4. A new mobile phone is released for sale.

The total sales $N$ of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where $A$ is a constant and $t$ is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of $A$.

**(1)**

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of $T$. Give your answer to 2 decimal places.

**(3)**

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(2)**

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

**(1)**



3. The share price, £$V$, of a company is being monitored.

A graph is drawn of $\log_{10} V$ against $t$, where $t$ is the number of years after monitoring began.

The graph is a straight line passing through the points $(0, 2)$ and $(5, 2.25)$

Using this information,

(a) find an equation for the line in the form

$$\log_{10} V = mt + c$$

where $m$ and $c$ are constants.

**(2)**

(b) Write the answer to part (a) in the form

$$V = ab^t$$

where $a$ and $b$ are constants to be found.

Give the exact value of $a$ and the value of $b$ to 3 significant figures.

**(3)**

When $t = T$, the rate of increase in the share price of the company was £50 per year.

(c) Find the value of $T$, giving your answer to the nearest integer.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(4)**

**5.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The temperature, $T$ °C, of the air in a room $t$ minutes after a heat source is switched off, is modelled by the equation

$$T = 10 + Ae^{-Bt}$$

where $A$ and $B$ are constants.

Given that the temperature of the air in the room at the instant the heat source was switched off was 18 °C,

(a) find the value of $A$

**(1)**

Given also that, exactly 45 minutes after the heat source was switched off, the temperature of the air in the room was 16 °C,

(b) find the value of $B$ to 3 significant figures.

**(3)**

Using the values for $A$ and $B$,

(c) find, according to the model, the rate of change of the temperature of the air in the room exactly two minutes after the heat source was switched off.
Give your answer in $°\text{C}\,\text{min}^{-1}$ to 3 significant figures.

**(2)**

(d) Explain why, according to the model, the temperature of the air in the room cannot fall to 5 °C

**(1)**

7. A scientist is studying two different populations of bacteria.

The number of bacteria $N$ in the first population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where $A$ and $k$ are positive constants and $t$ is the time in hours from the start of the study.

Given that
- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of $A$ and the value of $k$ to 4 significant figures.

**(3)**

The number of bacteria $N$ in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \qquad t \geqslant 0$$

where $t$ is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

**(2)**

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of $T$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(3)**

**3.** (i) The variables $x$ and $y$ are connected by the equation

$$y = \frac{10^6}{x^3} \qquad x > 0$$

Sketch the graph of $\log_{10} y$ against $\log_{10} x$

Show on your sketch the coordinates of the points of intersection of the graph with the axes.

**(3)**

(ii)

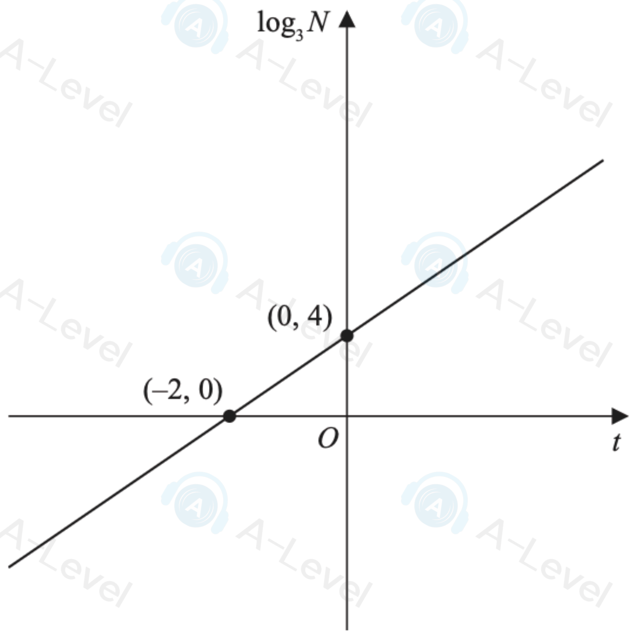


**Figure 2**

Figure 2 shows the linear relationship between $\log_3 N$ and $t$.

Show that $N = ab^t$ where $a$ and $b$ are constants to be found.

**(3)**

**8.**

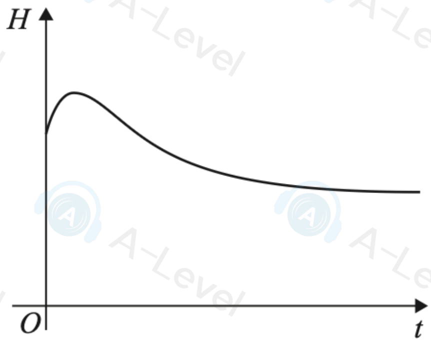


**Figure 4**

The heart rate of a horse is being monitored.
The heart rate $H$, measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20\,e^{-0.9t}$$

where $t$ minutes is the time after monitoring began.

Figure 4 is a sketch of $H$ against $t$.

**Use the equation of the model to answer parts (a) to (e).**

(a) State the initial heart rate of the horse.

**(1)**

In the long term, the heart rate of the horse approaches $L$ bpm.

(b) State the value of $L$.

**(1)**

The heart rate of the horse reaches its maximum value after $T$ minutes.

(c) Find the value of $T$, giving your answer to 3 decimal places.

(*Solutions based entirely on calculator technology are not acceptable*.)

**(5)**

The heart rate of the horse is 37 bpm after $M$ minutes.

(d) Show that $M$ is a solution of the equation

$$t = 5\ln\left(\frac{8}{1 + 4e^{-0.9t}}\right)$$

**(2)**

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{8}{1 + 4e^{-0.9t_n}}\right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of $t_2$

(ii) find, to 4 decimal places, the value of $M$

**(3)**

**4.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

(a) $$f(x) = \sqrt{3}\sin 2x - 3\cos 2x$$

Express $f(x)$ in the form $R\sin(2x - \alpha)$, where $R$ and $\alpha$ are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of $R$ and the exact value of $\alpha$.

**(3)**

(b) $$g(x) = \frac{18}{f(3x) + 4\sqrt{3}} \qquad x > 0$$

Using the answer to part (a),

(i) write down the exact minimum value of $g(x)$,

(ii) find the smallest value of $x$ for which this minimum value occurs.

You must make your method clear.

**(3)**



**2.**

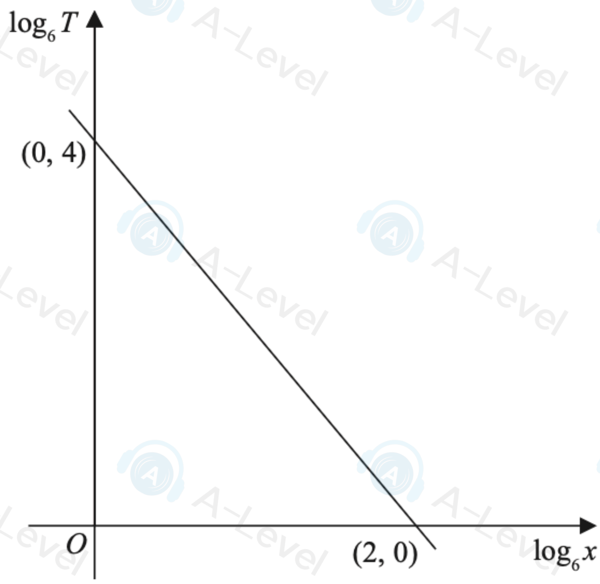


**Figure 1**

Figure 1 shows the linear relationship between $\log_6 T$ and $\log_6 x$

The line passes through the points (0, 4) and (2, 0) as shown.

(a) (i) Find an equation linking $\log_6 T$ and $\log_6 x$

(ii) Hence find the exact value of $T$ when $x = 216$

**(3)**

(b) Find an equation, not involving logs, linking $T$ with $x$

**(3)**