7.

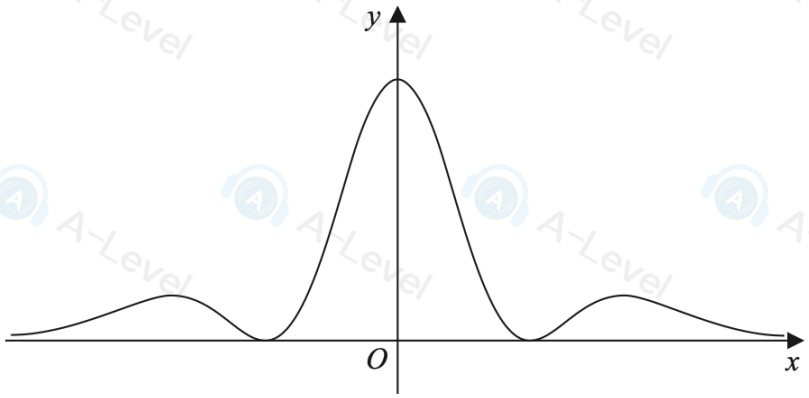


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \mathrm{e}^{-x^2}(2x^2 - 3)^2$$

(a) Find the range of f

(2)

(b) Show that

$$\mathrm{f}'(x) = 2x(2x^2 - 3)\mathrm{e}^{-x^2}(A - Bx^2)$$

where A and B are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

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3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x + 1}{(x + 3)^2} \qquad x \neq -3 \qquad x \in \mathbb{R}$$

Use calculus to find the range of values of x for which y is increasing.

(6)

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 2xe^{x^2 + (3k-2)x}$$

where k is a constant.

Given that C has two distinct turning points, find the range of possible values of k.

(7)

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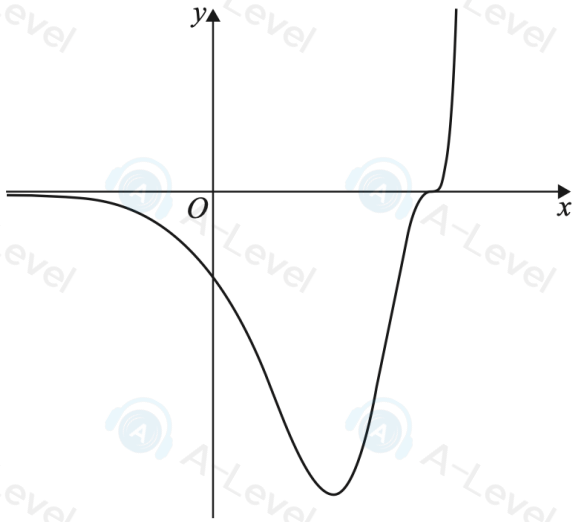


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (2x - 3)^3 \mathrm{e}^{4x-2}$$

(a) Show that

$$\mathrm{f}'(x) = 2(Px + Q)(2x - 3)^n \mathrm{e}^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = \mathrm{f}(x)$,

(ii) the range of the function g defined by

$$\mathrm{g}(x) = 8(2x - 3)^3 \mathrm{e}^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

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10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P

(2)

(b) Find $\frac{dx}{dy}$

(2)

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x + s)^2}}$$

where q, r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,

(ii) state the value of $\frac{dy}{dx}$ at this point.

(2)

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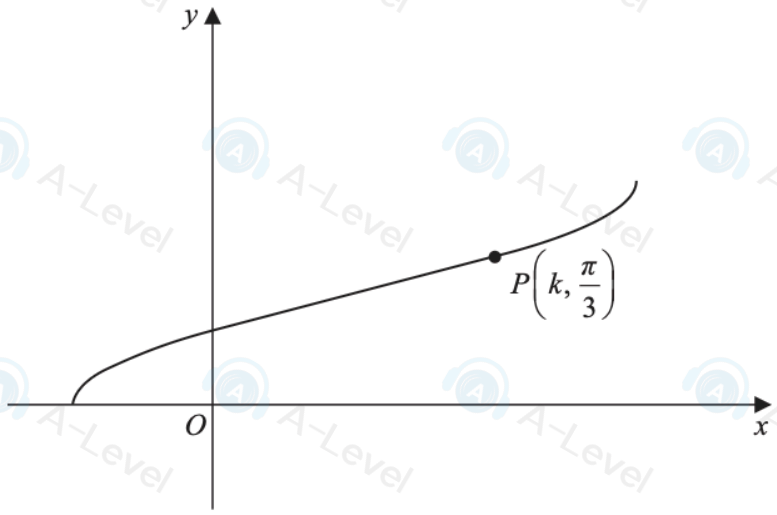


Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x-axis at the point N.

(c) Find the exact area of triangle OPN, where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

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5. The curve C has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

(c) find the upper limit to the value for k.

(1)

2.

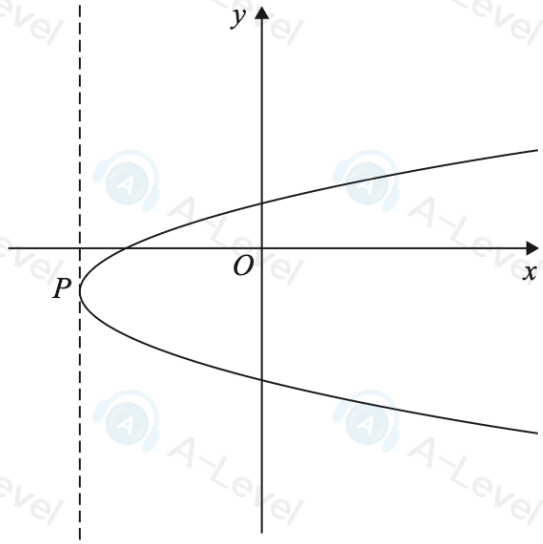


Figure 1

Figure 1 shows a sketch of the curve with equation

$$x = 2y^2 + 5y - 6$$

(a) Find $\frac{dy}{dx}$ in terms of y.

(2)

The point P lies on the curve and is shown in Figure 1.

Given that the tangent to the curve at P is parallel to the y-axis,

(b) find the coordinates of P.

(3)

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10.

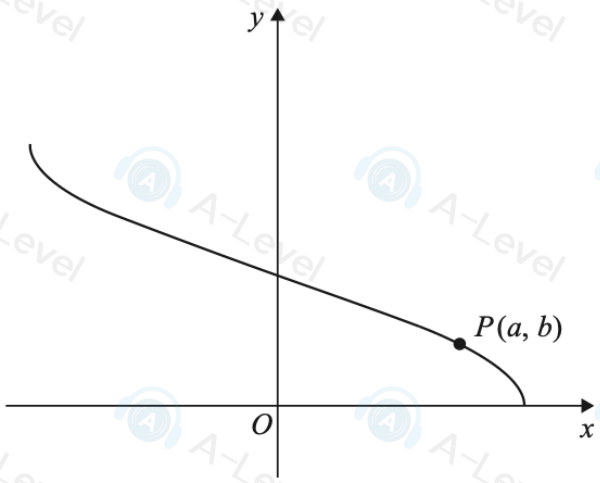


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\dfrac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)

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