Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2}\mathrm{d}x = \int x^{-2}\ln x \,\mathrm{d}x = \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$	M1A1
	$= \frac{x^{-1}}{-1}\ln x + \int x^{-2}\,\mathrm{d}x$	
	$= \frac{x^{-1}}{-1}\ln x + \frac{x^{-1}}{-1}(+c)$	dM1A1
	$\int_1^{e} \frac{\ln x}{x^2}\mathrm{d}x = \left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^{e} = \left(\frac{-1}{e}\ln e - \frac{1}{e}\right) - \left(\frac{-1}{1}\ln 1 - \frac{1}{1}\right)$	ddM1
	$= 1 - \frac{2}{e}$	A1
		(6)

Question Number	Scheme	Marks
3	$8y\frac{\mathrm{d}y}{\mathrm{d}x} = xe^{-y^2} \to \int 8ye^{y^2}\,\mathrm{d}y = \int x\,\mathrm{d}x$	B1
	$4e^{y^2} = \frac{x^2}{2}\ (+c)$	M1A1
	$4 = \frac{5^2}{2} + c \Rightarrow c = \ldots$	**d**M1
	$y^2 = \ln\left(\frac{x^2-17}{8}\right)$	A1
		(5 marks)

B1: Separates the variables correctly with some indication of integration

Question Number	Scheme	Marks
8(a)	$\frac{1}{\sqrt{4+x}} = 4^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$	M1
	$\left(1+\frac{x}{4}\right)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2+\ldots$	M1
	$\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2+\ldots$	A1A1
		(4)
(b)	$\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2+\ldots$	B1ft
		(1)
(c)	$\left(\frac{1}{2}-\frac{1}{16}x+\frac{3}{256}x^2\right)\left(\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2\right)=\frac{1}{4}+\frac{1}{128}x^2$	M1A1
		(2)
(d)	$\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{3}}=\text{"}\frac{1}{4}\text{"}+\text{"}\frac{1}{128}\text{"}=\text{"}\frac{33}{128}\text{"}$	M1
	$\text{"}\frac{128}{33}\text{"}\times 3=\frac{128}{11}$ (see alt (d) for alternative answer)	dM1A1
		(3)
Alt(d)	$\frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{3}}=\text{"}\frac{1}{4}\text{"}+\text{"}\frac{1}{128}\text{"}=\text{"}\frac{33}{128}\text{"}$	M1
	$\frac{1}{\sqrt{15}}\times\frac{\sqrt{15}}{\sqrt{15}}=\frac{\sqrt{15}}{15}\approx\text{"}\frac{33}{128}\text{"}\Rightarrow\sqrt{135}=3\sqrt{15}\approx 3\times 15\times\text{"}\frac{33}{128}\text{"}=\frac{1485}{128}$	dM1A1
		(10 marks)

Question Number	Scheme	Marks
5(a)	$y^3 - x^2 + 4x^2y = k$	
	$y^3 \to 3y^2\dfrac{dy}{dx}$	M1
	$4x^2y \to 8xy + 4x^2\dfrac{dy}{dx}$	M1
	$y^3 - x^2 + 4x^2y = k \Rightarrow 3y^2\dfrac{dy}{dx} - 2x + 8xy + 4x^2\dfrac{dy}{dx} = 0$	A1
	$\Rightarrow \left(3y^2 + 4x^2\right)\dfrac{dy}{dx} = 2x - 8xy$	M1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{2x - 8xy}{3y^2 + 4x^2}$	A1
		(5)
(b)	$\dfrac{dy}{dx} = -1$ at P	M1
	Uses $\dfrac{dy}{dx} = \pm 1$ and $y = x$ to set up and solve equation in x, y or 'p' $\Rightarrow -1 = \dfrac{2p - 8p^2}{3p^2 + 4p^2} \Rightarrow p = 2$	M1 A1
	e.g. $k = "2"^3 - "2"^2 + 4 \times "2"^3$	**dd**M1
	$k = 36$	A1
		(5)
		(10 marks)

Question Number	Scheme	Marks
2 (a)	E.g. $x = \dfrac{t-1}{2t+1} \Rightarrow t = \dfrac{x+1}{1-2x}$ or $y = \dfrac{6}{2t+1} \Rightarrow t = \dfrac{6-y}{2y}$	**M1**
	E.g. $y = \dfrac{6}{2t+1} \Rightarrow y = \dfrac{6}{2\times\left(\dfrac{x+1}{1-2x}\right)+1}$ or $t = \dfrac{6-y}{2y} \Rightarrow x = \dfrac{\dfrac{6-y}{2y} - 1}{2\times\dfrac{6-y}{2y} + 1}$	**A1**
	E.g. $y = \dfrac{6}{2\times\left(\dfrac{x+1}{1-2x}\right)+1} \Rightarrow y = \dfrac{6(1-2x)}{2\times(x+1) + 1(1-2x)} = ax + b$	**dM1**
	E.g. $y = \dfrac{6(1-2x)}{3}$, $y = 2(1-2x)$ oe so linear *	**A1***
		(4)
(b)	$y = 2(1-2x)$ and $y = x + 12 \Rightarrow 2(1-2x) = x + 12 \Rightarrow x = \ldots$	**M1**
	$x = -2$	**A1cao**
		(2)

Question	Scheme	Marks
4(a)	Area of one face $=\frac{1}{2}x\times x\times\sin 60° = \frac{1}{2}x^2\frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20\times\frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx}=10\sqrt{3}x$ and $\frac{dV}{dx}=\frac{15}{12}\left(3+\sqrt{5}\right)x^2$	M1
	$\frac{dV}{dA}=\frac{dV}{dx}\times\frac{dx}{dA}=\frac{dV}{dx}\div\frac{dA}{dx}=...$ (oe)	M1
	$=\frac{15\left(3+\sqrt{5}\right)x^2}{12\times10\sqrt{3}x}=\frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt}=0.025$	B1
	$\frac{dV}{dt}=\frac{dV}{dA}\times\frac{dA}{dt}=\frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}\times0.025=...$ When $x=2$, $\frac{dV}{dt}=\frac{2\left(3+\sqrt{5}\right)}{8\sqrt{3}}\times0.025=...$	M1
	awrt 0.019 (cm^3s^{-1})	A1
		(3)
		(8 marks)

Question Number	Sceme	Marks
3	$y\cos^2(2x)\frac{dy}{dx}=3\sin 2x\rightarrow\int y\,dy=\int 3\frac{\sin 2x}{\cos^2 2x}dx$	M1
	$\int y\,dy=\int 3\sec(2x)\tan(2x)\,dx$	A1
	$\frac{y^2}{2}=\frac{3}{2}\sec 2x\ (+c)$	M1A1
	$\frac{16}{2}=\frac{3}{2}\sec\left(2\frac{\pi}{6}\right)+c\Rightarrow c=...(=5)$	dM1
	$y^2=3\sec 2x+10$	A1
		(6)

8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \ldots$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overline{AC}.\overline{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overline{AC}.\overline{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = \ldots$ $\left(NB \quad \sqrt{172} = 2\sqrt{43},\ \sqrt{189} = 3\sqrt{21} \right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2,\ BC^2 = 6^2 + 4^2 + 3^2,\ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC \cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172}\cos\theta \Rightarrow \cos\theta = \ldots$	ddM1
	$\theta =$ awrt $33.7°$	A1
(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

Question	Scheme	Marks
9(a)	$\frac{d}{dy}(1+2\ln y)^{-2}=\alpha(1+2\ln y)^{-3}\times\frac{\beta}{y}=\frac{K}{y(1+2\ln y)^3}$	M1
	$=\frac{-4}{y(1+2\ln y)^3}$ oe	A1
		(2)
(b)	$3\operatorname{cosec}(2x)\frac{dy}{dx}=y(1+2\ln y)^3$ $\Rightarrow\int ...dy=\int k\sin(2x)\,dx$ **or** $\int\frac{k}{y(1+2\ln y)^3}dy=\int ...dx$ **and** $\Rightarrow\int ...dy=\int k\sin(2x)\,dx\Rightarrow ...=A\cos 2x$ oe **or** $\Rightarrow\int\frac{k}{y(1+2\ln y)^3}dy=\int ...dx\Rightarrow\frac{A}{(1+2\ln y)^2}=...$	M1
	$\Rightarrow\int\frac{A}{y(1+2\ln y)^3}dy=\int B\sin(2x)\,dx\Rightarrow\frac{C}{(1+2\ln y)^2}=D\cos 2x$ oe	M1
	One side integrated **correctly** $\int\frac{3}{y(1+2\ln y)^3}dy=-\frac{3}{4(1+2\ln y)^2}$, $\int\frac{1}{y(1+2\ln y)^3}dy=-\frac{1}{4(1+2\ln y)^2}$ or $\int\frac{1}{3}\sin(2x)\,dx=-\frac{1}{6}\cos(2x)$ oe, $\int\sin(2x)\,dx=-\frac{1}{2}\cos(2x)$ oe	A1
	$-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)(+c)$ or $-\frac{1}{4(1+2\ln y)^2}=-\frac{1}{6}\cos(2x)(+c)$	A1
		(4)
(c)	$y=1, x=\frac{\pi}{6}\Rightarrow-\frac{3}{4(1+2\ln 1)}=-\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+c\Rightarrow c=...$	M1
	$-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)-\frac{1}{2}$ or $-\frac{1}{4(1+2\ln y)^2}=-\frac{1}{6}\cos(2x)-\frac{1}{6}$	A1
	$-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)-\frac{1}{2}\Rightarrow\frac{3}{4(1+2\ln y)^2}=\frac{1}{2}+\frac{1}{2}\cos(2x)=\cos^2 x$	M1
	$\Rightarrow(1+2\ln y)^2=\frac{3}{4}\sec^2 x\Rightarrow\ln y=\frac{1}{2}\left(\frac{\sqrt{3}}{2}\sec x-1\right)$ **Depends on both previous method marks.**	ddM1
	$\Rightarrow y=e^{\frac{\sqrt{3}}{4}\sec x-\frac{1}{2}}$	A1
		(5)

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=8(1-2x)^{-\frac{3}{2}}$	B1
	$(1-2x)^{-\frac{3}{2}}=1+(-\tfrac{3}{2})(-2x)+\frac{-\frac{3}{2}(-\frac{3}{2}-1)}{2!}(-2x)^2+\frac{-\frac{3}{2}(-\frac{3}{2}-1)(-\frac{3}{2}-2)}{3!}(-2x)^3+\ldots$	M1 A1
	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=8+24x+60x^2+140x^3+\ldots$	A1, A1
		(5)
(b)	$n=2$	B1
		(1)
(c)	$\left(\frac{1}{4}-\frac{1}{2}x\right)^2=\frac{1}{16}-\frac{1}{4}x+\frac{1}{4}x^2$	B1
	$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}=\left(\frac{1}{16}-\frac{1}{4}x+\frac{1}{4}x^2\right)(8+24x+60x^2+140x^3+\ldots)$ $=8\times\frac{1}{16}+24x\times\frac{1}{16}+60x^2\times\frac{1}{16}-8\times\frac{1}{4}x-24x\times\frac{1}{4}x+8\times\frac{1}{4}x^2$	M1
	$=\frac{1}{2}-\frac{1}{2}x-\frac{1}{4}x^2+\ldots$	A1
		(3)
		Total 9

Question	Scheme	Marks
4(a)	$(l=)\sqrt{25+r^2}$	**B1**
		(1)
(b)	$(S=\pi r^2+)\pi r\sqrt{25+r^2}\Rightarrow\left(\frac{dS}{dr}=2\pi r+\right)\underline{\pi\sqrt{25+r^2}+\pi r.\frac{1}{2}(25+r^2)^{-\frac{1}{2}}.2r}$ Or $(S=\pi r^2+)\pi\sqrt{25r^2+r^4}\rightarrow\left(\frac{dS}{dr}=2\pi r+\right)\underline{\pi.\frac{1}{2}(25r^2+r^4)^{-\frac{1}{2}}\times(50r+4r^3)}$	**M1** **A1**
	$(S=\pi(l^2-25))+\pi l\sqrt{l^2-25}\Rightarrow\left(\frac{dS}{dl}=2\pi l+\right)\underline{\pi\sqrt{l^2-25}+\pi l.\frac{1}{2}(l^2-25)^{-\frac{1}{2}}.2l}$	**(M1** **A1)**
	$\frac{dS}{dt}=\frac{dS}{dr}\times\frac{dr}{dt}=..\times3$	**M1**
	$=81.5\ (\text{cm}^2/\text{min})$	**A1**
		(4)
		(5 marks)