

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{2(1-t) - (-1)(3+2t)}{(1-t)^2} = \frac{5}{(1-t)^2} \quad \text{and} \quad \frac{dy}{dt} = -2t$ $\text{At } t=2 \Rightarrow \frac{dy}{dx} = \frac{(-1)^2}{5} \times (-2(2)) = -\frac{4}{5}$ $(-7, -3)$ $y+3 = \frac{5}{4}(x+7)$ $5x - 4y + 23 = 0$	<p>M1A1</p> <p>B1 M1</p> <p>A1</p> <p>(5)</p>
(b)	$x = \frac{3+2t}{1-t} \rightarrow t = \frac{x-3}{x+2}$ $y = 1 - \left(\frac{x-3}{x+2}\right)^2 = \frac{(x+2)^2 - (x-3)^2}{(x+2)^2} = \frac{10x-5}{(x+2)^2}$ $x \neq -2$	<p>M1</p> <p>dM1A1</p> <p>B1</p> <p>(4)</p>
		(9 marks)

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Question	Scheme	Marks
9(a)	$\frac{dx}{dt} = \sec t \tan t \quad \text{and} \quad \frac{dy}{dt} = \sqrt{3} \sec^2 \left(t + \frac{\pi}{3} \right)$	B1
	$\left(\frac{dy}{dx} \right) \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sqrt{3} \sec^2 \left(t + \frac{\pi}{3} \right)}{\sec t \tan t} \quad \text{oe}$	M1 A1
		(3)

(b)	$t = \frac{\pi}{3} \Rightarrow x = 2, y = -3$	B1
	$t = \frac{\pi}{3} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{3} \sec^2 \frac{2\pi}{3}}{\sec \frac{\pi}{3} \tan \frac{\pi}{3}} = \dots (=2)$	M1
	Tangent is $y + 3 = 2(x - 2) \Rightarrow y = 2x - 7$ cso	M1A1
		(4)
(c)	$y = \sqrt{3} \frac{\tan t \pm \tan \frac{\pi}{3}}{1 \pm \tan t \tan \frac{\pi}{3}}$	M1
	$x^2 = \sec^2 t = 1 + \tan^2 t \Rightarrow \tan t = \sqrt{x^2 - 1} \Rightarrow y = \sqrt{3} \frac{\sqrt{x^2 - 1} + \sqrt{3}}{1 - \sqrt{3}\sqrt{x^2 - 1}}$	M1 A1
	$= \sqrt{3} \frac{\sqrt{x^2 - 1} + \sqrt{3}}{1 - \sqrt{3}\sqrt{x^2 - 1}} \times \frac{1 + \sqrt{3}\sqrt{x^2 - 1}}{1 + \sqrt{3}\sqrt{x^2 - 1}} = \dots$	
	$= \sqrt{3} \frac{\sqrt{x^2 - 1} + \sqrt{3}(x^2 - 1) + \sqrt{3} + 3\sqrt{x^2 - 1}}{1 - (3x^2 - 3)} = \frac{\dots \sqrt{x^2 - 1} + \dots x^2}{4 - 3x^2}$	M1
	$= \frac{3x^2 + 4\sqrt{3x^2 - 3}}{4 - 3x^2}$	A1
		(5)

Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1} + \frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3) + B(2x-1) \Rightarrow A = \dots B = \dots$	M1
	$A=1$ or $B=2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1} + \frac{2}{4x-3}$	A1
		(3)
(b)	$\int \left(\frac{1}{2x-1} + \frac{2}{4x-3} \right) dx = \frac{1}{2} \ln 2x-1 + \frac{1}{2} \ln 4x-3 (+c)$ <p>Follow through their A and B</p>	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2} \ln(2x-1) + \frac{1}{2} \ln(4x-3) \right]_k^{3k} = \frac{1}{2} \ln(6k-1)(12k-3) - \frac{1}{2} \ln(2k-1)(4k-3)$ $= \frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2 - 170k + 57 = 0$	dM1A1
	$88k^2 - 170k + 57 = 0 \Rightarrow k = \dots$	ddM1
	$k = \frac{3}{2}$	A1
		(5)
	Total 11	

Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2}$ and $y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6\left(\frac{1}{2}\right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = " \frac{38}{13} " \left(x - \frac{1}{2} \right)$	M1
$38x - 13y + 33 = 0$	A1	
	(6)	
	(8 marks)	

Question Number	Scheme	Marks
6	$u = 3 + 4 \sin x \Rightarrow \frac{du}{dx} = 4 \cos x$ $\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{32 \sin x \cos x}{(3 + 4 \sin x)^2} dx = \int \frac{2(u-3)}{u^2} du$ $= \int \frac{2}{u} - \frac{6}{u^2} du = 2 \ln u + \frac{6}{u}$ <p>Uses limits of 5 and 7 $\Rightarrow 2 \ln 7 + \frac{6}{7} - 2 \ln 5 - \frac{6}{5} = -\frac{12}{35} + \ln \frac{49}{25}$</p>	B1 M1 A1 dM1 A1 M1 A1 (7 marks)

Question	Scheme	Marks
7(a)	$u = 4x + 2 \sin 2x \Rightarrow \frac{du}{dx} = 4 + 4 \cos 2x \quad \text{oe}$	M1 A1
	$\frac{du}{dx} = 4 + 4 \cos 2x = 8 \cos^2 x \Rightarrow \int_{(0)}^{\left(\frac{\pi}{2}\right)} e^{4x+2\sin 2x} \cos^2 x \, dx = \int_{(0)}^{(2\pi)} \frac{e^u}{8} \, du$	M1
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi} \quad \text{or} \quad \left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	A1ft
	$\int e^{4x+2\sin 2x} \cos^2 x \, dx = \int e^{4x+2\sin 2x} \times \frac{1}{2} (1 + \cos 2x) \, dx$	(M1 A1)
	$= k e^{4x+2\sin 2x}; = \left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	(M1; A1ft)
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi} \quad \text{or} \quad \left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}} = \frac{1}{8} (e^{2\pi} - 1) *$	A1*cs0
(b)	$V \left[= \pi \int_0^{\frac{\pi}{2}} y^2 \, dx = \pi \int_0^{\frac{\pi}{2}} (6e^{2x+\sin 2x} \cos x)^2 \, dx \right] = 36\pi \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx$	B1
	$K \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx = \frac{K}{8} (e^{2\pi} - 1)$	M1
	$= \frac{9\pi}{2} (e^{2\pi} - 1)$	A1
		(3)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ <p>Sets $-\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$</p> $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{t+1} + \frac{B}{2t+5} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{t+1} - \frac{\frac{4}{3}}{2t+5}$ $\int \left(\frac{\frac{2}{3}}{t+1} - \frac{\frac{4}{3}}{2t+5} \right) dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
3(a)	$x = \frac{t+15}{t+4} \Rightarrow t = \frac{15-4x}{x-1}$ $y = \frac{5}{t+2} \Rightarrow y = \frac{5}{\frac{15-4x}{x-1} + 2}$ $y = \frac{5(x-1)}{15-4x+2(x-1)}$ $g(x) = \frac{5x-5}{13-2x} \quad 1 < x < \frac{15}{4}$	M1 M1 dM1 A1B1 (5)
(b)	$0 < g(x) < \frac{5}{2}$	M1A1 (2) (7 marks)

Question	Scheme	Marks
4(a)	$(l =) \sqrt{25+r^2}$	B1 (1)
(b)	$(S = \pi r^2 +) \pi r \sqrt{25+r^2} \Rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \sqrt{25+r^2} + \pi r \cdot \frac{1}{2} (25+r^2)^{-\frac{1}{2}} \cdot 2r$ $\text{Or } (S = \pi r^2 +) \pi \sqrt{25r^2+r^4} \rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \cdot \frac{1}{2} (25r^2+r^4)^{-\frac{1}{2}} \times (50r+4r^3)$	M1 A1
	$(S = \pi(l^2 - 25)) + \pi l \sqrt{l^2 - 25} \Rightarrow \left(\frac{dS}{dl} = 2\pi l + \right) \pi \sqrt{l^2 - 25} + \pi l \cdot \frac{1}{2} (l^2 - 25)^{-\frac{1}{2}} \cdot 2l$	(M1) (A1)
	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = \dots \times 3$	M1
	$= 81.5 \text{ (cm}^2 \text{ / min)}$	A1
		(4) (5 marks)

Question Number	Scheme	Marks
3	Differentiates wrt x $3^x \ln 3 + 6 \frac{dy}{dx} = \frac{3}{2} y^2 + 3xy \frac{dy}{dx}$ Substitutes (2, 3) AND rearranges to get $\frac{dy}{dx}$ or vice versa $\Rightarrow 9 \ln 3 + 6 \frac{dy}{dx} = \frac{27}{2} + 18 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{9 \ln 3 - \frac{27}{2}}{12} = \frac{6 \ln 3 - 9}{8} = \frac{-9 + \ln 729}{8}$	B1 <u>B1</u> , <u>M1</u> , A1 M1 A1, A1 (7) (7 marks)

Question Number	Scheme	Marks
5 (a)	$2y \frac{dy}{dx} = e^{-2x} \frac{dy}{dx} - 2ye^{-2x} - 3$ $(e^{-2x} - 2y) \frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} *$	B1 M1 A1 <u> </u> A1* (4)
(b)	Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$ Attempts tangent at (0, 0) or (0, 1) $y = 3x$ or $y = -5x + 1$ Solves $y = 3x$ with $y = -5x + 1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	B1 M1 A1 dM1 A1 (5) (9 marks)