Question Number	Scheme	Marks
9	$\begin{pmatrix}2-\lambda\\8+2\lambda\\10+3\lambda\end{pmatrix}=\begin{pmatrix}-4+5\mu\\-1+4\mu\\2+8\mu\end{pmatrix}$	
	Attempts to solve any two of the three equations	
	Either (1) and (2) $\left.\begin{array}{l}2-\lambda=-4+5\mu\\8+2\lambda=-1+4\mu\end{array}\right\}\Rightarrow\lambda=-\frac{3}{2},\mu=\frac{3}{2}$	
	(1) and (3) $\left.\begin{array}{l}2-\lambda=-4+5\mu\\10+3\lambda=2+8\mu\end{array}\right\}\Rightarrow\lambda=\frac{8}{23},\mu=\frac{26}{23}$	M1, A1
	(2) and (3) $\left.\begin{array}{l}8+2\lambda=-1+4\mu\\10+3\lambda=2+8\mu\end{array}\right\}\Rightarrow\lambda=-10,\mu=-\frac{11}{4}$	
	Substitutes their values of λ and μ into both sides of the "third" equation	dM1
	E.g. $\lambda=-\frac{3}{2}$ into $10+3\lambda=\frac{11}{2}$ and $\mu=\frac{3}{2}$ into $2+8\mu=14$	
	Concludes that lines don't intersect with correct calculations and minimal reason	A1
	Additionally states that $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ is not parallel to $\begin{pmatrix}5\\4\\8\end{pmatrix}$ with a minimal reason	A1*
	So lines are skew CSO *	
		(5) **(5 marks)**

Question Number	Scheme	Marks
6(a)	Attempts $\pm\left((5\mathbf{i}+2\mathbf{j}+\mathbf{k})-(\mathbf{i}+2\mathbf{j}-3\mathbf{k})\right)=\pm(4\mathbf{i}+4\mathbf{k})$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{4^2+4^2}$ or $\lvert\overrightarrow{AC}\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{32}$ and $\lvert\overrightarrow{AC}\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Question	Scheme	Marks
2	$\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\quad\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\quad\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$	
(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=...$	**M1**
	$=\begin{pmatrix}5\\6\\-2\end{pmatrix}$, $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$	**A1**
		(2)
(b)	$\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=...$	**M1**
	$\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$	**dM1**
	$a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=...$	**ddM1**
	$a=-1,\ 6$	**A1**
		(4)
		(6 marks)

Question Number	Scheme	Marks
6(a)	Attempts $\pm((5\mathbf{i}+2\mathbf{j}+\mathbf{k})-(\mathbf{i}+2\mathbf{j}-3\mathbf{k}))=\pm(4\mathbf{i}+4\mathbf{k})$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{4^2+4^2}$ or $\lvert\overrightarrow{AC}\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{32}$ and $\lvert\overrightarrow{AC}\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Question	Scheme	Marks
6(a)	$(3\mathbf{i}+p\mathbf{j}+7\mathbf{k})+\lambda(2\mathbf{i}-5\mathbf{j}+4\mathbf{k})=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$ $3+2\lambda=8+4\mu \quad (1)$ $\Rightarrow p-5\lambda=-2+\mu \quad (2)$ $7+4\lambda=5+2\mu \quad (3)$	M1
	e.g. $(3)-2\times(1): 1=-11-6\mu \Rightarrow \mu=-2 \quad \left(\text{or } \lambda=-\frac{3}{2}\right)$	M1
	$\mu=-2,\ \lambda=-\frac{3}{2} \Rightarrow p=-2-2-\frac{15}{2}=\ldots$	M1
	$p=-\frac{23}{2}$	A1
		(4)
(b)	Intersect at $(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+-2(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=..$	M1
	$=-4\mathbf{j}+\mathbf{k}$	A1
		(2)

(c)	$(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=2\times4-5\times1+4\times2=\ldots$	M1
	$\cos\theta=\frac{"11"}{\sqrt{2^2+5^2+4^2}\sqrt{4^2+1^2+2^2}}=\ldots\left(=\frac{11}{3\sqrt{105}}=0.3578\ldots\right)$	M1
	$\theta=\cos^{-1}\frac{11}{3\sqrt{105}}=69.0°$	A1
		(3)
(d)	$\lambda=2\Rightarrow\overrightarrow{OA}=\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)$	B1ft
	$\overrightarrow{AB}=\pm\left((8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})-\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)\right)$ $=\pm\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right)$	M1
	$\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=0\Rightarrow\mu=..\left(=-\frac{1}{6}\right)$	M1
	$\overrightarrow{OB}=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+"-\frac{1}{6}"(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=\ldots$	dM1
	$=\frac{22}{3}\mathbf{i}-\frac{13}{6}\mathbf{j}+\frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3},-\frac{13}{6},\frac{14}{3}\right)$	A1
		(5)

Question	Scheme	Marks
2	$\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\quad\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\quad\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$	
(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=\ldots$	M1
	$=\begin{pmatrix}5\\6\\-2\end{pmatrix}$, $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$	A1
		(2)
(b)	$\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=\ldots$	M1
	$\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$	dM1
	$a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=\ldots$	ddM1
	$a=-1,\ 6$	A1
		(4)
		(6 marks)

Question	Scheme	Marks
8(a)	Attempts $\pm\left(\begin{pmatrix}6\\0\\7\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}\right)=\pm\begin{pmatrix}0\\-6\\5\end{pmatrix}$	M1
	$\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ oe	A1
		(2)
(b)	Meet if $\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}=\begin{pmatrix}3\\1\\4\end{pmatrix}+\mu\begin{pmatrix}1\\5\\9\end{pmatrix}$ so $\begin{cases}6=3+\mu\\6-6\lambda=1+5\mu\\2+5\lambda=4+9\mu\end{cases}$	M1
	From first equation $\mu=3$ Alt: solves equations 2 and 3 to give either $\mu=\frac{13}{79}$ or $\lambda=\frac{55}{79}$	A1
	Then need $\lambda=\frac{5-5\times3}{6}=-\frac{5}{3}$ from 2nd equation, $\lambda=\frac{2+9\times3}{5}=\frac{29}{5}$ from 3rd equation Alt: checks their μ in the first equation	M1
	Values of λ do not agree and hence lines do not meet.	A1
		(4)
(c)	$\overrightarrow{OC}=\begin{pmatrix}3\\1\\4\end{pmatrix}-\begin{pmatrix}1\\5\\9\end{pmatrix}\left(=\begin{pmatrix}2\\-4\\-5\end{pmatrix}\right)$	M1
	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}2\\-4\\-5\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}=\begin{pmatrix}-4\\-10\\-7\end{pmatrix}$ Accept $\pm$. **Depends on first mark.**	dM1
	$\frac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\lvert\overrightarrow{AC}\rvert\lvert\mathbf{i}+5\mathbf{j}+9\mathbf{k}\rvert}=\pm\frac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}$ **Depends on both previous marks.**	ddM1A1
	$\Rightarrow\theta=\arccos\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$ or e.g. $\Rightarrow\theta=90°-\arcsin\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$	A1
		(5)
		(11 marks)

Question Number	Scheme	Marks
9(a)	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$	
	Either $A: \lambda = 1$ or $B: \lambda = 3$	M1
	leading to $a = -3,\ b = 5$	A1, A1
		(3)
(b)	$\overrightarrow{AB} = = \pm\left((\text{'}5\mathbf{i}\text{'} - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ \text{'}-3\mathbf{j}\text{'} + 5\mathbf{k})\right); = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1, A1
		(2)
(c)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$ or $\overrightarrow{CA} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$	M1
	$\cos C\hat{A}B = \dfrac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2}\,.\sqrt{(3)^2 + (0)^2 + (-3)^2}}$	dM1
	$\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24}.\sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$	A1
		(3)
(d)	$\text{Area } CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ; = 3\sqrt{3}$	M1, A1
		(2)
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1
	For both of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1
		(4)
		(14 marks)

Question	Scheme	Marks
6(a)	$\left\|\overrightarrow{OA}\right\|^2=(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2$	M1
	$\left\|\overrightarrow{OA}\right\|=5\sqrt{10}\Rightarrow(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2=250$	M1
	$\Rightarrow 64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ $\Rightarrow 81\lambda^2+52\lambda-220=0*$	A1*
		(3)
(b)	$81\lambda^2+52\lambda-220=0\Rightarrow(81\lambda-110)(\lambda+2)=0\Rightarrow\lambda=...\left(-2,\frac{110}{81}\right)$ $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1+8\times\text{"their}\lambda\text{"}\\2-\text{"their}\lambda\text{"}\\5+4\times\text{"their}\lambda\text{"}\end{pmatrix}$	M1
	E.g $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1\\2\\5\end{pmatrix}-2\begin{pmatrix}8\\-1\\4\end{pmatrix}=\begin{pmatrix}-15\\4\\-3\end{pmatrix}*$; or $\overrightarrow{OA}=\begin{pmatrix}1+8\times\frac{110}{81}\\2-\frac{110}{81}\\5+4\times\frac{110}{81}\end{pmatrix}=\begin{pmatrix}\frac{961}{81}\\\frac{52}{81}\\\frac{845}{81}\end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos\theta=\pm\dfrac{\begin{pmatrix}-15\\4\\-3\end{pmatrix}.\begin{pmatrix}8\\-1\\4\end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}}=...$	M1
	$=\pm\dfrac{-136}{45\sqrt{10}}\ (=\pm0.9557...)$ or e.g. $\theta=$ awrt $0.299(rad)/17.1°$	A1
	Area $OAB=\frac{1}{2}5\sqrt{10}\times4\sqrt{10}\sin\theta=...$	M1
	= awrt 29.4	A1
		(4)
		(10 marks)

Question Number	Scheme	Notes	Marks
5(a)	$4+2\lambda=13+5\mu$ $4-3\lambda=-1+\mu$ $-5+6\lambda=4-3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4+2\lambda=13+5\mu$ $4-3\lambda=-1+\mu$ $\Rightarrow \lambda=...$ or $\mu=...$	Full method for finding λ or μ	M1
	$\lambda=2,\ \mu=-1$	Both correct values	A1
	$-5+6\lambda=-5+12=7$ $4-3\mu=4+3=7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda=2 \rightarrow (4+4)\mathbf{i}+(4-6)\mathbf{j}+(-5+12)\mathbf{k}$ **or** $\mu=-1\rightarrow(13-5)\mathbf{i}+(-1-1)\mathbf{j}+(4+3)\mathbf{k}$	Uses their λ or μ to find A.	M1
	$8\mathbf{i}-2\mathbf{j}+7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix}2\\-3\\6\end{pmatrix}\bullet\begin{pmatrix}5\\1\\-3\end{pmatrix}=10-3-18=\sqrt{2^2+3^2+6^2}\sqrt{5^2+1^2+3^2}\cos\theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos\theta=\pm\dfrac{11}{7\sqrt{35}}$	Correct magnitude for cos θ (may be implied by e.g. $\theta=105.4...$ or $74.6...$	A1
	$\theta=74.6°$	Awrt 74.6	A1
			(3)
(c)	$\lvert 2\mathbf{i}-3\mathbf{j}+6\mathbf{k}\rvert=\sqrt{2^2+3^2+6^2}=7$	Finds the magnitude of the direction of l_1	M1
	$35\div7=5\Rightarrow\lambda=5$ $8\mathbf{i}-2\mathbf{j}+7\mathbf{k}\pm5(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18,-17,37)$ or $(-2,13,-23)$	One correct point (ignore labels)	A1
	$P(18,-17,37)$ and $Q(-2,13,-23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question Number	Scheme	Marks
2(a)	$\overrightarrow{BA}.\overrightarrow{BC}=-6\times2+2\times5-3\times8=(-26)$	M1
	Uses $\overrightarrow{BA}.\overrightarrow{BC}=\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta\Rightarrow-26=\sqrt{49}\times\sqrt{93}\cos\theta\Rightarrow\theta=...$	dM1
	$\theta=112.65°$	A1
		(3)
(b)	Attempts to use $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)