

6.

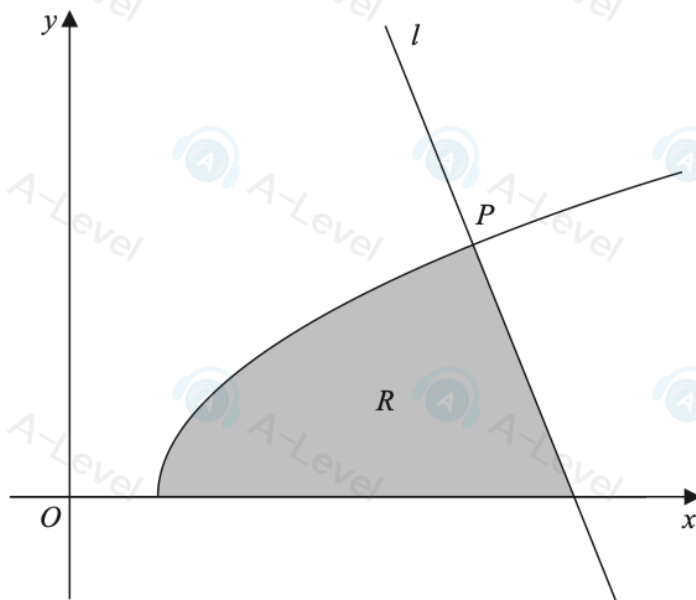


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l , shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0 \quad (5)$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and l .

(b) Use algebraic integration to find the exact area of R .

(4)

9.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \equiv \operatorname{cosec} x \quad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\left(\frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} \right)^2 = 6 \cot \theta - 4$$

giving your answers to 3 significant figures as appropriate.

(5)

(c) Using the result from part (a), or otherwise, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} \right) \cot x \, dx \quad (2)$$

6:

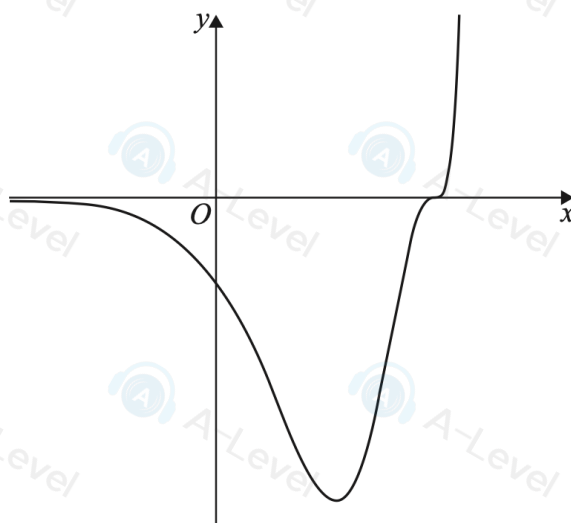


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (2x - 3)^3 e^{4x-2}$$

- (a) Show that

$$f'(x) = 2(Px + Q)(2x - 3)^n e^{4x-2}$$

where P , Q and n are constants to be found.

(4)

- (b) Hence find

- the x coordinates of the stationary points on the curve with equation $y = f(x)$,
- the range of the function g defined by

$$g(x) = 8(2x - 3)^3 e^{4x-2} \quad x \leq \frac{3}{2}$$

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

8.

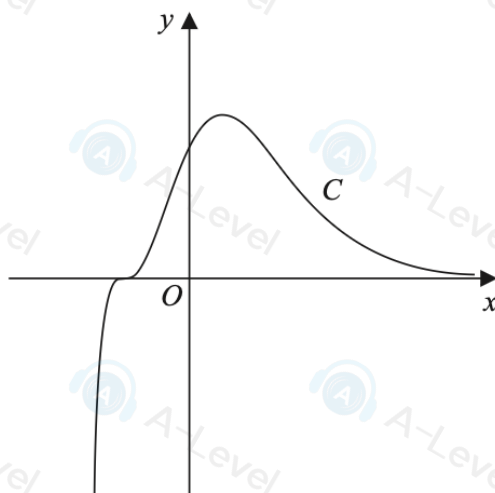


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (2x + 1)^3 e^{-4x}$$

- (a) Show that

$$f'(x) = A(2x + 1)^2 (1 - 4x) e^{-4x}$$

where A is a constant to be found.

(4)

- (b) Hence find the exact coordinates of the two stationary points on C .

(3)

The function g is defined by

$$g(x) = 8f(x - 2)$$

- (c) Find the coordinates of the maximum stationary point on the curve with equation $y = g(x)$.

(2)

10.

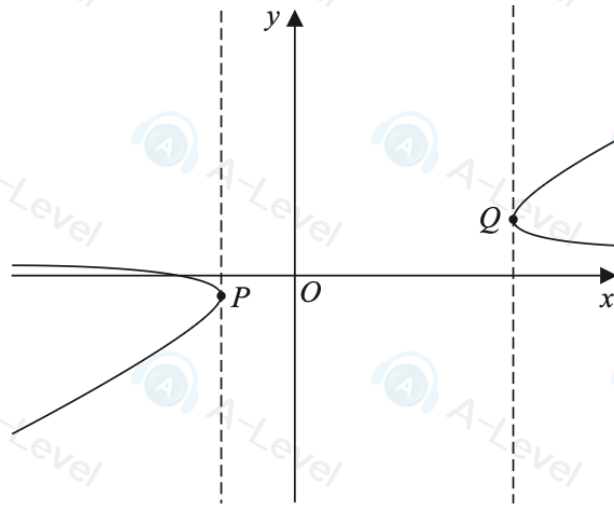
**Figure 4**

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

- (a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y -axis, as shown in Figure 4.

- (b) Use the answer to part (a) to find the equations of these two tangents.

(4)

7.

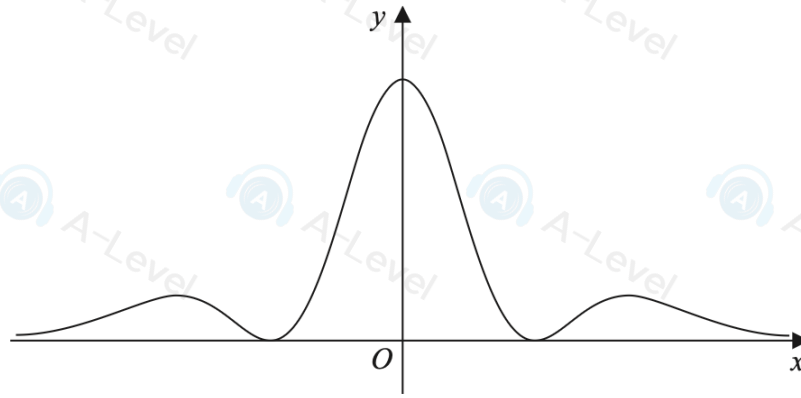


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = e^{-x^2} (2x^2 - 3)^2$$

(a) Find the range of f

(2)

(b) Show that

$$f'(x) = 2x(2x^2 - 3)e^{-x^2}(A - Bx^2)$$

where A and B are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

5:

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 5 \cos 2x - 12 \sin 2x \quad -\pi < x < \pi$$

The point A with x coordinate $\frac{\pi}{3}$ lies on C .

- (a) Use algebraic differentiation to find the gradient of the tangent to C at A .
Give the answer in simplest form.

(3)

- (b) Express $5 \cos 2x - 12 \sin 2x$ in the form

$$R \cos(2x + \alpha)$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Find the exact value of R and find the value of α to 3 decimal places.

(3)

A point P lies on C .

Given that the gradient of the tangent to C at P is 6

- (c) find the greatest possible value for the x coordinate of P .
Give the answer to 2 decimal places.

(4)

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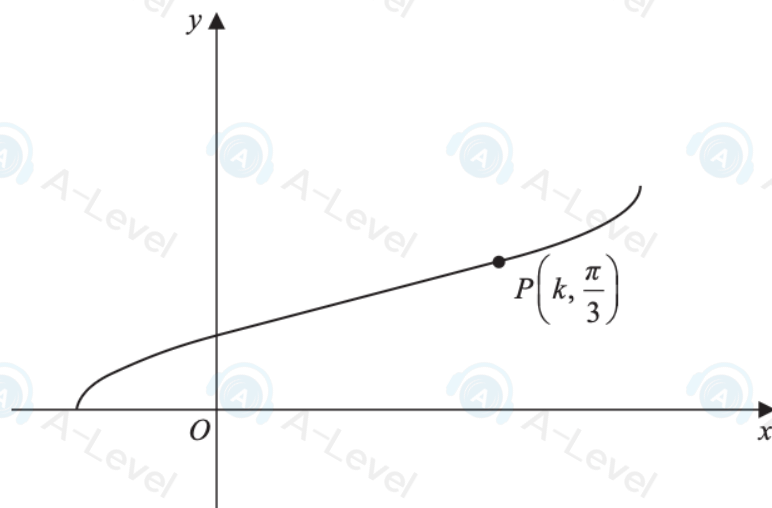


Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \quad 0 \leq y \leq \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$ (1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ (6)

The normal to the curve at P cuts the x -axis at the point N .

(c) Find the exact area of triangle OPN , where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants. (3)

2.

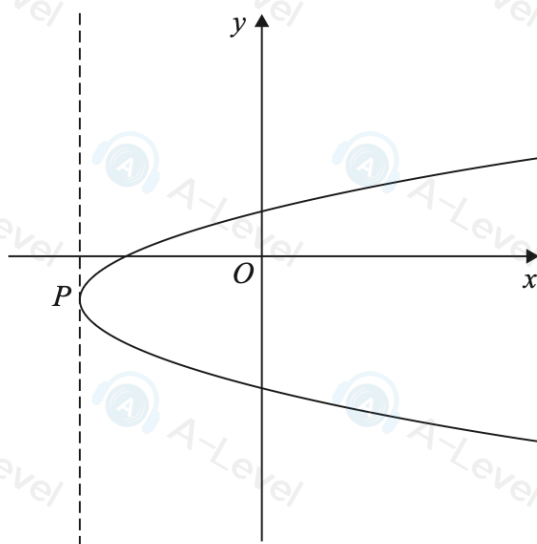


Figure 1

Figure 1 shows a sketch of the curve with equation

$$x = 2y^2 + 5y - 6$$

- (a) Find $\frac{dy}{dx}$ in terms of y .

(2)

The point P lies on the curve and is shown in Figure 1.

Given that the tangent to the curve at P is parallel to the y -axis,

- (b) find the coordinates of P .

(3)

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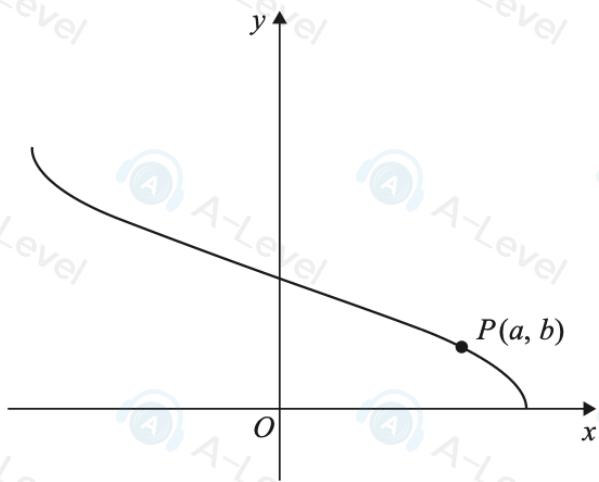


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3 \cos 2y \quad -3 \leq x \leq 3 \quad 0 \leq y \leq \frac{\pi}{2}$$

(a) Find $\frac{dx}{dy}$ in terms of y .

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b .

(4)

3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \quad y \neq -\frac{1}{2}$$

- (a) Find $\frac{dy}{dx}$ in terms of y , giving the answer in simplest form.

(4)

A point P lies on C .

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x -axis

- (b) find the coordinates of P .

(Solutions relying entirely on calculator technology are not acceptable.)

(4)