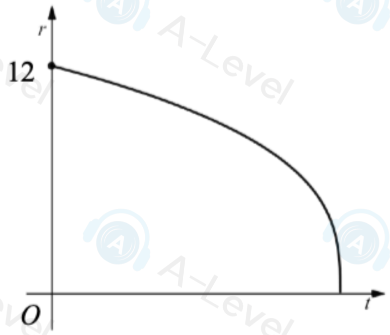
Question Number	Scheme	Marks
3.	States or implies Volume = $\int_{\sqrt{5}}^{5} \pi \left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 dx$	**B1**
	$\int \left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 dx = \int \frac{3x}{(3x^2+5)} dx = \frac{1}{2}\ln(3x^2+5)$	**M1A1**
	Volume = $\{\pi\}\left(\frac{1}{2}\ln(3\times 25+5) - \frac{1}{2}\ln(3\times 5+5)\right)$	**M1**
	$= \pi \ln 2$	**A1**
		(5 marks)

Question Number	Scheme	Marks
8(a)	*A* and *B* are where *y* = 0 so $t^3 - 9t = 0 \Rightarrow t(t^2-9) = 0 \Rightarrow t = 3 \quad (0 \text{ and } -3)$ Substitutes $t = 3$ in $x = 3^2 + 2\times3 = 15$ *B* (15, 0) $A = (3, 0)$	M1 A1* B1 **(3)**
8(a) ALT	Uses answer *x* = 15: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ *B* (15, 0) $A = (3, 0)$	M1 A1* B1 **(3)**
(b)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2-9}{2t+2}$ Substitutes $t = 3$ into $\frac{dy}{dx} = \frac{3t^2-9}{2t+2} \Rightarrow \frac{dy}{dx} = \frac{9}{4}$ Uses their $\frac{9}{4}$ with $(15, 0)$ to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 **(4)**
(c)	Substitutes $x = t^2 + 2t,\ y = t^3 - 9t,$ into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2+2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\frac{15}{4}$ Coordinates of *X* are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$	M1 dM1 A1 dM1A1 **(5)** **(12 marks)**

Question Number	Scheme	Marks
4(i)	$\frac{dV}{dt} = 70\pi$	B1
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 70\pi = 4\pi \times 5^2 \times \frac{dr}{dt}$ or e.g. $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \Rightarrow \frac{dr}{dt} = 70\pi \times \frac{1}{4\pi \times 5^2}$	M1
	$\left(\frac{dr}{dt}\right) = 0.7$ (cm s^{-1}) oe e.g. $\frac{7}{10}$	A1
		(4)

(ii)	$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int h^3\,dh = \int k\,dt \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0,\ h = 4 \Rightarrow \frac{1}{4} \times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5,\ h = 6 \Rightarrow \frac{1}{4} \times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = \ldots$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
10 (a)	States or uses $\frac{dr}{dt} \propto \pm\frac{1}{r^2}$ or $\frac{dr}{dt} = \pm\frac{k}{r^2}$	B1
	$\frac{dr}{dt} = \pm\frac{k}{r^2} \Rightarrow \int r^2 dr = \int \pm k dt$	
	$\frac{1}{3}r^3 = \pm kt + c$	M1 A1
	Substitutes $t = 0, r = 12 \Rightarrow c = (576)$	
	AND $t = 15, r = 6$ and their $c = (576) \Rightarrow k = (\pm 33.6)$	M1
	$\frac{1}{3}r^3 = -33.6t + 576$ or $r^3 = -100.8t + 1728$ o.e.	A1
		(5)
(b)	Sets $r = 0 \Rightarrow -100.8t + 1728 = 0$	M1
	$\frac{120}{7}$ minutes or awrt 17.1 minutes	A1
		(2)
(c)		B1
		(1)
		(8 marks)

Question Number	Scheme	Marks
5(i)	$\int x^2 e^x dx = x^2 e^x - \int 2xe^x dx(+c)$	M1A1
	$\int xe^x dx = xe^x - \int e^x dx(+c)$	M1
	$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x (+c)$ Also allow $\int x^2 e^x dx = x^2 e^x - (2xe^x - 2e^x)(+c)$	A1
		(4)

(ii)	$u=(1-3x)^{\frac{1}{2}} \Rightarrow u^2=1-3x \Rightarrow 2u\frac{\mathrm{d}u}{\mathrm{d}x}=-3$ or $u=(1-3x)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=-\frac{3}{2}(1-3x)^{-\frac{1}{2}}$	B1
	$\int \frac{27x}{\sqrt{1-3x}}\mathrm{d}x = 27\int \frac{\frac{1-u^2}{3}}{u}\left(-\frac{2u}{3}\right)\mathrm{d}u$ or $\int \frac{27x}{\sqrt{1-3x}}\mathrm{d}x = 27\int \frac{\frac{1-u^2}{3}}{\sqrt{1-3x}}\left(-\frac{2}{3}\sqrt{1-3x}\right)\mathrm{d}u$	M1
	$6\int (u^2-1)\mathrm{d}u$ or $-6\int (1-u^2)\mathrm{d}u$	A1
	$6\left(\frac{u^3}{3}-u\right)(+k)$	A1ft
	$6\left(\frac{(1-3x)^{\frac{3}{2}}}{3}-(1-3x)^{\frac{1}{2}}\right)(+k)=2(1-3x)^{\frac{1}{2}}(1-3x-3)(+k)$	M1
	$=-2(1-3x)^{\frac{1}{2}}(2+3x)(+k)$ or $=-2(1-3x)^{\frac{1}{2}}(3x+2)(+k)$	A1
		(6)
		Total 10

Question Number	Scheme	Marks
2(a)	$x=2\Rightarrow 4-8y+y^2=13\Rightarrow\left(y^2-8y-9=0\Rightarrow\right)y=...$	M1
	$y=9$	A1
		(2)
(b)	$2^x\rightarrow 2^x\ln 2$	B1
	$-4xy\rightarrow -4x\frac{dy}{dx}-4y$ **OR** $y^2\rightarrow 2y\frac{dy}{dx}$	M1
	$2^x\ln 2-4x\frac{dy}{dx}-4y+2y\frac{dy}{dx}=0$	A1
	$\frac{dy}{dx}(2y-4x)=4y-2^x\ln 2\Rightarrow\frac{dy}{dx}=...$	M1
	$\frac{dy}{dx}=\frac{4y-2^x\ln 2}{2y-4x}$ or $\frac{dy}{dx}=\frac{2^x\ln 2-4y}{4x-2y}$ or $\frac{dy}{dx}=\frac{\ln 2e^{x\ln 2}-4y}{4x-2y}$	A1
		(5)
(c)	$(2,9)\rightarrow\frac{dy}{dx}=\frac{4(9)-2^2\ln 2}{2(9)-4(2)}$ $\Rightarrow y-"9"="\frac{36-4\ln 2}{10}"(x-2)$	M1
	$y=0\Rightarrow 0-"9"="\frac{36-4\ln 2}{10}"(x-2)\Rightarrow x=...$	dM1
	$x=\frac{4\ln 2+9}{2\ln 2-18}$ o.e. e.g. $x=\frac{-8\ln 2-18}{-4\ln 2+36}$.	A1
		(3)
		Total 10

Question Number	Scheme	Marks
3(i)	$\int x^2e^{2x}\,dx=\frac{1}{2}x^2e^{2x}-\int xe^{2x}\,dx$	M1
	$=\frac{1}{2}x^2e^{2x}-\left\{\frac{1}{2}xe^{2x}-\int\frac{e^{2x}}{2}\,dx\right\}$	M1
	$=\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{e^{2x}}{4}$	A1
	$\int_0^4 x^2e^{2x}\,dx=\left[\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{e^{2x}}{4}\right]_0^4=8e^8-2e^8+\frac{e^8}{4}-\frac{1}{4}$	M1
	$=\frac{25e^8}{4}-\frac{1}{4}$	A1
		(5)

(ii)	**1.** $u=2x-1\Rightarrow\frac{du}{dx}=2$ or e.g. $du=2dx$ **OR** **2.** $u=(2x-1)^2\Rightarrow\frac{du}{dx}=4(2x-1)$ or e.g. $du=4(2x-1)dx$ **OR** **3.** $u=2x\Rightarrow\frac{du}{dx}=2$ or e.g. $du=2dx$	B1
	1. $u=2x-1\Rightarrow\int\frac{4x}{(2x-1)^2}dx=\int\frac{2u+2}{u^2}\times\frac{1}{2}du$ **OR** **2.** $u=(2x-1)^2\Rightarrow\int\frac{4x}{(2x-1)^2}dx=\int\frac{2\sqrt{u}+2}{u}\times\frac{1}{4\sqrt{u}}du$ **OR** **3.** $u=2x\Rightarrow\int\frac{4x}{(2x-1)^2}dx=\int\frac{2u}{(u-1)^2}\times\frac{1}{2}du$	M1 A1
	1. $\int\left(\frac{1}{u}+\frac{1}{u^2}\right)du=\ln u-\frac{1}{u}$ **OR** **2.** $\frac{1}{2}\int\left(\frac{1}{u}+\frac{1}{u^{\frac{3}{2}}}\right)du=\frac{1}{2}\ln u-\frac{1}{\sqrt{u}}$ **OR 3.** $\int\frac{u}{(u-1)^2}du=-u(u-1)^{-1}+\int\frac{1}{u-1}du=-u(u-1)^{-1}+\ln(u-1)$	dM1 A1
	1. Uses limits $u=5$ to $u=20$: $\left(\ln 20-\frac{1}{20}\right)-\left(\ln 5-\frac{1}{5}\right)$ **OR** **2.** Uses limits $u=25$ to $u=400$: $\left(\frac{1}{2}\ln 400-\frac{1}{20}\right)-\left(\frac{1}{2}\ln 25-\frac{1}{5}\right)$ **OR** **3.** Uses limits $u=6$ to $u=21$: $\left(\ln 20-\frac{21}{20}\right)-\left(\ln 5-\frac{6}{5}\right)$	M1
	$=\frac{3}{20}+\ln 4$	A1
		(7)
		(12 marks)

Question Number	Scheme	Marks
8(a)	*A* and *B* are where *y* = 0 so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3 \quad (0 \text{ and } -3)$ Substitutes $t = 3$ in $x = 3^2 + 2\times 3 = 15$ *B* (15, 0) $A = (3, 0)$	M1 A1* B1 **(3)**
8(a) ALT	Uses answer *x* = 15: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ *B* (15, 0) $A = (3, 0)$	M1 A1* B1 **(3)**
(b)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 9}{2t + 2}$ Substitutes $t = 3$ into $\frac{dy}{dx} = \frac{3t^2 - 9}{2t + 2} \Rightarrow \frac{dy}{dx} = \frac{9}{4}$ Uses their $\frac{9}{4}$ with $(15, 0)$ to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 **(4)**
(c)	Substitutes $x = t^2 + 2t,\ y = t^3 - 9t,$ into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\frac{15}{4}$ Coordinates of *X* are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$	M1 dM1 A1 dM1A1 **(5)** **(12 marks)**

Question Number	Scheme	Notes	Marks
9(a)	$V=\pi\int y^2\,\mathrm{d}x=\pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ $=\pi\int(3\sin\theta-\sin 2\theta)^2(-5\sin\theta)\mathrm{d}\theta$	Applies $V=\pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$ with or without the π	M1
	$=\pi\int(3\sin\theta-2\sin\theta\cos\theta)^2(-5\sin\theta)\mathrm{d}\theta$	Applies $\sin 2\theta=2\sin\theta\cos\theta$	M1
		Fully correct integral in terms of $\sin\theta$ and $\cos\theta$ only (π not needed)	A1
	$=\pi\int\sin^2\theta(3-2\cos\theta)^2(-5\sin\theta)\mathrm{d}\theta$ $V=-5\pi\int\sin^3\theta(3-2\cos\theta)^2\,\mathrm{d}\theta$ $V=-5\pi\int_{\pi}^{0}\sin^3\theta(3-2\cos\theta)^2\,\mathrm{d}\theta$ $V=5\pi\int_{0}^{\pi}\sin^3\theta(3-2\cos\theta)^2\,\mathrm{d}\theta$*	Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout.	A1*
			(4)
(b)	$u=\cos\theta\Rightarrow V=5\pi\int\sin^3\theta(3-2u)^2\frac{\mathrm{d}u}{-\sin\theta}$	Applies the substitution correctly	M1
	$\theta=0\Rightarrow u=1,\ \theta=\pi\Rightarrow u=-1$	Attempts to change θ limits to u limits	M1
	$V=-5\pi\int\sin^2\theta(3-2u)^2\,\mathrm{d}u=-5\pi\int(1-u^2)(3-2u)^2\mathrm{d}u$ Correct integral in terms of u only		A1
	$(1-u^2)(3-2u)^2=(1-u^2)(9-12u+4u^2)$	Attempt to expand	M1
	$=9-12u-5u^2+12u^3-4u^4$	Correct expansion	A1
	$V=5\pi\int_{-1}^{1}(9-12u-5u^2+12u^3-4u^4)\mathrm{d}u$ $=5\pi\left[9u-6u^2-\frac{5u^3}{3}+3u^4-\frac{4u^5}{5}\right]_{-1}^{1}=\ldots$ Integrates and applies their u limits		M1
	$=\frac{196}{3}\pi$	Cao	A1
			(7)
			Total 11

Question Number	Scheme	Marks
3 (a)	Attempts to find $\dfrac{\frac{1}{3}\times 20^2\times 24}{160}=20$ seconds	M1 A1
		(2)
(b)	Attempts $\dfrac{dV}{dh}=h^2+\dfrac{8}{3}h$	M1
	Attempts to use $\dfrac{dV}{dt}=\dfrac{dV}{dh}\times\dfrac{dh}{dt}\Rightarrow 160=\left(h^2+\dfrac{8}{3}h\right)\times\dfrac{dh}{dt}$	M1 A1
	Substitutes $h=5\Rightarrow 160=\left(5^2+\dfrac{40}{3}\right)\times\dfrac{dh}{dt}\Rightarrow\dfrac{dh}{dt}=\dfrac{96}{23}$ (4.2) cm s $^{-1}$	dM1 A1
		(5)
		(7 marks)

Question Number	Scheme	Marks
6	$u=\sqrt{x^3+1}\Rightarrow u^2=x^3+1\Rightarrow 2u\dfrac{du}{dx}=3x^2$ or $u=\sqrt{x^3+1}\Rightarrow u^2=x^3+1\Rightarrow 2u=3x^2\dfrac{dx}{du}$ or $u=\sqrt{x^3+1}\Rightarrow\dfrac{du}{dx}=\dfrac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}\times 3x^2$ or $x=(u^2-1)^{\frac{1}{3}}\Rightarrow\dfrac{dx}{du}=\dfrac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}dx=\int\frac{9x^5}{u}\frac{2u}{3x^2}du=\int 6x^3\,du=6\int\left(u^2-1\right)du$ or e.g. $\displaystyle\int\frac{9x^5}{\sqrt{x^3+1}}dx=\int\frac{9x^5}{\sqrt{x^3+1}}\frac{2\sqrt{x^3+1}}{3x^2}du=\int 6x^3\,du=6\int\left(u^2-1\right)du$	M1A1
	$6\displaystyle\int\left(u^2-1\right)du=2u^3-6u\ (+c)=2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}\ (+c)$	M1
	$=2\left(x^3+1\right)^{\frac{1}{2}}\left[\left(x^3+1\right)-3\right]+c=2\left(x^3+1\right)^{\frac{1}{2}}\left(x^3-2\right)+c$	A1
		(5)
		Total 5