10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P

(2)

(b) Find $\frac{dx}{dy}$

(2)

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x+s)^2}}$$

where q, r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,

(ii) state the value of $\frac{dy}{dx}$ at this point.

(2)

3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \qquad y \neq -\frac{1}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of y, giving the answer in simplest form.

(4)

A point P lies on C.

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x-axis

(b) find the coordinates of P.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

10.

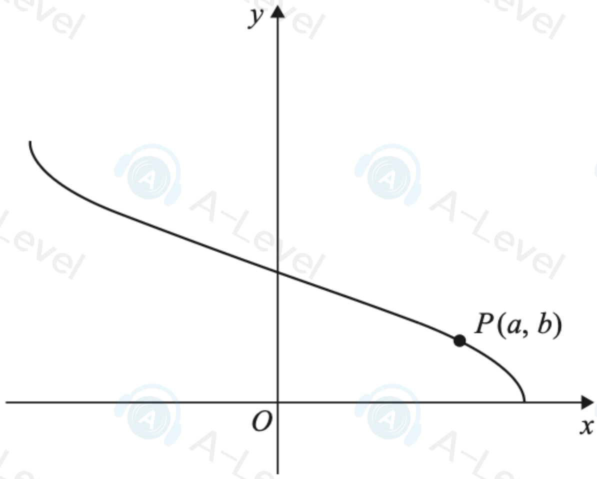

Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\dfrac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\dfrac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)

6.

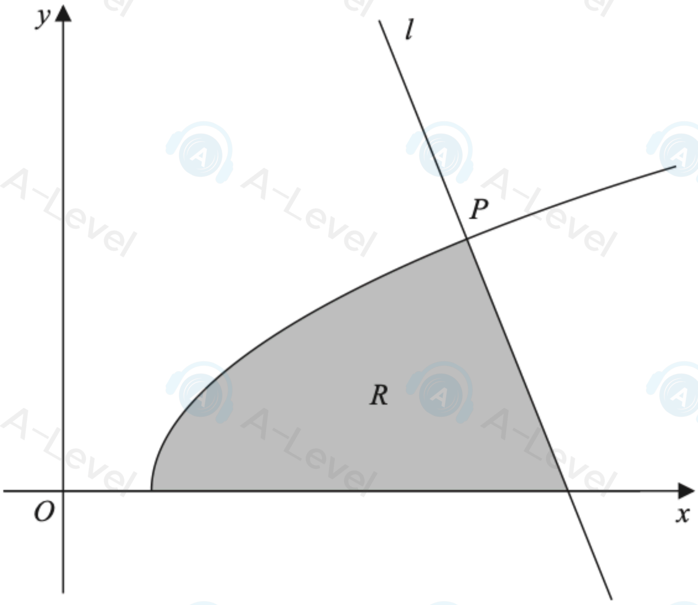

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given $y = \ln(2x^2 + 5)$ find $\dfrac{dy}{dx}$

(2)

(ii) A curve C has equation $y = f(x)$ where

$$f(x) = \frac{21x}{3x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive integer, $k > 1$

Given that

$$\int_1^k f(x)\,dx < 7\ln 8$$

find the greatest possible value of k

(5)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{16}{9(3x - k)} \qquad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

(a) Find $\dfrac{dy}{dx}$ giving your answer in simplest form in terms of k.

(2)

The point P with x coordinate 1 lies on C.

Given that the gradient of the curve at P is -12

(b) find the two possible values of k

(3)

Given also that $k < 3$

(c) find the equation of the normal to C at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(d) show, using algebraic integration that,

$$\int_1^3 \frac{16}{9(3x - k)}\,dx = \lambda \ln 10$$

where λ is a constant to be found.

(4)

6:

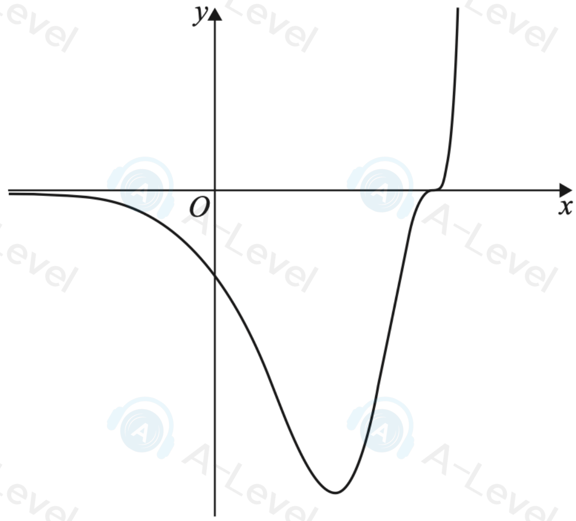

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$\mathrm{f}'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = \mathrm{f}(x)$,

(ii) the range of the function g defined by

$$\mathrm{g}(x) = 8(2x - 3)^3 e^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta}\right)^2 = 6\cot\theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right)\cot x \, dx$$

(2)

8.

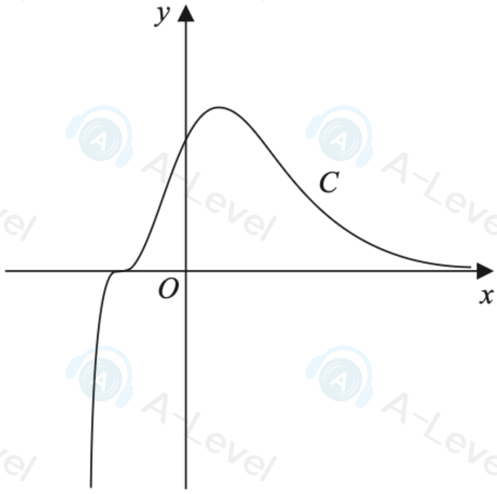

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (2x + 1)^3 e^{-4x}$$

(a) Show that

$$f'(x) = A(2x + 1)^2 (1 - 4x) e^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$g(x) = 8f(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = g(x)$.

(2)

9.

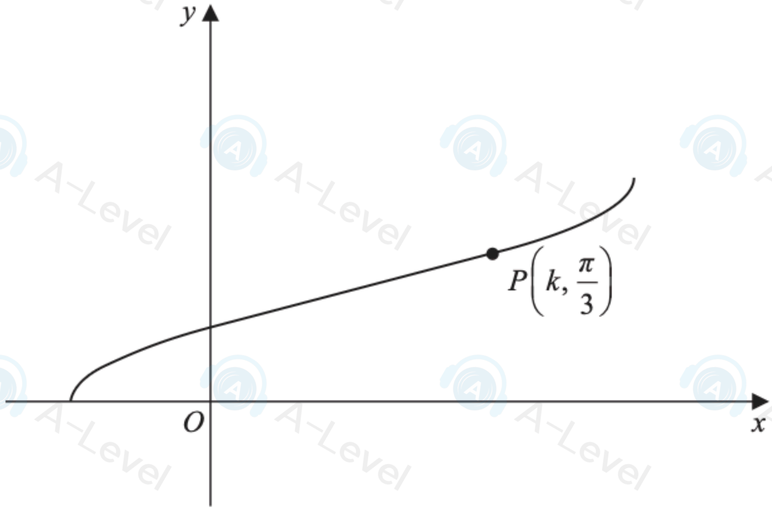

Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y

(ii) Hence show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x-axis at the point N.

(c) Find the exact area of triangle OPN, where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

9.

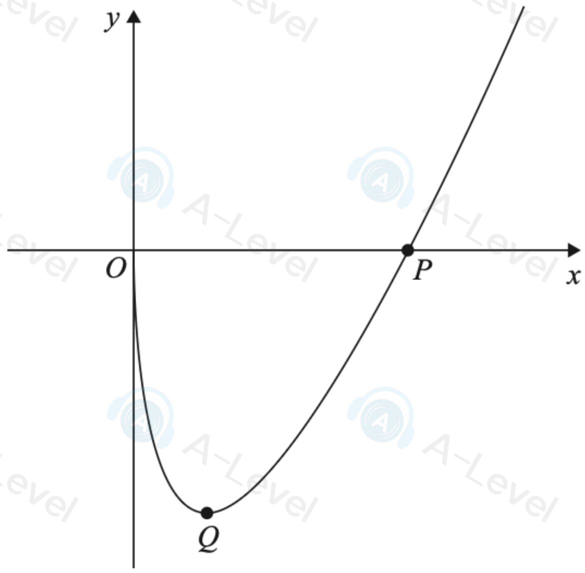

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

DO NOT WRITE IN THIS AREA