \in \mathbb{R}$$

(a) Solve the equation

$$f(|x|) = 48 \quad (3)$$

(b) Find the set of values of x for which

$$|f(x)| \geq \frac{5}{2}x \quad (4)$$

5.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**

- (a) Show that $\sin 3x$ can be written in the form

$$P \sin x + Q \sin^3 x$$

where P and Q are constants to be found.

(4)

- (b) Hence or otherwise, solve, for $0 < \theta \leq 360^\circ$, the equation

$$2 \sin 3\theta = 5 \sin 2\theta$$

giving your answers, in degrees, to one decimal place as appropriate.

(4)

3.

In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

A curve has equation

$$y = \frac{4x+1}{(x+3)^2} \quad x \neq -3 \quad x \in \mathbb{R}$$

Use calculus to find the range of values of x for which y is increasing.

(6)

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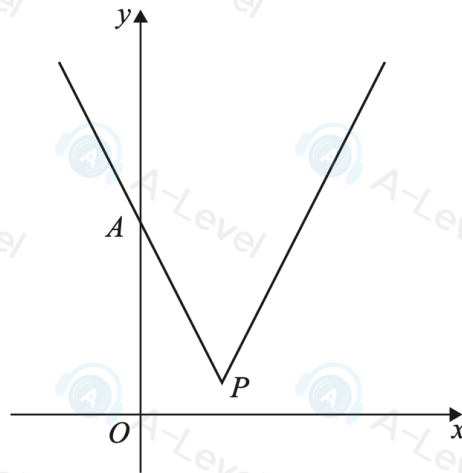


Figure 3

Figure 3 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 10| + k \quad x \in \mathbb{R}$$

and k is a positive constant.

The graph

- cuts the y -axis at the point A
- has a vertex at the point P

(a) Find, in simplest form in terms of k ,

- the y coordinate of A
- the coordinates of P

(3)

(b) Find, in terms of k , the range of values of x which satisfy

$$|kx - 10| + k \geq 2k$$

(3)

Given that the line with equation $y = 3x + 1$ intersects the graph of $y = f(x)$ at 2 distinct points,

(c) find the range of values of k .

(4)

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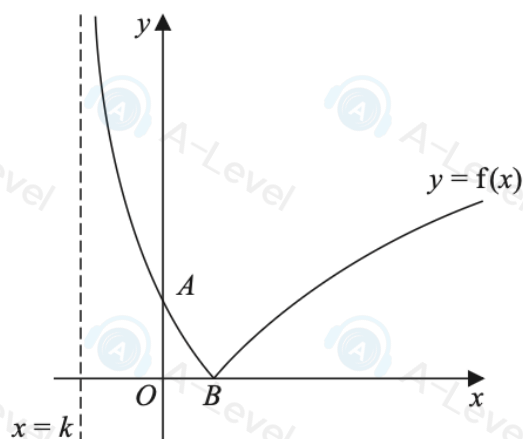


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \quad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y -axis at point A
- meets the x -axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A (ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x + 1)| > 3$$

(5)

7. The curve C has equation

$$x = 3 \tan\left(y - \frac{\pi}{6}\right) \quad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C .

Given that the tangent to C at P crosses the x -axis at the point Q .

(b) find, in simplest form, the exact x coordinate of Q .

(5)

6. An area of sea floor is being monitored.

The area of the sea floor, $S \text{ km}^2$, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where p and q are constants and t is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

(a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$

(2)

(b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of p and the value of q each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant q

(1)

9:

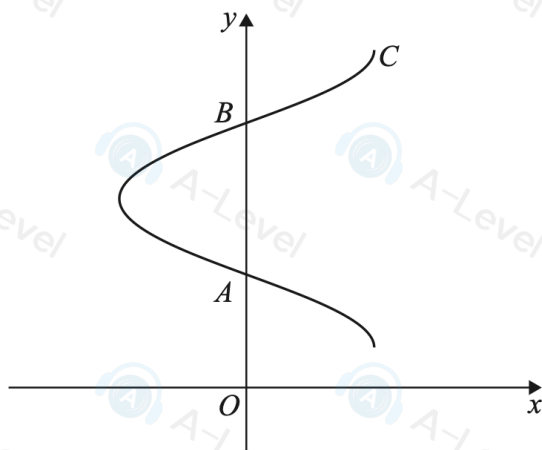


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$x = \frac{2}{3} \sin\left(3y + \frac{\pi}{4}\right) \quad \frac{\pi}{12} < y < \frac{3\pi}{4}$$

The curve intersects the y -axis at the points A and B as shown.

(a) Find the exact value of the y coordinate of

- point A
- point B

(3)

(b) Show that

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{p - qx^2}$$

where p and q are integers to be found.

(4)

The **normal** to C at A and the **tangent** to C at B intersect at the point D .

Using

- the answer to part (b)
- the sketch of curve C in Figure 2

(c) find, in simplest form, the exact x coordinate of D .

(4)

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2. The weed on the surface of a pond is being monitored.

The surface area of the pond covered by the weed, $A \text{ m}^2$, is modelled by the equation

$$\log_{10} A = 1 + 0.03t$$

where t is the number of weeks after monitoring began.

Use the equation of the model to answer parts (a) and (b).

- (a) Find the surface area of the pond initially covered by the weed.

(1)

After T weeks, 25 m^2 of the pond is covered by the weed.

- (b) Find the value of T , giving your answer to 2 decimal places.

(2)

2:

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

$$f(x) = 7 \cos x - 24 \sin x$$

- (a) Express $f(x)$ in the form $R \cos(x + \alpha)$ where R and α are constants, $R > 0$

$$\text{and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α , in radians, to 3 decimal places.

(3)

$$g(x) = \frac{5}{90 - 3f(2x)}$$

- (b) Using the answer to part (a), find

- (i) the minimum value of $g(x)$, giving your answer as a fully simplified fraction,
(ii) the smallest positive value of x for which this minimum value occurs, giving your answer to 3 decimal places.

(4)

DO NOT WRITE IN THIS AREA

DO NOT

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4 \cot 2x \operatorname{cosec} 2x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4 \cot 2\theta \operatorname{cosec} 2\theta = 2 \tan^2 \theta$$

giving your answers to 2 decimal places.

(5)

9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} \right)^2 = 6 \cot \theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \right) \cot x \, dx$$

(2)

2:

$$f(x) = 4x^3 + 2x^2 - 12$$

The equation $f(x) = 0$ has a single root α .

(a) Show that α lies in the interval $[1, 2]$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{k}{2x+1}}$$

where k is a constant to be found.

(2)

(c) Using the iteration formula

$$x_{n+1} = \sqrt{\frac{k}{2x_n+1}}$$

with $x_1 = 1$ and the value of k found in part (b), find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α .

(3)

10.

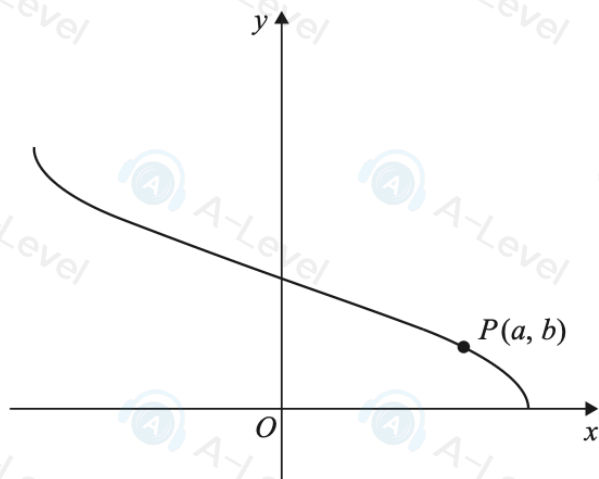


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3 \cos 2y \quad -3 \leq x \leq 3 \quad 0 \leq y \leq \frac{\pi}{2}$$

(a) Find $\frac{dx}{dy}$ in terms of y .

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b .

(4)

8.

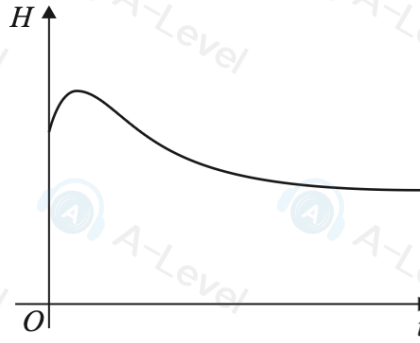


Figure 4

The heart rate of a horse is being monitored.

The heart rate H , measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t .

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L .

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T , giving your answer to 3 decimal places.

(Solutions based entirely on calculator technology are not acceptable.)

(5)

The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t}} \right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5 \ln \left(\frac{8}{1 + 4e^{-0.9t_n}} \right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)

4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A , B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} dx$$

giving your answer in the form $p + q \ln 2$, where p and q are integers.

(5)

7.

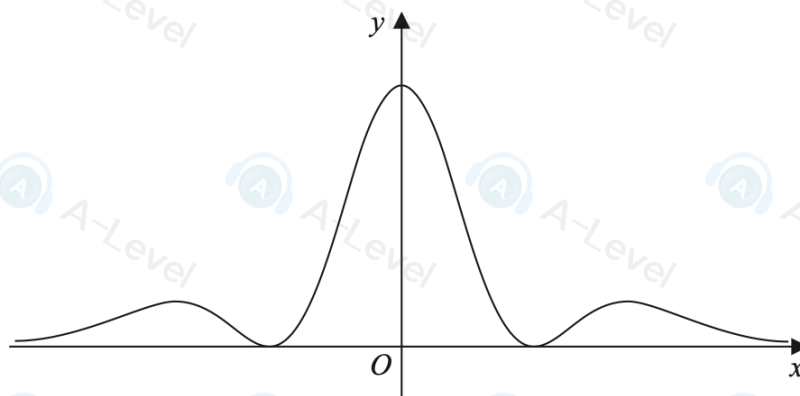


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = e^{-x^2} (2x^2 - 3)^2$$

(a) Find the range of f

(2)

(b) Show that

$$f'(x) = 2x(2x^2 - 3)e^{-x^2} (A - Bx^2)$$

where A and B are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

6. The functions f and g are defined by

$$f(x) = 6 - \frac{21}{2x+3} \quad x \geq 0$$

$$g(x) = x^2 + 5 \quad x \in \mathbb{R}$$

(a) Find $gf(2)$

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$gg(x) = 126$$

(3)

2.

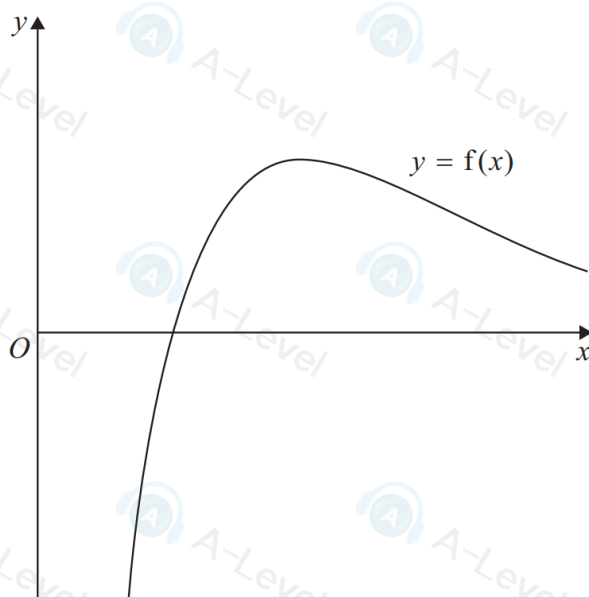


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = \frac{2x^2 + 3x - 4}{e^x} - \frac{1}{x^2} \quad x \in \mathbb{R} \quad x \neq 0$$

(a) Show that $f(x) = 0$ has a root α in the interval $[1, 2]$

(2)

(b) Show that the equation $f(x) = 0$ can be written in the form

$$x = \sqrt[3]{\frac{e^x + 4x^2}{2x + 3}}$$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[3]{\frac{e^{x_n} + 4x_n^2}{2x_n + 3}} \quad \text{with } x_1 = 1$$

find, to 4 decimal places,

(c) (i) the value of x_3

(ii) the value of α

(3)

1.

$$f(x) = 2\sec x + 6x - 3$$

$$0 < x < \frac{\pi}{2}$$

The equation $f(x) = 0$ has a single root α

(a) Show that $0.1 < \alpha < 0.2$

(2)

(b) Show that α is a solution of

$$x = \frac{1}{2} - \frac{1}{3\cos x}$$

(1)

The iterative formula

$$x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$$

is used to find α

(c) Starting with $x_1 = 0.15$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

6.

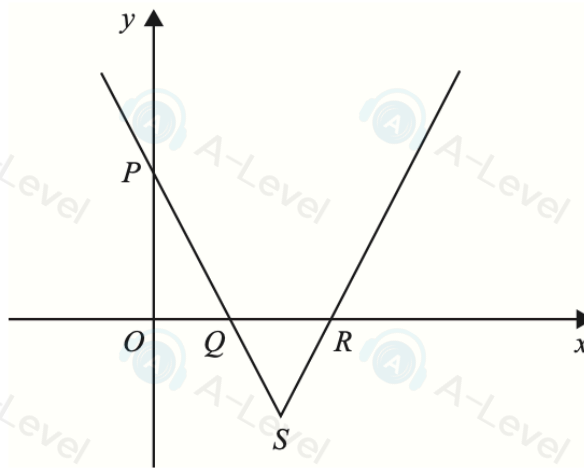


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y -axis at the point P
- cuts the x -axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a , the coordinates of

(i) point P

(ii) points Q and R

(iii) point S

(4)

(b) Find, in simplest form in terms of a , the values of x for which

$$|3x - 5a| - 2a = |x - 2a|$$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT

3. (i) The variables x and y are connected by the equation

$$y = \frac{10^6}{x^3} \quad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

(3)

(ii)

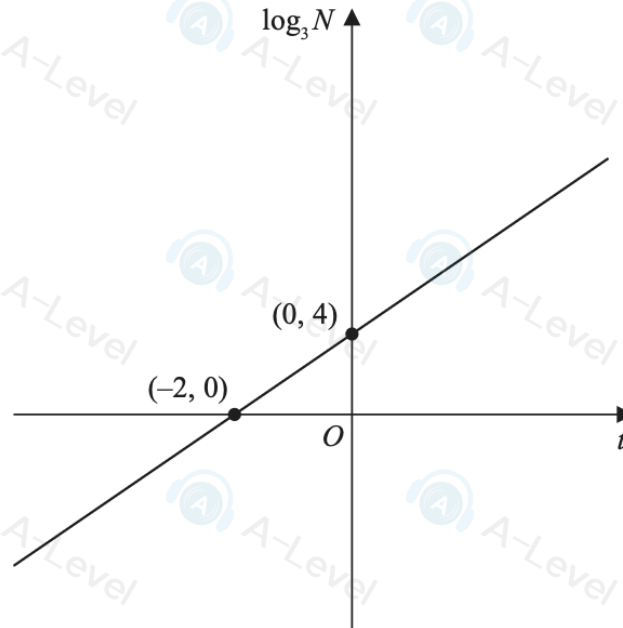


Figure 2

Figure 2 shows the linear relationship between $\log_3 N$ and t .

Show that $N = ab^t$ where a and b are constants to be found.

(3)

3. The amount of money raised for a charity is being monitored.

The total amount raised in the t months after monitoring began, £ D , is modelled by the equation

$$\log_{10} D = 1.04 + 0.38t$$

- (a) Write this equation in the form

$$D = ab^t$$

where a and b are constants to be found. Give each value to 4 significant figures.

(3)

When $t = T$, the total amount of money raised is £45 000

According to the model,

- (b) find the value of T , giving your answer to 3 significant figures.

(2)

The charity aims to raise a total of £350 000 within the first 12 months of monitoring.

According to the model,

- (c) determine whether or not the charity will achieve its aim.

(2)

8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (i) Solve, for $0 < \theta < \pi$

$$3 \operatorname{cosec} \theta = 8 \cos \theta$$

giving your answers, in radians, to 3 significant figures.

(5)

- (ii) Solve, for $0 < x < 180^\circ$

$$\frac{\tan 2x - \tan 70^\circ}{1 + \tan 2x \tan 70^\circ} = -\frac{3}{8}$$

giving your answers, in degrees, to one decimal place.

(4)

1:

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

The functions f and g are defined by

$$f(x) = \frac{2x}{3x+1} \quad x \in \mathbb{R} \quad x \geq 0$$

$$g(x) = 4 - x^2 \quad x \in \mathbb{R} \quad x \geq 0$$

(a) Find the value of $gf(1)$

(2)

(b) Find the range of f

(2)

(c) Find $f^{-1}(x)$

(2)

(d) Solve $f^{-1}(x) = f(x)$

(2)

7.

In this question you must show all stages of your working.**Solutions relying entirely on calculator technology are not acceptable.**The curve C has equation

$$y = \frac{16}{9(3x - k)} \quad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

- (a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of k .

(2)

The point P with x coordinate 1 lies on C .Given that the gradient of the curve at P is -12

- (b) find the two possible values of k

(3)

Given also that $k < 3$

- (c) find the equation of the normal to C at P , writing your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(3)

- (d) show, using algebraic integration that,

$$\int_1^3 \frac{16}{9(3x - k)} dx = \lambda \ln 10$$

where λ is a constant to be found.

(4)

3. The share price, £ V , of a company is being monitored.

A graph is drawn of $\log_{10} V$ against t , where t is the number of years after monitoring began.

The graph is a straight line passing through the points $(0, 2)$ and $(5, 2.25)$

Using this information,

- (a) find an equation for the line in the form

$$\log_{10} V = mt + c$$

where m and c are constants.

(2)

- (b) Write the answer to part (a) in the form

$$V = ab^t$$

where a and b are constants to be found.

Give the exact value of a and the value of b to 3 significant figures.

(3)

When $t = T$, the rate of increase in the share price of the company was £50 per year.

- (c) Find the value of T , giving your answer to the nearest integer.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

4. The function f is defined by

$$f(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \quad x \in \mathbb{R} \quad x > 2$$

- (a) Show that $f(x) = \frac{2x}{3x - 5}$

(3)

- (b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear.

(3)

The function g is defined by

$$g(x) = 3 + 2 \ln x \quad x \geq 1$$

- (c) Find g^{-1}

(3)

- (d) Find the exact value of a for which

$$gf(a) = 5$$

(4)

8.

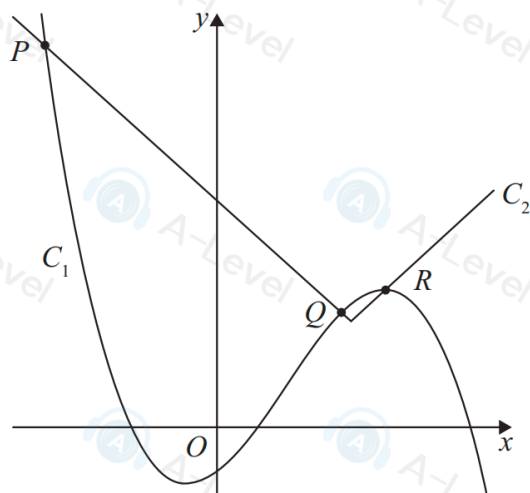


Figure 2

Figure 2 shows a sketch of the graph C_1 with equation

$$y = -2x^3 + 5x^2 + 4x - 3$$

and a sketch of the graph C_2 with equation

$$y = a + |5x + b|$$

where a and b are constants.

Given that P has coordinates $(-2, 25)$

(a) show that

$$a = 15 + b$$

(2)

Given also that R has coordinates $(2, 9)$

(b) find the value of a and the value of b

(3)

Using the answer to part (b),

(c) state the coordinates of the vertex of C_2

(2)

(d) Find, using algebra, the coordinates of Q . Show each stage of your working.

(Solutions relying on calculator technology are not acceptable.)

(6)

1.

$$g(x) = x^6 + 2x - 1000$$

- (a) Show that $g(x) = 0$ has a root α in the interval $[3, 4]$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \quad \text{with } x_1 = 3$$

- (b) (i) find, to 4 decimal places, the value of x_2
(ii) find, by repeated iteration, the value of α .
Give your answer to 4 decimal places.

(3)

9.

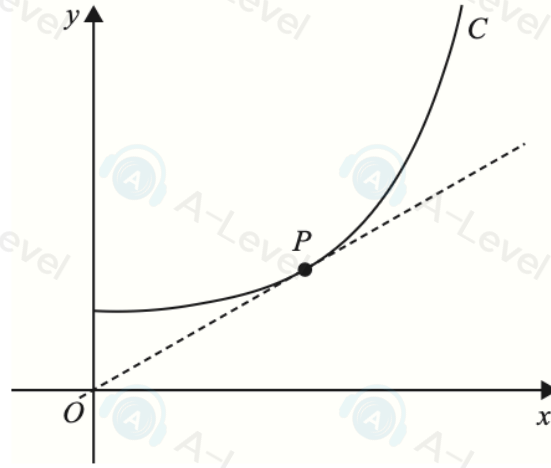


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \quad x \geq 0$$

- (a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

The point P with x coordinate α lies on C .

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

- (b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

- (c) Hence show that α lies between 1 and 2

(2)

- (d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

(1)

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for α .

- (e) Use the iteration formula to find, to 4 decimal places, the value of

(i) x_3

(ii) α

1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $0 < \theta \leq 360^\circ$, the equation

$$3 \tan^2 \theta + 7 \sec \theta - 3 = 0$$

giving your answers to one decimal place.

(5)

6. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Given $y = \ln(2x^2 + 5)$ find $\frac{dy}{dx}$

(2)

(ii) A curve C has equation $y = f(x)$ where

$$f(x) = \frac{21x}{3x^2 + k} \quad x \in \mathbb{R}$$

where k is a positive integer, $k > 1$

Given that

$$\int_1^k f(x) dx < 7 \ln 8$$

find the greatest possible value of k

(5)

10.

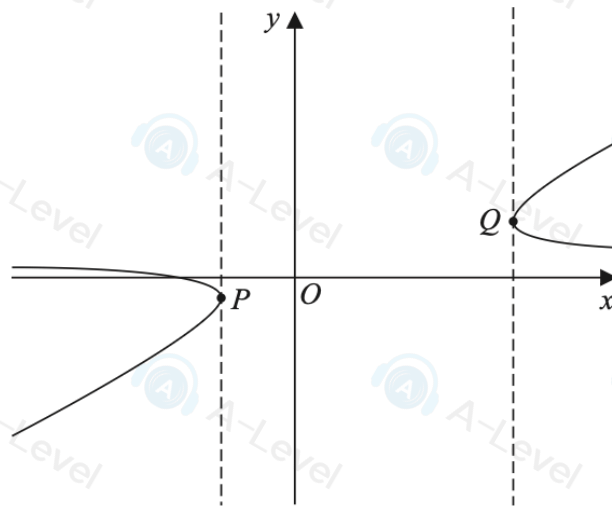


Figure 4

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

- (a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y -axis, as shown in Figure 4.

- (b) Use the answer to part (a) to find the equations of these two tangents.

(4)

2.

$$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

- (a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where A , B and C are integers to be found.

(3)

- (b) Hence use algebraic integration to show that

$$\int_4^8 g(x) dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

4:

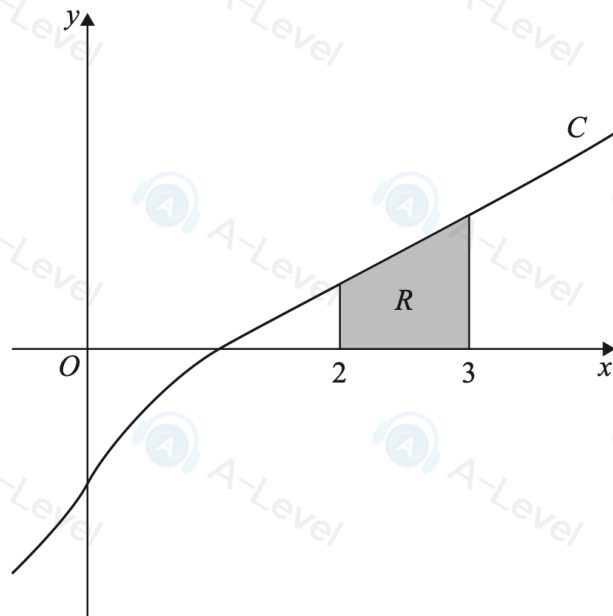


Figure 1

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 1 shows a sketch of part of the curve C with equation $y = g(x)$ where

$$g(x) = \frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1}$$

Given that

$$\frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1} \equiv Ax + B + \frac{Cx + D}{x^2 + 1}$$

(a) (i) find the values of the constants A , B and C

(ii) show that $D = 0$

(4)

The finite region R , shown shaded in Figure 1, is bounded by C , the x -axis and the lines with equations $x = 2$ and $x = 3$

(b) Find the exact area of region R using algebraic integration.

Give the answer in the form $\alpha + \ln \beta$ where α and β are constants to be found.

(4)

1. A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \quad x \in \mathbb{R}$$

- (a) Show that the equation $f(x) = 0$ has a root, α , in the interval $[1, 2]$

(2)

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root α .

- (b) (i) Find the value of x_2 to 4 decimal places.

- (ii) Find, by repeated iteration, the value of α , giving your answer to 4 decimal places.

(3)

1.

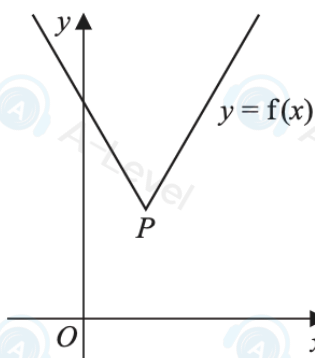


Figure 1

Figure 1 shows a sketch of the graph with equation $y = f(x)$ where

$$f(x) = 2|x - 5| + 10$$

The point P , shown in Figure 1, is the vertex of the graph.

- (a) State the coordinates of P

(2)

- (b) Use algebra to solve

$$2|x - 5| + 10 > 6x$$

(Solutions relying on calculator technology are not acceptable.)

(2)

- (c) Find the point to which P is mapped, when the graph with equation $y = f(x)$ is transformed to the graph with equation $y = 3f(x - 2)$

(2)

8.

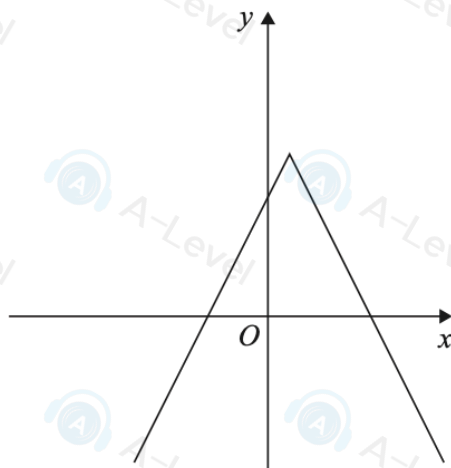


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b ,

(i) the coordinates of the maximum point of the graph,

(ii) the coordinates of the point of intersection of the graph with the y -axis,

(iii) the coordinates of the points of intersection of the graph with the x -axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)

7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \quad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

- (a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \quad t \geq 0$$

where t is the time in hours from the start of the study.

- (b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

- (c) Find the value of T , giving your answer to 3 significant figures.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

7:

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

- (a) Write $\sin 4\theta$ in the form

$$\sin \theta \cos \theta (P + Q \sin^n \theta)$$

where P , Q and n are constants to be found.

(3)

- (b) Use the result from part (a) to show that, for $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$, the equation

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

can be written in the form

$$4 \sec^2 x - 5 \tan x - 8 \tan^2 x = 0$$

(3)

- (c) Use the result from part (b) to solve, for $0 < x < \pi$, $x \neq \frac{\pi}{2}$, the equation

$$\sec x \sin 4x = 5 \sin^3 x \cot x$$

giving the answers in radians to 3 significant figures.

(4)

2. A curve has equation $y = f(x)$ where

$$f(x) = x^4 - 5x^2 + 4x - 7 \quad x \in \mathbb{R}$$

- (a) Show that the equation $f(x) = 0$ has a root, α , in the interval $[2, 3]$

(2)

- (b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$$

(1)

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{5x_n^2 - 4x_n + 7}{x_n}}$$

is used to find α

- (c) Starting with $x_1 = 2$ and using the iterative formula,

- (i) find, to 4 decimal places, the value of x_2

- (ii) find, to 4 decimal places, the value of α

(3)

1.

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

The functions f and g are defined by

$$f(x) = \ln(x^2 + 3)$$

$$x \in \mathbb{R}$$

$$g(x) = \frac{3 + 5x}{x + 2}$$

$$x \in \mathbb{R} \quad x > -2$$

- (a) State the range of f

(1)

- (b) Find g^{-1}

(3)

- (c) Find $fg(0)$

(2)

- (d) Find the exact value of a for which

$$g(e^{2a}) = f(\sqrt{e^4 - 3})$$

(3)

8: The functions f and g are defined by

$$f(x) = \ln(2x - 3) \quad x \in \mathbb{R} \quad x > 2$$

$$g(x) = \frac{3x - 2a}{x - a} \quad x \in \mathbb{R} \quad x \neq a$$

where a is a positive constant.

(a) Find the exact value of $fg(2a)$

(2)

(b) Find f^{-1}

(3)

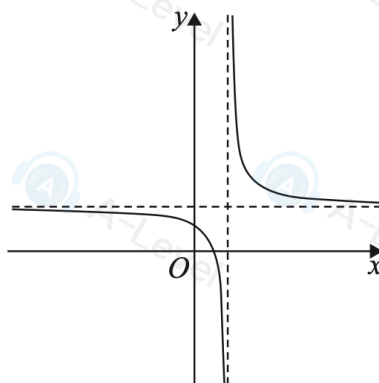


Figure 3

Figure 3 shows a sketch of the curve with equation $y = g(x)$

(c) Sketch a graph of the curve with equation $y = |g(x)|$

Indicate clearly on your sketch the equation of the **horizontal** asymptote.

(3)

The equation $|g(x)| = 10$ has two solutions.

Given that the larger solution is 8

(d) (i) find the value of a ,

(ii) find the value of the smaller solution.

(4)

4.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a)

$$f(x) = \sqrt{3} \sin 2x - 3 \cos 2x$$

Express $f(x)$ in the form $R \sin(2x - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and the exact value of α .

(3)

(b)

$$g(x) = \frac{18}{f(3x) + 4\sqrt{3}} \quad x > 0$$

Using the answer to part (a),

(i) write down the exact minimum value of $g(x)$,

(ii) find the smallest value of x for which this minimum value occurs.

You must make your method clear.

(3)

8.

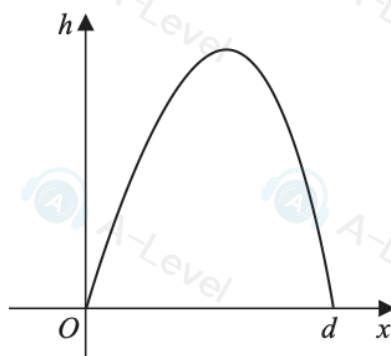


Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5xe^{0.02x} \quad 0 \leq x \leq d$$

Use the model to answer parts (a), (b) and (c).

- (a) Find the value of d , giving your answer to 2 decimal places.

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

- (b) Show that the maximum value of h occurs when

$$x = 50 \ln \left(\frac{150}{x + 50} \right)$$

(4)

Using the iteration formula

$$x_{n+1} = 50 \ln \left(\frac{150}{x_n + 50} \right) \quad \text{with } x_1 = 30$$

- (c) (i) find the value of x_2 to 2 decimal places,
 (ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

(3)

5: Find

(i) $\int \sin^2 3x \, dx$

(2)

(ii) $\int x(x^2 + 4)^{\frac{3}{2}} \, dx$

(2)

DO NOT WRITE IN THIS AREA

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8.

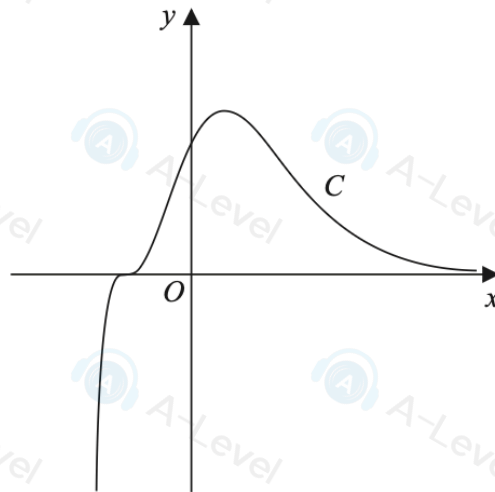


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (2x + 1)^3 e^{-4x}$$

(a) Show that

$$f'(x) = A(2x + 1)^2 (1 - 4x) e^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C .

(3)

The function g is defined by

$$g(x) = 8f(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = g(x)$.

(2)

6:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta^{\circ}\text{C}$, of a computer processor, t minutes after the computer is switched off, is modelled by the equation

$$\theta = 21 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the processor was 75°C when the computer was switched off,

(a) find the value of A .

(2)

Given also that it takes 5 minutes for the temperature of the processor to decrease from 75°C to 25°C ,

(b) find the value of k , giving your answer to 3 significant figures.

(3)

At time T minutes, the temperature of the processor is decreasing at a rate of 9°C per minute.

(c) Find the value of T according to the model, giving your answer to 2 decimal places.

(3)

1. The point $P(-4, -3)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(2x)$

(1)

(b) $y = 3f(x - 1)$

(2)

(c) $y = |f(x)|$

(1)

10.

In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**A curve C has equation

$$x = \sin^2 4y \quad 0 \leq y \leq \frac{\pi}{8} \quad 0 \leq x \leq 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C (a) Find the exact y coordinate of P

(2)

(b) Find $\frac{dx}{dy}$

(2)

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x + s)^2}}$$

where q , r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,(ii) state the value of $\frac{dy}{dx}$ at this point.

(2)

5. The curve C has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \quad x \in \mathbb{R}$$

where k is a positive constant.

- (a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

- (b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

- (c) find the upper limit to the value for k .

(1)

4. A new mobile phone is released for sale.

The total sales N of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \quad t \geq 0$$

where A is a constant and t is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

- (a) state the value of A .

(1)

Given that when $t = T$ the total sales of the phone was 100 000

- (b) find, according to the model, the value of T . Give your answer to 2 decimal places.

(3)

- (c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(Solutions relying entirely on calculator technology are not acceptable.)

(2)

The total sales of the mobile phone is expected to reach 150 000

Using this information,

- (d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

(1)

5.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The number of squirrels in a forest is being studied.

The number of squirrels, N , in the forest, t years after the start of the study, is modelled by the equation

$$N = \frac{4000e^{0.1t}}{19 + e^{0.2t}} \quad t \geq 0$$

Use the equation of the model to answer parts (a), (b), (c) and (d).

(a) Find the number of squirrels in the forest at the start of the study.

(1)

(b) Find $\frac{dN}{dt}$

(2)

The number of squirrels in the forest is at a maximum when $t = T$

Using the answer to part (b),

(c) show that $e^{0.2T} = A$, where A is a constant to be found.

(2)

(d) Hence find the maximum number of squirrels in the forest.

Show your working and give your answer to the nearest whole number.

(3)

5. The functions f and g are defined by

$$f(x) = 2 + 5 \ln x \quad x > 0$$

$$g(x) = \frac{6x - 2}{2x + 1} \quad x > \frac{1}{3}$$

(a) Find $f^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

8:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\tan 3x \equiv \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \quad x \neq (2n+1)\frac{\pi}{6} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = 2 \sec^2 3\theta - 8$$

giving your answers to 2 decimal places.

(5)

6.

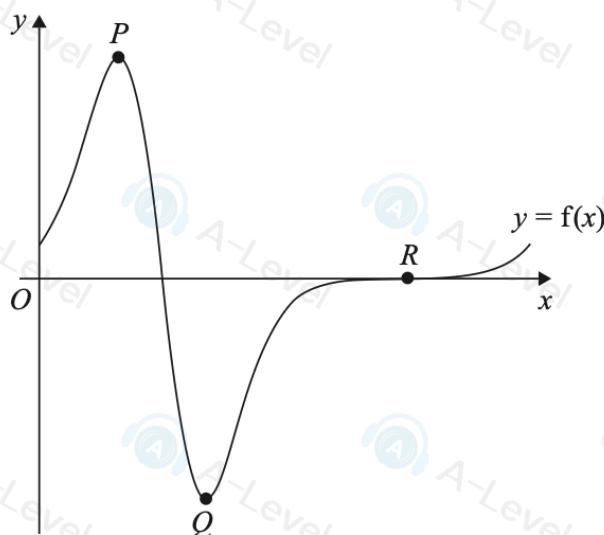


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 2e^{3\sin x} \cos x \quad 0 \leq x \leq 2\pi$$

The curve intersects the x -axis at point R , as shown in Figure 1.

(a) State the coordinates of R

(1)

The curve has two turning points, at point P and point Q , also shown in Figure 1.

(b) Show that, at points P and Q ,

$$a \sin^2 x + b \sin x + c = 0$$

where a , b and c are integers to be found.

(4)

(c) Hence find the x coordinate of point Q , giving your answer to 3 decimal places.

(2)

7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y -axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

giving any solution for x in terms of a and b .

(2)

1. The functions f and g are defined by

$$f(x) = 9 - x^2 \quad x \in \mathbb{R} \quad x \geq 0$$

$$g(x) = \frac{3}{2x+1} \quad x \in \mathbb{R} \quad x \geq 0$$

(a) Write down the range of f

(1)

(b) Find the value of $fg(1.5)$

(2)

(c) Find g^{-1}

(3)

5:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 5 \cos 2x - 12 \sin 2x \quad -\pi < x < \pi$$

The point A with x coordinate $\frac{\pi}{3}$ lies on C .

- (a) Use algebraic differentiation to find the gradient of the tangent to C at A .
Give the answer in simplest form.

(3)

- (b) Express $5 \cos 2x - 12 \sin 2x$ in the form

$$R \cos(2x + \alpha)$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Find the exact value of R and find the value of α to 3 decimal places.

(3)

A point P lies on C .

Given that the gradient of the tangent to C at P is 6

- (c) find the greatest possible value for the x coordinate of P .
Give the answer to 2 decimal places.

(4)

6. The function f is defined by

$$f(x) = \frac{4x + 3}{x - 2} \quad x \neq 2$$

- (a) Find f^{-1}

(3)

- (b) Show that

$$ff(x) = \frac{ax + b}{cx + d}$$

where a , b , c and d are integers to be found.

(3)

The point $P(3, 15)$ lies on the curve with equation $y = f(x)$.

- (c) Find the point to which P is mapped when $y = f(x)$ is transformed to the curve with equation $y = 2f(3x) + 8$

(2)

3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \quad y \neq -\frac{1}{2}$$

- (a) Find $\frac{dy}{dx}$ in terms of y , giving the answer in simplest form.

(4)

A point P lies on C .

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x -axis

- (b) find the coordinates of P .

(Solutions relying entirely on calculator technology are not acceptable.)

(4)