

Question Number	Scheme	Marks
<p><b>5.</b></p> <p><b>Main</b></p>	<p>One of <math>\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2</math></p> <p>Two of <math>\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2</math></p> <p>All three of <math>\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2</math></p> <p>Full attempt to find <math>\frac{dV}{dt}</math></p> <p>E.g. <math>\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times \frac{dA}{dt} = 18\pi x^2 \times \frac{1}{2\pi x} \times \frac{\pi}{20}</math></p> <p>Finds <math>\frac{dV}{dt}</math> at <math>x = 2 \Rightarrow \frac{dV}{dt} = 18\pi 2^2 \times \frac{1}{80} = \frac{9}{10}\pi</math></p> <p>Note that the M1 may be found in two steps:</p> <p>E.g. I Finds <math>\frac{dx}{dt}</math> from <math>\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}</math> followed by <math>\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}</math></p> <p>E.g. II Finds <math>\frac{dV}{dA}</math> from <math>\frac{dV}{dx} = \frac{dV}{dA} \times \frac{dA}{dx}</math> followed by <math>\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>dM1A1</p> <p><b>(6 marks)</b></p>
<p><b>Alt</b></p>	<p><math>\frac{dA}{dt} = \frac{\pi}{20}</math></p> <p><math>V = 6\pi x^3, A = \pi x^2 \Rightarrow V = 6\pi \left(\frac{A}{\pi}\right)^{\frac{3}{2}} \frac{dV}{dA} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}</math></p> <p><math>\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} \Rightarrow \frac{dV}{dt} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}} \times \frac{\pi}{20}</math></p> <p><math>x = 2 \Rightarrow A = 4\pi \Rightarrow \frac{dV}{dt} \Big _{A=4\pi} = \frac{9}{10}\pi</math></p>	<p>B1</p> <p>B1, B1</p> <p>M1</p> <p>dM1A1</p> <p><b>(6 marks)</b></p>

Question Number	Scheme	Marks
<b>10 (a)</b>	Writes $2(4p^3 + 6p^2 + 3p) + 1$ which is odd	B1
<b>(b)</b>	Assumption: E.g. States that there exists integers $p$ and $q$ such that $\sqrt[3]{2} = \frac{p}{q}$ (where $\frac{p}{q}$ is in its simplest form) and then cubes to get $2 = \frac{p^3}{q^3}$  $2 = \frac{p^3}{q^3} \Rightarrow p^3 = 2q^3$ and concludes that $p^3$ is even so therefore $p$ is even  If $p$ is even then it can be written $p = 2m$ so $(2m)^3 = 2q^3$  States that $q^3 = 4m^3$ and concludes that $q^3$ is even so therefore $q$ is even This contradicts our initial statement, as if they both have a factor of 2 it means that $\frac{p}{q}$ is not in its simplest form, so $\sqrt[3]{2}$ is irrational *	M1 A1 M1 A1 A1*
		(5) <b>(6 marks)</b>

Question Number	Scheme	Marks
<b>6(a)</b>	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{(\theta - 15)^2} = \int -k dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = -(\theta - 15)^{-1}$	M1
	$-\frac{1}{\theta - 15} = -kt + c$	A1
	$t = 0, \theta = 85 \Rightarrow -\frac{1}{70} = c$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25} = 10k + \frac{1}{70} \Rightarrow k = \dots \left( \frac{9}{3500} \right)$	M1
	$\frac{1}{\theta - 15} = \frac{9t}{3500} + \frac{1}{70} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1
		(7)
<b>(b)</b>	$20 = \frac{135t + 4250}{9t + 50} \Rightarrow t = \dots$	M1
	$t = \text{awrt } 72$	A1
		(2)
		<b>Total 9</b>

Question Number	Scheme	Marks
<b>1 (a)</b>	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (kx)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (kx)^3 \dots$	
<b>(i)</b>	$\frac{1}{2}k = \frac{1}{8} \Rightarrow k = \frac{1}{4}$	M1A1
<b>(ii)</b>	$A = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times "k"^{n^2} = -\frac{1}{128} \quad B = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times "k"^{n^3} = \frac{1}{1024}$	M1 A1 A1
<b>(b)</b>	Substitutes $x = 0.6 \Rightarrow \sqrt{1.15} = 1 + \frac{1}{8} \times 0.6 - \frac{1}{128} \times 0.6^2 + \frac{1}{1024} \times 0.6^3 = 1.072398$	M1 A1
		(5)
		(2)
		<b>(7 marks)</b>
<b>1(b) alt</b>	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) - \left(\frac{1}{8}\right) \times (kx)^2 + \left(\frac{1}{16}\right) \times (kx)^3$ Substitutes "kx" = 0.15 $\Rightarrow \sqrt{1.15} = 1 + \frac{1}{2} \times 0.15 - \frac{1}{8} \times 0.15^2 + \frac{1}{16} \times 0.15^3 = 1.072398$	M1A1
		(2)

Question Number	Scheme	Marks
<b>4 (a)</b>	$\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-2x)^2$	M1A1
	$\frac{2+3x}{\sqrt{1-2x}} = (2+3x)\left(1+x+\frac{3}{2}x^2+\dots\right)$ $= 2+5x+6x^2 \quad *$	dM1 A1*
<b>(b)</b>	Sub $x = \frac{1}{20}$ into both sides of $\frac{2+3x}{\sqrt{1-2x}} = 2+5x+6x^2$ $\frac{\frac{43}{20}}{\sqrt{\frac{9}{10}}} = \frac{453}{200} \Rightarrow \sqrt{10} = \dots$ $\sqrt{10} = \frac{1359}{430}$	M1 dM1 A1
		(4)
		(3)
		<b>(7 marks)</b>

Question Number	Scheme	Marks
<b>1(a)</b>	$(2-5x)^{-2} = \frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$ or e.g. $\frac{8}{4\left(1-\frac{5x}{2}\right)^2}$	B1
	$= 8 \times \frac{1}{4} \left( 1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3 \dots \right)$	M1A1
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
		(4)
<b>Alt (a) by direct expansion</b>	$= 8 \times \left( 2^{-2} + (-2)2^{-3}(-5x)^1 + \frac{-2 \times -3}{2!} 2^{-4}(-5x)^2 + \frac{-2 \times -3 \times -4}{3!} 2^{-5}(-5x)^3 \right)$	B1, <u>M1A1</u>
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
<b>(b)</b>	$ x  < \frac{2}{5}$ o.e.	B1
		(1)
		(5 marks)

Question Number	Scheme	Marks
<b>2</b>	$\frac{dV}{dt} = \pm k$	B1
	$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$	B1
	$\pm k = \lambda r^2 \times \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2}$ ✓ *	A1*
		(4 marks)

Question Number	Scheme	Marks
<b>5(i)</b>	$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx (+c)$	M1A1
	$\int x e^x dx = x e^x - \int e^x dx (+c)$	M1
	$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x (+c)$	A1
	Also allow $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x)(+c)$	
		(4)

(ii)	$u = (1-3x)^{\frac{1}{2}} \Rightarrow u^2 = 1-3x \Rightarrow 2u \frac{du}{dx} = -3$ <p style="text-align: center;">or</p> $u = (1-3x)^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$	B1
	$\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{u} \left(-\frac{2u}{3}\right) du$ <p style="text-align: center;">or</p> $\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{\sqrt{1-3x}} \left(-\frac{2}{3}\sqrt{1-3x}\right) du$	M1
	$6 \int (u^2 - 1) du \text{ or } -6 \int (1 - u^2) du$	A1
	$6 \left( \frac{u^3}{3} - u \right) (+k)$	A1ft
	$6 \left( \frac{(1-3x)^{\frac{3}{2}}}{3} - (1-3x)^{\frac{1}{2}} \right) (+k) = 2(1-3x)^{\frac{1}{2}}(1-3x-3)(+k)$	M1
	$= -2(1-3x)^{\frac{1}{2}}(2+3x)(+k) \text{ or } = -2(1-3x)^{\frac{1}{2}}(3x+2)(+k)$	A1
		<b>(6)</b>
		<b>Total 10</b>

Question Number	Scheme	Marks
9(a)	$\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3} \text{ (or } 60^\circ) \text{ (Allow } x = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \text{ (or } 60^\circ))$	B1
	$V = (\pi) \int y^2 dx = (\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta \text{ oe}$	M1A1
	$4(\pi) \int \sin^2 2\theta \sec^2 \theta d\theta = 4(\pi) \int 4 \sin^2 \theta \cos^2 \theta \times \frac{1}{\cos^2 \theta} d\theta$	dM1
	$= 16(\pi) \int \sin^2 \theta d\theta \text{ oe e.g. } 16(\pi) \int (1 - \cos^2 \theta) d\theta$	A1
	$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \Rightarrow 16(\pi) \int \sin^2 \theta d\theta = 16(\pi) \int \frac{1 - \cos 2\theta}{2} d\theta$	dM1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta$	A1 Cso
		<b>(7)</b>
(b)	$\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$	B1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi(1 - \cos 2\theta) d\theta = [8\pi\theta - 4\pi \sin 2\theta]_0^{\frac{\pi}{3}} = \frac{8}{3}\pi^2 - 2\sqrt{3}\pi$	M1 A1
		<b>(3)</b>
		<b>(10 marks)</b>

Question Number	Scheme	Marks
<b>7. (a)</b>	Attempts $\overrightarrow{BA} \cdot \overrightarrow{BC} \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix} = 24 + 6 + 15 = 45$	M1
	Attempts to apply $\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}   \mathbf{b}  \cos \theta \Rightarrow 45 = 7 \times \sqrt{50} \cos \theta$ $\cos \theta = \frac{9\sqrt{2}}{14} \text{ oe}$	dM1 A1
<b>(b)</b>	Attempts $\sin \theta = \sqrt{1 - \left(\frac{9\sqrt{2}}{14}\right)^2}$	(3) M1
	Attempts to apply $ \mathbf{a}   \mathbf{b}  \sin \theta = 7 \times \sqrt{50} \times \frac{\sqrt{34}}{14}$	M1
	Area ABCD = $5\sqrt{17}$	A1 (3) (6 marks)

Question Number	Scheme	Marks
<b>6</b>	$u = \sqrt{x^3 + 1} \Rightarrow u^2 = x^3 + 1 \Rightarrow 2u \frac{du}{dx} = 3x^2$ or $u = \sqrt{x^3 + 1} \Rightarrow u^2 = x^3 + 1 \Rightarrow 2u = 3x^2 \frac{dx}{du}$ or $u = \sqrt{x^3 + 1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(x^3 + 1)^{-\frac{1}{2}} \times 3x^2$ or $x = (u^2 - 1)^{\frac{1}{3}} \Rightarrow \frac{dx}{du} = \frac{2}{3}u(u^2 - 1)^{-\frac{2}{3}}$	B1
	e.g. $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = \int \frac{9x^5}{u} \frac{2u}{3x^2} du = \int 6x^3 du = 6 \int (u^2 - 1) du$ or e.g. $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = \int \frac{9x^5}{\sqrt{x^3 + 1}} \frac{2\sqrt{x^3 + 1}}{3x^2} du = \int 6x^3 du = 6 \int (u^2 - 1) du$	M1A1
	$6 \int (u^2 - 1) du = 2u^3 - 6u (+c) = 2(x^3 + 1)^{\frac{3}{2}} - 6(x^3 + 1)^{\frac{1}{2}} (+c)$	M1
	$= 2(x^3 + 1)^{\frac{1}{2}} [(x^3 + 1) - 3] + c = 2(x^3 + 1)^{\frac{1}{2}} (x^3 - 2) + c$	A1
		(5) <b>Total 5</b>