

Question Number	Scheme	Marks
7	Assume that $\sqrt{7}$ is rational so that $\sqrt{7} = \frac{a}{b} \Rightarrow 7 = \frac{a^2}{b^2}$ (where a and b have no factors in common)	M1
	$7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2$ So a^2 is a multiple of 7 which means a is a multiple of 7	A1
	$a = 7k \Rightarrow 7b^2 = 49k^2 \Rightarrow b^2 = 7k^2$	M1
	So b^2 is a multiple of 7 which means b is a multiple of 7. As a and b are both multiples of 7, this contradicts the fact that a and b have no factors in common. Hence $\sqrt{7}$ must be irrational.	A1
		(4)
		Total 4

Question Number	Scheme	Marks
7	Assume that $\sqrt{7}$ is rational so that $\sqrt{7} = \frac{a}{b} \Rightarrow 7 = \frac{a^2}{b^2}$ (where a and b have no factors in common)	M1
	$7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2$ So a^2 is a multiple of 7 which means a is a multiple of 7	A1
	$a = 7k \Rightarrow 7b^2 = 49k^2 \Rightarrow b^2 = 7k^2$	M1
	So b^2 is a multiple of 7 which means b is a multiple of 7. As a and b are both multiples of 7, this contradicts the fact that a and b have no factors in common. Hence $\sqrt{7}$ must be irrational.	A1
		(4)
		Total 4

Question	Scheme	Marks
8	$y = 2x + x^3 + \cos x$	
	Assume (there is a stationary point so) $\frac{dy}{dx} = 0$ for some value of x	B1
	So for this x $\frac{dy}{dx} = 2 + 3x^2 - \sin x$	M1
	$\frac{dy}{dx} = 0 \Rightarrow \sin x = 3x^2 + 2 \dots 2$	dM1
	This is a <u>contradiction</u> as $ \sin x \leq 1$ for all x , hence the assumption is false and so the function has <u>no stationary points</u> .	A1cso
		(4)
		(4 marks)

Question Number	Scheme	Marks
4(a)	Assume that there exists a positive number k such that $k + \frac{9}{k} < 6$	B1
	$k + \frac{9}{k} < 6 \Rightarrow k^2 + 9 < 6k \Rightarrow k^2 - 6k + 9 < 0$ or $k + \frac{9}{k} < 6 \Rightarrow k + \frac{9}{k} - 6 < 0 \Rightarrow (\sqrt{k} \dots)(\sqrt{k} \dots) < 0$ or $k + \frac{9}{k} < 6 \Rightarrow \left(k + \frac{9}{k}\right)^2 < 36 \Rightarrow k^2 + 18 + \frac{81}{k^2} - 36 < 0$	M1
	$\Rightarrow (k-3)^2 < 0$ or $\Rightarrow \left(\sqrt{k} - \frac{3}{\sqrt{k}}\right)^2 < 0$ or $\Rightarrow \left(k - \frac{9}{k}\right)^2 < 0$	A1
	But numbers squared are ≥ 0 , hence $k + \frac{9}{k} \geq 6$	A1*
		(4)
(b)	E.g. When $k = -3$, $-3 + \frac{9}{-3} (= -6)$ which is not ≥ 6	B1
		(1)

Question	Scheme	Marks
9	For question 9 many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader. M1: Will be scored for setting up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage. A1: Scored for a correct statement from which it is possible to draw a contradiction. dM1: For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions. A1: Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technicality a correct assumption as long as a relevant assumption has been made. E.g. Accept "Assume $n^2 - 2$ is divisible by 4 for all n"	
9	(Assume that there is an n with $n^2 - 2$ is divisible by 4 so) $n^2 - 2 = 4k$	M1
	then $n^2 = 4k + 2 = 2(2k + 1)$ (so is even)	A1
	Hence n^2 is even so $n (=2m)$ is even hence n^2 is a multiple of 4 As n^2 is a multiple of 4 then $n^2 - 2 = 4m^2 - 2 = 2(2m^2 - 1)$ cannot be a multiple of 4 (as $2m^2 - 1$ is odd) so there is a contradiction.	dM1
	So the original assumption has been shown false. Hence " $n^2 - 2$ is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Question Number	Scheme	Marks
10	Assume there is an angle x, $90^\circ < x < 180^\circ$, $\left \frac{\cos 2x}{\cos x - \sin x} \right \dots 1$ $\frac{\cos 2x}{\cos x - \sin x} \equiv \frac{(\cos x + \sin x) (\cancel{\cos x - \sin x})}{\cancel{\cos x - \sin x}}$ $ \cos x + \sin x \dots 1 \Rightarrow (\cos x + \sin x)^2 \dots 1 \Rightarrow 1 + 2 \sin x \cos x \dots 1 \Rightarrow \sin 2x \dots 0$ This is a contradiction, as $\sin \alpha$ is negative for all angles $180^\circ < \alpha < 360^\circ$, hence true *	B1 M1 dM1 A1*
		(4 marks)

Question Number	Scheme	Marks
10 Main	Assume that $p^2 - 4q + 2 = 0$	B1
	$p^2 = 4q - 2$	M1
	$p^2 = 4q - 2 (= 2(2q - 1))$ So p^2 is even so p is even oe e.g. so $p = 2k$	A1
	If $p = 2k$ then e.g. $4k^2 = 4q - 2$ or e.g. $4k^2 = 4q - 2 \Rightarrow q = k^2 + \frac{1}{2} \text{ or e.g. } q = \frac{2k^2 + 1}{2}$ or e.g. $4k^2 = 4q - 2 \Rightarrow k^2 = q - \frac{1}{2} \text{ or e.g. } k^2 = \frac{2q - 1}{2}$ or e.g. Let $4k^2 = 4q - 2 \Rightarrow 2q - 2k^2 = 1$	dM1
	$4k^2$ is a multiple of 4 and $4q - 2$ isn't a multiple of 4 or e.g. $q = k^2 + \frac{1}{2} \text{ which is not an integer}$ or e.g. $k^2 = q - \frac{1}{2} \text{ which is not an integer}$ or e.g. $2q - 2k^2 = 1 \text{ which is not possible}$ So $p^2 - 4q + 2 \neq 0$	A1
		(5 marks)

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Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4 + 10x^2 + 8 = 2x^2 - 7$ (and has real solutions).	B1
	$x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 5)(x^2 + 3) = 0 \Rightarrow x^2 = \dots$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow x^2 = \frac{-8 \pm \sqrt{8^2 - 4 \times 15}}{2}$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 4)^2 - 1 = 0 \Rightarrow x^2 = \dots$	M1
	$x^2 = -5, -3$	A1
	e.g. The two values of $x^2 < 0$ (so $x^4 + 8x^2 + 15 = 0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Question Number	Scheme	Marks
8	$(3x - y) = 25$ and $(x + y) = 1$ Solves one of or $(3x - y) = 5$ and $(x + y) = 5$	M1
	Correct solution of one. Either $\left. \begin{matrix} 3x - y = 25 \\ x + y = 1 \end{matrix} \right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, (y = -5.5)$	A1
	Or $\left. \begin{matrix} 3x - y = 5 \\ x + y = 5 \end{matrix} \right\} \Rightarrow 4x = 10 \Rightarrow x = 2.5, (y = 2.5)$	
	Solves both equations Both solved correctly with a minimal reason given for the contradiction e.g "not integers" with conclusion "hence there are no integers x and y such that $3x^2 + 2xy - y^2 = 25$ "	dM1 A1 (4)

Question Number	Scheme	Marks
10 (a)	Writes $2(4p^3 + 6p^2 + 3p) + 1$ which is odd	B1
(b)	Assumption: E.g. States that there exists integers p and q such that $\sqrt[3]{2} = \frac{p}{q}$ (where $\frac{p}{q}$ is in its simplest form) and then cubes to get $2 = \frac{p^3}{q^3}$ $2 = \frac{p^3}{q^3} \Rightarrow p^3 = 2q^3$ and concludes that p^3 is even so therefore p is even If p is even then it can be written $p = 2m$ so $(2m)^3 = 2q^3$ States that $q^3 = 4m^3$ and concludes that q^3 is even so therefore q is even This contradicts our initial statement, as if they both have a factor of 2 it means that $\frac{p}{q}$ is not in its simplest form, so $\sqrt[3]{2}$ is irrational *	(1) M1 A1 M1 A1 A1* (5) (6 marks)

Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) "$n^2 - 4n + 5$ is even and n is even"	B1
	Sets e.g. $n = 2k$ and attempts $(2k)^2 - 4(2k) + 5$	M1
	$4k^2 - 8k + 5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4(k^2 - 2k) + 5$ or e.g. $4(k^2 - 2k + 1) + 1$ or e.g. $2(2k^2 - 4k + 2) + 1$ or e.g. $(2k - 2)^2 + 1$ Condone $4k^2 - 8k + 5 = \text{even} - \text{even} + \text{odd} = \text{odd}$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4 + 10x^2 + 8 = 2x^2 - 7$ (and has real solutions).	B1
	$x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 5)(x^2 + 3) = 0 \Rightarrow x^2 = \dots$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow x^2 = \frac{-8 \pm \sqrt{8^2 - 4 \times 15}}{2}$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 4)^2 - 1 = 0 \Rightarrow x^2 = \dots$	M1
	$x^2 = -5, -3$	A1
	e.g. The two values of $x^2 < 0$ (so $x^4 + 8x^2 + 15 = 0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) " $n^2 - 4n + 5$ is even and n is even"	B1
	Sets e.g. $n = 2k$ and attempts $(2k)^2 - 4(2k) + 5$	M1
	$4k^2 - 8k + 5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4(k^2 - 2k) + 5$ or e.g. $4(k^2 - 2k + 1) + 1$ or e.g. $2(2k^2 - 4k + 2) + 1$ or e.g. $(2k - 2)^2 + 1$ Condone $4k^2 - 8k + 5 = \text{even} - \text{even} + \text{odd} = \text{odd}$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Question	Scheme	Marks
8(a)	$(8 - 3x)^{\frac{4}{3}} = 16 \left(1 - \frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1 - kx)^{\frac{4}{3}}, k \neq 3$	M1
	"Correct" term 3 or term 4 in $(1 - kx)^{\frac{4}{3}} = 1 \pm \frac{4}{3}kx + \frac{\frac{4}{3} \times \frac{1}{3}}{2}(\pm kx)^2 + \frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6}(\pm kx)^3 + \dots$	M1
	Two terms correct of $16 - 8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$ following M1, M1 and $k = \pm \frac{3}{8}$	A1
	$= 16 - 8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$	A1
		(4)

(b)	Assume the curves meet so $8x - \frac{15x^2}{2} + 8 = "16 - 8x + \frac{x^2}{2} + \frac{x^3}{24}"$ (for some x)	M1
	$\Rightarrow 0 = 8 - 16x + 8x^2 + \frac{x^3}{24} + \dots \Rightarrow 0 = 8(x^2 - 2x) + 8 + \frac{x^3}{24} + \dots \Rightarrow 8(x-1)^2 \pm \dots$	M1
	$\Rightarrow 0 = 8(x-1)^2 + \frac{x^3}{24}$	A1
	Requires - Argument that $8(x-1)^2 + \frac{x^3}{24} > 0$. E.g. $8(x-1)^2 \geq 0$ and $\frac{x^3}{24} > 0$ (for $0 < x < \frac{8}{3}$) so $8(x-1)^2 + \frac{x^3}{24} > 0$ - Minimal statement (contradiction) and conclusion	A1
		(4)
(8 marks)		

Question Number	Scheme	Marks
1	Assume that there exists a number m such that when m^3 is even, m is odd	B1
	If m is odd then $m = 2p + 1$ (where p is an integer) and $m^3 = (2p + 1)^3 = \dots$	M1
	$= 8p^3 + 12p^2 + 6p + 1$	A1
	$2 \times (4p^3 + 6p^2 + 3p) + 1$ is odd and hence we have a contradiction so if n^3 is even, then n is even.	A1
		(4) (4 marks)

Question Number	Scheme	Marks
10 Main	Assume that $p^2 - 4q + 2 = 0$	B1
	$p^2 = 4q - 2$	M1
	$p^2 = 4q - 2 (= 2(2q - 1))$ So p^2 is even so p is even oe e.g. so $p = 2k$	A1
	If $p = 2k$ then e.g. $4k^2 = 4q - 2$ or e.g. $4k^2 = 4q - 2 \Rightarrow q = k^2 + \frac{1}{2}$ or e.g. $q = \frac{2k^2 + 1}{2}$ or e.g. $4k^2 = 4q - 2 \Rightarrow k^2 = q - \frac{1}{2}$ or e.g. $k^2 = \frac{2q - 1}{2}$ or e.g. Let $4k^2 = 4q - 2 \Rightarrow 2q - 2k^2 = 1$	dM1
	$4k^2$ is a multiple of 4 and $4q - 2$ isn't a multiple of 4 or e.g. $q = k^2 + \frac{1}{2}$ which is not an integer or e.g. $k^2 = q - \frac{1}{2}$ which is not an integer or e.g. $2q - 2k^2 = 1$ which is not possible So $p^2 - 4q + 2 \neq 0$	A1
		(5 marks)

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Question Number	Scheme	Marks
9 (a)	Let $p = 3k + 2$ then $(3k + 2)^3 = 27k^3 + 54k^2 + 36k + 8$	M1
	$= 3 \times (9k^3 + 18k^2 + 12k + 3) - 1$ not a multiple of 3	A1
	So p cannot be of form $3k + 1$ or $3k + 2$, since p^3 is a multiple of 3. Hence p must be a multiple of 3, a contradiction of our assumption, hence for all integers p , when p^3 is a multiple of 3, then p is a multiple of 3	A1
		(3)
(b)	Assumption: there exist (integers) p and q such that $\sqrt[3]{3} = \frac{p}{q}$ (where p and q have no (non-trivial) common factors.)	B1
	Then $\sqrt[3]{3} = \frac{p}{q} \Rightarrow p^3 = 3q^3$	M1
	So p^3 is a multiple of 3 and (so) p is a multiple of 3	A1
	But $p = 3k \Rightarrow 27k^3 = 3q^3 \Rightarrow q^3 = 9k^3$	dM1
	Hence q^3 is a multiple of 3 so q is a multiple of 3, but as p and q have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.*	A1*
		(5)
		(8 marks)

Question	Scheme	Marks
6	Assume the sequence is geometric	B1
	So $(r =) \frac{1+2k}{k} = \frac{3+3k}{1+2k}$	M1
	$\Rightarrow (1+2k)^2 = k(3+3k) \Rightarrow k^2 + k + 1 = 0$	A1
	But $k^2 + k + 1 = \left(k + \frac{1}{2}\right)^2 - \frac{1}{4} + 1 \geq \frac{3}{4} > 0$ (since $\left(k + \frac{1}{2}\right)^2 \geq 0$ for all (real) k)	dM1
	This is a contradiction and hence the original assumption is not true. The sequence is not geometric.	A1
		(5)
(5 marks)		

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dt} = 6t$ o.e.	B1
	Area = $\left(\int \sin t \sin 2t \times 6t \, dt \right) \int \sin t \times 2 \sin t \cos t \times 6t \, dt$	M1
	$= 12 \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$	A1
		(3)
(b)	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - (12) \int_0^{\frac{\pi}{2}} \frac{1}{3} \sin^3 t \, dt$	M1
	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3} \right) \int_0^{\frac{\pi}{2}} \sin t (1 - \cos^2 t) \, dt$	M1
	$= \left[4t \sin^3 t + 4 \cos t - \frac{4}{3} \cos^3 t \right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3} \right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4 + 10x^2 + 8 = 2x^2 - 7$ (and has real solutions).	B1
	$x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 5)(x^2 + 3) = 0 \Rightarrow x^2 = \dots$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow x^2 = \frac{-8 \pm \sqrt{8^2 - 4 \times 15}}{2}$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 4)^2 - 1 = 0 \Rightarrow x^2 = \dots$	M1
	$x^2 = -5, -3$	A1
	e.g. The two values of $x^2 < 0$ (so $x^4 + 8x^2 + 15 = 0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Question Number	Scheme	Marks
3	States the largest odd number and an odd number that is greater E.g. odd number n and $n + 2$	M1
	Fully correct proof including - the assumption: there exists a greatest odd number "n" - a correct statement that their second odd number is greater than their assumed greatest odd number - a minimal conclusion "this is a contradiction, hence proven" You can ignore any spurious information e.g. $n > 0, n + 2 > 0$ etc.	A1*
		(2)
		(2 marks)

Question Number	Scheme	Marks
1	Assume there are positive numbers a and b such that $\frac{9a}{b} + \frac{4b}{a} < 12$	B1
	Multiplies by $ab \Rightarrow 9a^2 + 4b^2 < 12ab$	M1
	$\Rightarrow 9a^2 - 12ab + 4b^2 < 0 \Rightarrow (3a - 2b)^2 < 0$	A1
	Square numbers cannot be negative, so we have a contradiction and hence, if a and b are positive real numbers, then $\frac{9a}{b} + \frac{4b}{a} \geq 12$	A1*
		(4 marks)

Question Number	Scheme	Marks
1	Assume there are positive numbers a and b such that $\frac{9a}{b} + \frac{4b}{a} < 12$	B1
	Multiplies by $ab \Rightarrow 9a^2 + 4b^2 < 12ab$	M1
	$\Rightarrow 9a^2 - 12ab + 4b^2 < 0 \Rightarrow (3a - 2b)^2 < 0$	A1
	Square numbers cannot be negative, so we have a contradiction and hence, if a and b are positive real numbers, then $\frac{9a}{b} + \frac{4b}{a} \geq 12$	A1*
		(4 marks)

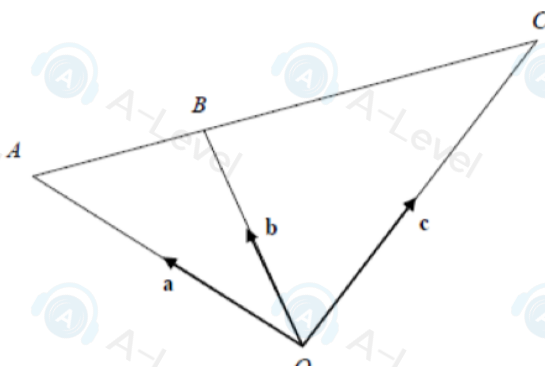
Question Number	Scheme	Notes	Marks
8	Assume that there exist positive real numbers x and y such $\frac{9x}{y} + \frac{y}{x} < 6$	Starts the proof by contradicting the given statement	B1
	$\frac{9x}{y} + \frac{y}{x} < 6 \Rightarrow 9x^2 + y^2 < 6xy$ as x and y are both positive	Multiplies through by xy	M1
	$\Rightarrow 9x^2 + y^2 - 6xy < 0$ $\Rightarrow (3x - y)^2 < 0$	Reaches a correct contradictory statement	A1
	As x and y are positive real numbers, this is a contradiction and so $\frac{9x}{y} + \frac{y}{x} < 6$ must be incorrect and so $\frac{9x}{y} + \frac{y}{x} \dots 6^*$	Makes a suitable conclusion	A1*
			(4)
			Total 4

Question Number	Scheme	Marks
4(a)	Assume that there exists a positive number k such that $k + \frac{9}{k} < 6$	B1
	$k + \frac{9}{k} < 6 \Rightarrow k^2 + 9 < 6k \Rightarrow k^2 - 6k + 9 < 0$ or $k + \frac{9}{k} < 6 \Rightarrow k + \frac{9}{k} - 6 < 0 \Rightarrow (\sqrt{k} \dots)(\sqrt{k} \dots) < 0$ or $k + \frac{9}{k} < 6 \Rightarrow \left(k + \frac{9}{k}\right)^2 < 36 \Rightarrow k^2 + 18 + \frac{81}{k^2} - 36 < 0$	M1
	$\Rightarrow (k-3)^2 < 0$ or $\Rightarrow \left(\sqrt{k} - \frac{3}{\sqrt{k}}\right)^2 < 0$ or $\Rightarrow \left(k - \frac{9}{k}\right)^2 < 0$	A1
	But numbers squared are ≥ 0 , hence $k + \frac{9}{k} \geq 6$	A1*
		(4)
(b)	E.g. When $k = -3$, $-3 + \frac{9}{-3} = -6$ which is not ≥ 6	B1
		(1)

Question Number	Scheme	Marks
9 (a)	Let $p = 3k + 2$ then $(3k + 2)^3 = 27k^3 + 54k^2 + 36k + 8$	M1
	$= 3 \times (9k^3 + 18k^2 + 12k + 3) - 1$ not a multiple of 3	A1
	So p cannot be of form $3k + 1$ or $3k + 2$, since p^3 is a multiple of 3. Hence p must be a multiple of 3, a contradiction of our assumption, hence for all integers p , when p^3 is a multiple of 3, then p is a multiple of 3	A1
		(3)
(b)	Assumption: there exist (integers) p and q such that $\sqrt[3]{3} = \frac{p}{q}$ (where p and q have no (non-trivial) common factors.)	B1
	Then $\sqrt[3]{3} = \frac{p}{q} \Rightarrow p^3 = 3q^3$	M1
	So p^3 is a multiple of 3 and (so) p is a multiple of 3	A1
	But $p = 3k \Rightarrow 27k^3 = 3q^3 \Rightarrow q^3 = 9k^3$	dM1
	Hence q^3 is a multiple of 3 so q is a multiple of 3, but as p and q have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.*	A1*
		(5)
		(8 marks)

Question	Scheme	Marks
8(a)	$(8-3x)^{\frac{4}{3}} = 16\left(1-\frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1-kx)^{\frac{4}{3}}, k \neq 3$	M1
	"Correct" term 3 or term 4 in $(1-kx)^{\frac{4}{3}} = 1 \pm \frac{4}{3}kx + \frac{\frac{4}{3} \times \frac{1}{3}}{2}(\pm kx)^2 + \frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6}(\pm kx)^3 + \dots$	M1
	Two terms correct of $16-8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$ following M1, M1 and $k = \pm \frac{3}{8}$	A1
	$= 16-8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$	A1
		(4)
(b)	Assume the curves meet so $8x - \frac{15x^2}{2} + 8 = "16-8x + \frac{x^2}{2} + \frac{x^3}{24}"$ (for some x)	M1
	$\Rightarrow 0 = 8-16x+8x^2 + \frac{x^3}{24} + \dots \Rightarrow 0 = 8(x^2-2x)+8 + \frac{x^3}{24} + \dots \Rightarrow 8(x-1)^2 \pm \dots + \dots$	M1
	$\Rightarrow 0 = 8(x-1)^2 + \frac{x^3}{24}$	A1
	Requires - Argument that $8(x-1)^2 + \frac{x^3}{24} > 0$. E.g. $8(x-1)^2 \geq 0$ and $\frac{x^3}{24} > 0$ (for $0 < x < \frac{8}{3}$) so $8(x-1)^2 + \frac{x^3}{24} > 0$ - Minimal statement (contradiction) and conclusion	A1
		(4)
(8 marks)		

Question Number	Scheme	Marks
9 (i)	Attempts two of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$ either way around Attempts $\mathbf{c} - \mathbf{b} = 2 \times (\mathbf{b} - \mathbf{a})$ or such as $\mathbf{c} - \mathbf{a} = 3 \times (\mathbf{b} - \mathbf{a})$ $\Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}^*$	M1 dM1 A1 * (3)
(ii)	Assume that there exists a number n that isn't a multiple of 3 yet n^2 is a multiple of 3 If n is not a multiple of 3 then $m = 3p + 1$ or $m = 3p + 2$ ($p \in \mathbb{N}$) giving $m^2 = (3p + 1)^2 = 9p^2 + 6p + 1$ Or $m^2 = (3p + 2)^2 = 9p^2 + 12p + 4 = 3(3p^2 + 4p + 1) + 1$ $(3p + 1)^2 = 9p^2 + 6p + 1 = 3(3p^2 + 2p) + 1$ is one more than a multiple of 3 $(3p + 2)^2 = 9p^2 + 12p + 4$ is not a multiple of 3 as 3 does not divide into 4 (exactly) Hence if n is a multiple of 3 then n^2 is a multiple of 3	B1 M1 M1 A1 A1 (5) (8 marks)



Question	Scheme	Marks
8	$y = 2x + x^3 + \cos x$	
	Assume (there is a stationary point so) $\frac{dy}{dx} = 0$ for some value of x	B1
	So for this x $\frac{dy}{dx} = 2 + 3x^2 - \sin x$	M1
	$\frac{dy}{dx} = 0 \Rightarrow \sin x = 3x^2 + 2 \dots 2$	dM1
	This is a <u>contradiction</u> as $ \sin x \leq 1$ for all x , hence the assumption is false and so the function has <u>no stationary points</u> .	A1cso
		(4)
(4 marks)		

Question Number	Scheme	Marks
10	Assume there is an angle x, $90^\circ < x < 180^\circ$, $\left \frac{\cos 2x}{\cos x - \sin x} \right \dots 1$ $\frac{\cos 2x}{\cos x - \sin x} \equiv \frac{(\cos x + \sin x) \cancel{(\cos x - \sin x)}}{\cancel{\cos x - \sin x}}$ $ \cos x + \sin x \dots 1 \Rightarrow (\cos x + \sin x)^2 \dots 1 \Rightarrow 1 + 2 \sin x \cos x \dots 1 \Rightarrow \sin 2x \dots 0$ This is a contradiction, as $\sin \alpha$ is negative for all angles $180^\circ < \alpha < 360^\circ$, hence true *	B1 M1 dM1 A1*
		(4 marks)