7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 2xe^{x^2+(3k-2)x}$$

where k is a constant.

Given that C has two distinct turning points, find the range of possible values of k.

(7)

2.

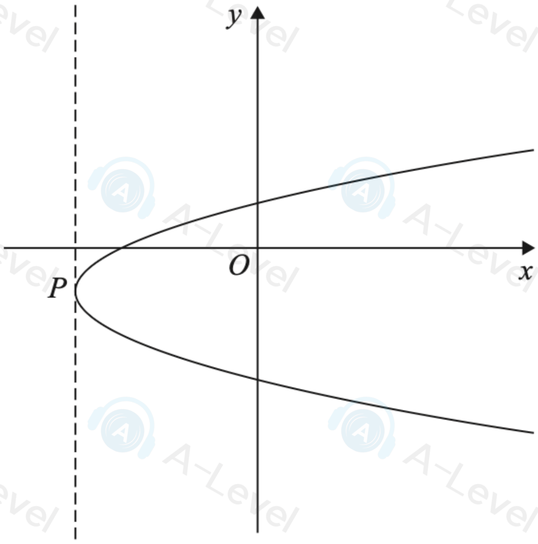

Figure 1

Figure 1 shows a sketch of the curve with equation

$$x = 2y^2 + 5y - 6$$

(a) Find $\frac{dy}{dx}$ in terms of y.

(2)

The point P lies on the curve and is shown in Figure 1.

Given that the tangent to the curve at P is parallel to the y-axis,

(b) find the coordinates of P.

(3)

4: The function f is defined by

$$f(x) = \frac{49x}{x^2 + x - 12} + \frac{7x}{x + 4} \qquad x > 3$$

(a) Show that

$$f(x) = \frac{7x}{x - 3} \qquad x > 3$$

(3)

(b) Hence find $f'(x)$ giving your answer in simplest form.

(2)

3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \qquad y \neq -\frac{1}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of y, giving the answer in simplest form.

(4)

A point P lies on C.

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x-axis

(b) find the coordinates of P.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

9.

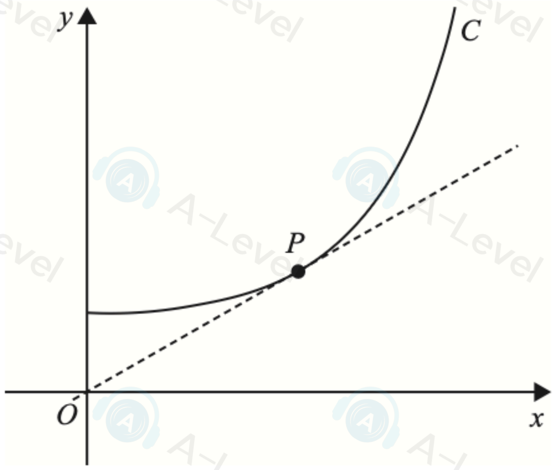

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

(2)

The point P with x coordinate α lies on C.

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

(b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

(c) Hence show that α lies between 1 and 2

(2)

(d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

(1)

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for α.

(e) Use the iteration formula to find, to 4 decimal places, the value of

(i) x_3

(ii) α

9.

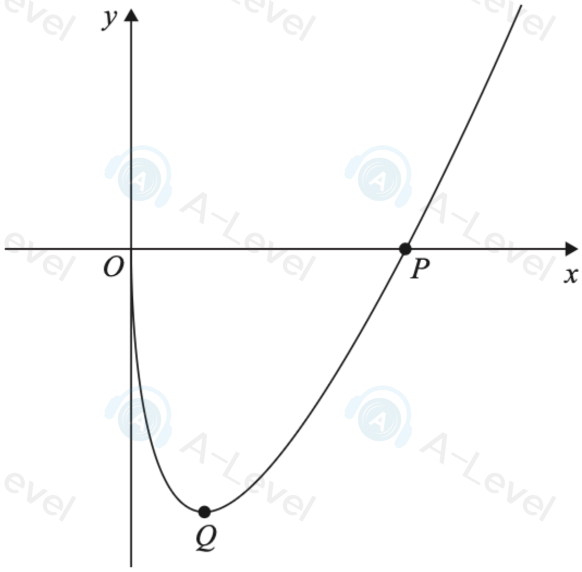

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

9.

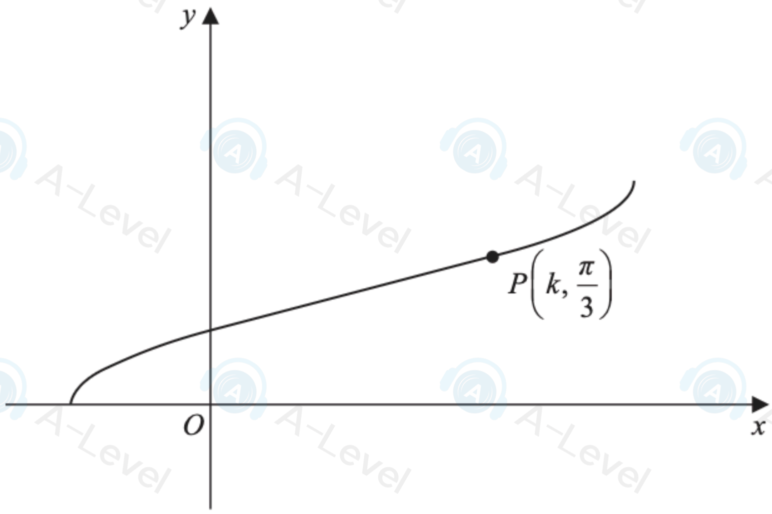

Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x-axis at the point N.

(c) Find the exact area of triangle OPN, where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin\theta} + \frac{\sin 2\theta}{\cos\theta}\right)^2 = 6\cot\theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x}\right)\cot x \, dx$$

(2)

6:

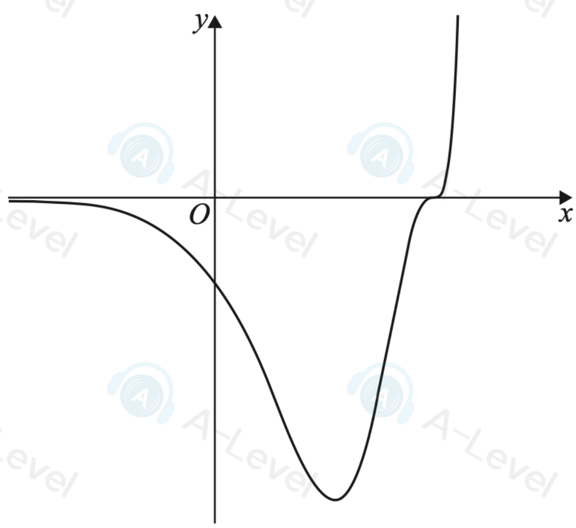

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

(a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = f(x)$,

(ii) the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

6.

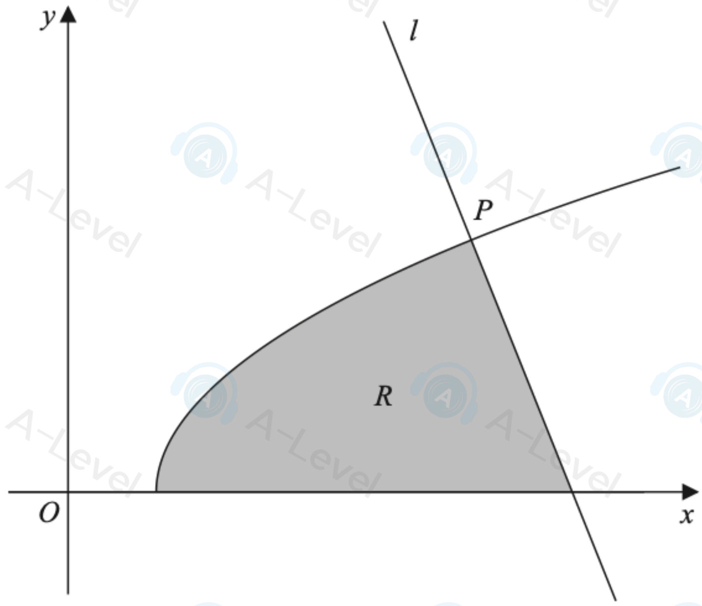

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

(4)

10.

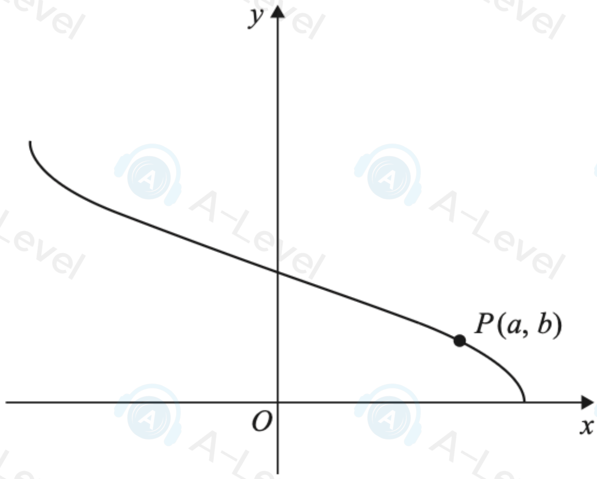

Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)