2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \ln(x^2 + 3) \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3+5x}{x+2} \qquad x \in \mathbb{R} \quad x > -2$$

(a) State the range of f

(1)

(b) Find g^{-1}

(3)

(c) Find $fg(0)$

(2)

(d) Find the exact value of a for which

$$g\left(e^{2a}\right) = f\left(\sqrt{e^4 - 3}\right)$$

(3)

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3.

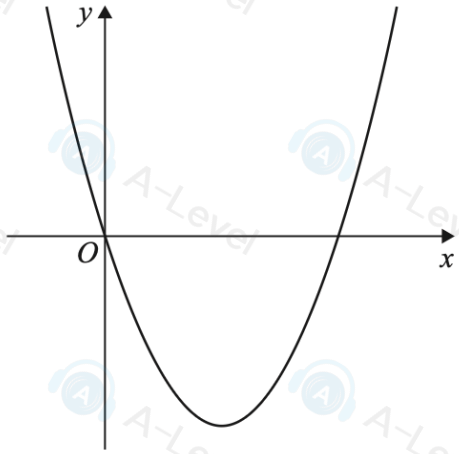


Figure 2

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x^2 - 10x \qquad x \in \mathbb{R}$$

(a) Solve the equation

$$\mathrm{f}(|x|) = 48$$

(3)

(b) Find the set of values of x for which

$$|\mathrm{f}(x)| \geqslant \frac{5}{2}x$$

(4)

7. Given that a and b are positive constants with $a > b$,

(a) sketch, on **separate** diagrams, the graph with equation

(i) $y = |3x - a|$

(ii) $y = |3x - a| - b$

Show on each sketch

- the coordinates of the minimum point on the graph
- the coordinates of the point at which the graph crosses the y-axis

(6)

(b) Solve the equation

$$|3x - a| - b = 5x$$

giving any solution for x in terms of a and b.

(2)

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5. The functions f and g are defined by

$$f(x) = 2 + 5\ln x \qquad x > 0$$

$$g(x) = \frac{6x - 2}{2x + 1} \qquad x > \frac{1}{3}$$

(a) Find $f^{-1}(22)$

(2)

(b) Use differentiation to prove that g is an increasing function.

(3)

(c) Find g^{-1}

(3)

(d) Find the range of fg

(2)

8.

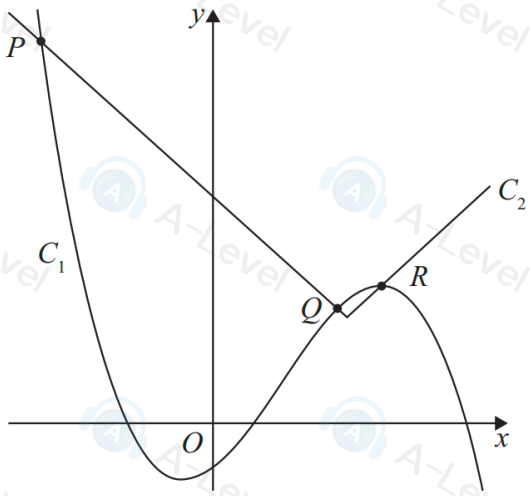


Figure 2

Figure 2 shows a sketch of the graph C_1 with equation

$$y = -2x^3 + 5x^2 + 4x - 3$$

and a sketch of the graph C_2 with equation

$$y = a + |5x + b|$$

where a and b are constants.

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Given that P has coordinates $(-2, 25)$

(a) show that

$$a = 15 + b$$

(2)

Given also that R has coordinates $(2, 9)$

(b) find the value of a and the value of b

(3)

Using the answer to part (b),

(c) state the coordinates of the vertex of C_2

(2)

(d) Find, using algebra, the coordinates of Q. Show each stage of your working.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

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