

Question Number	Scheme	Marks
5 (a)	$y^2 = 2x^2 + 15x + 10y \Rightarrow 2y \frac{dy}{dx} = 4x + 15 + 10 \frac{dy}{dx}$	M1 A1
	$(2y - 10) \frac{dy}{dx} = 4x + 15 \Rightarrow \frac{dy}{dx} = \frac{4x + 15}{2y - 10}$ oe	M1, A1
		(4)
(b)	Deduces that $2y - 10 = 0 \Rightarrow y = 5$	B1ft
	Substitutes $y = 5$ into $y^2 = 2x^2 + 15x + 10y \Rightarrow 2x^2 + 15x + 25 = 0$ and solves for x	M1
	$(p =) -5, (q =) -\frac{5}{2}$	A1
		(3)
		(7 marks)



Question Number	Scheme	Marks
4(i)	$\frac{dV}{dt} = 70\pi$	B1
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 70\pi = 4\pi \times 5^2 \times \frac{dr}{dt}$ or e.g. $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \Rightarrow \frac{dr}{dt} = 70\pi \times \frac{1}{4\pi \times 5^2}$	M1
	$\left(\frac{dr}{dt}\right) = 0.7 \text{ (cm s}^{-1}\text{)} \text{ oe e.g. } \frac{7}{10}$	A1
		(4)

(ii)	$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int h^3 dh = \int k dt \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0, h = 4 \Rightarrow \frac{1}{4} \times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5, h = 6 \Rightarrow \frac{1}{4} \times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = \dots$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
5 (a)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{16 \sec^2 t \tan t}{2 \sec^2 t} = 8 \tan t$ At $x = 3, \tan t = -1 \Rightarrow \text{Gradient} = -8$	M1 A1 dM1 A1 (4)
(b)	Attempts to use $1 + \tan^2 t = \sec^2 t \Rightarrow 1 + \frac{(x-5)^2}{4} = \frac{y}{8}$ $y = 2(x-5)^2 + 8$	M1 A1 A1 (3)
(c)	8,, f,, 32	M1 A1 (2) (9 marks)

Question Number	Scheme	Marks
6(a)	$4y^2 + 3x = 6ye^{-2x}$	
	$4y^2 + 3x \rightarrow 8y \frac{dy}{dx} + 3$	B1
	$6ye^{-2x} \rightarrow -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$	M1 A1
	$8y \frac{dy}{dx} + 3 = -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} \text{ oe}$	M1 A1
		(5)
(b)	Sets $x = 0$ in $4y^2 + 3x = 6ye^{-2x} \Rightarrow y = \frac{3}{2} \text{ oe}$	B1
	Substitutes $\left(0, \frac{3}{2}\right)$ in their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} = \left(\frac{7}{-2}\right)$	M1
	$m_N = -1 \div \frac{7}{-2} \Rightarrow y = \frac{2}{7}x + \frac{3}{2}$	dM1
	$y = \frac{2}{7}x + \frac{3}{2} \text{ oe e.g. } y = \frac{6}{21}x + \frac{3}{2}$	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
1	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $3x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	Correct differentiation $2 - 8y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 8x$	A1
	Substitutes $x = 3, y = 2 \Rightarrow \frac{dy}{dx} = -\frac{14}{11}$	M1, A1
	Correct method for normal at $(3, 2) \Rightarrow y - 2 = \frac{11}{14}(x - 3)$ $11x - 14y - 5 = 0$	dM1 A1 (7) (7 marks)

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	dM1A1
		(3)
Alternative 1		
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3}V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	dM1A1
Alternative 2		
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5 \frac{dt}{dx}$	dM1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)

Note this is now being marked as M1dM1A1 not R1M1A1

Question Number	Scheme	Marks
9 (a)	States or uses $V = \pi \times 4^2 \times h \Rightarrow \frac{dV}{dh} = 16\pi$	B1
	States or uses $\frac{dV}{dt} = 0.6\pi - 0.15\pi h$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 0.6\pi - 0.15\pi h = 16\pi \times \frac{dh}{dt}$	M1
	$\frac{dh}{dt} = \frac{12-3h}{320} *$	A1*
(4)		
(b)	$\frac{dh}{dt} = \frac{12-3h}{320} \Rightarrow \int \frac{dh}{12-3h} = \int \frac{1}{320} dt$	
	$-\frac{1}{3} \ln(12-3h) = \frac{1}{320} t + c$	M1 A1
	Uses $t = 0, h = 0.5 \quad -\frac{1}{3} \ln\left(\frac{21}{2}\right) = c \Rightarrow -\frac{1}{3} \ln(12-3h) = \frac{1}{320} t - \frac{1}{3} \ln\left(\frac{21}{2}\right)$	M1 A1
	Substitutes $h = 3.5 \quad -\frac{1}{3} \ln\left(\frac{3}{2}\right) = \frac{1}{320} t - \frac{1}{3} \ln\left(\frac{21}{2}\right) \Rightarrow t = \dots$ 208 minutes cso	dM1 A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2} \quad \text{and} \quad y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6\left(\frac{1}{2}\right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = " \frac{38}{13} " \left(x - \frac{1}{2} \right)$	M1
	$38x - 13y + 33 = 0$	A1
		(6)
		(8 marks)

Question Number	Scheme	Marks
4(a)	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $2xy \rightarrow \alpha x \frac{dy}{dx} + \beta y$	M1
	Correct differentiation $8x + 2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 24$ oe	A1
	Collects terms in $(2y - 2x) \frac{dy}{dx} = 24 - 8x + 2y$	M1
	$\left(\frac{dy}{dx} = \right) \frac{12 - 4x + y}{y - x}$ oe	A1
		(5)
(b)	Sets $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$	M1
	Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$	dM1
	$x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$	A1
	Solves $x = 6, y = 0$ or $x = 2, y = 8$	ddM1
	Chooses $a(x) = 2, b(y) = 8$ or states $(2, 8)$ only	A1
		(5)
		(10 marks)

Question	Scheme	Marks
3(a)	$y^2x + 3y = 4x^2 + k$	
	$y^2x \rightarrow \dots \times y \frac{dy}{dx} (+\dots)$ or $3y \rightarrow 3 \frac{dy}{dx}$	M1
	$y^2x \rightarrow 2xy \frac{dy}{dx} + y^2$	A1
	$\dots + 3y = 4x^2 + k \rightarrow \dots + 3 \frac{dy}{dx} = 8x$	A1
	$2xy \frac{dy}{dx} + y^2 + 3 \frac{dy}{dx} = 8x \Rightarrow \frac{dy}{dx} = \dots$	dM1
	$\frac{dy}{dx} = \frac{8x - y^2}{2xy + 3}$ oe	A1
		(5)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 8x - 4 = 0 \Rightarrow x = \dots$	M1
	$p = \frac{1}{2}$	A1
	$k = 2^2 \times \frac{1}{2} + 3 \times 2 - 4 \times \frac{1}{4} = \dots$	M1
	$k = 7$	A1
		(4)

Question Number	Scheme	Marks
2(a)	States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$	B1
	Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} \Rightarrow 4 = 12x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \dots$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
		(4)
(b)	States or uses $\frac{dV}{dx} = 3x^2$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{1}{3x} = x$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(3)

Question Number	Scheme	Marks
5 (a)	$2y \frac{dy}{dx} = e^{-2x} \frac{dy}{dx} - 2ye^{-2x} - 3$ $(e^{-2x} - 2y) \frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y} *$	B1 M1 A1 A1* (4)
(b)	Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$ Attempts tangent at $(0,0)$ or $(0,1)$ $y = 3x$ or $y = -5x + 1$ Solves $y = 3x$ with $y = -5x + 1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	B1 M1 A1 dM1 A1 (5) (9 marks)

Question Number	Scheme	Marks
8(a)	A and B are where $y = 0$ so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3$ (0 and -3) Substitutes $t = 3$ in $x = 3^2 + 2 \times 3 = 15$ B(15, 0) $A = (3, 0)$	M1 A1* B1 (3)
8(a) ALT	Uses answer $x = 15$: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ B(15, 0) $A = (3, 0)$	M1 A1* B1 (3)
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 9}{2t + 2}$ Substitutes $t = 3$ into $\frac{dy}{dx} = \frac{3t^2 - 9}{2t + 2} \Rightarrow \frac{dy}{dx} = \frac{9}{4}$ Uses their $\frac{9}{4}$ with (15, 0) to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 (4)
(c)	Substitutes $x = t^2 + 2t$, $y = t^3 - 9t$, into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\frac{15}{4}$ Coordinates of X are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$	M1 dM1 A1 dM1 A1 (5) (12 marks)

Question Number	Scheme	Marks
2(a)	$x = 2 \Rightarrow 4 - 8y + y^2 = 13 \Rightarrow (y^2 - 8y - 9 = 0 \Rightarrow) y = \dots$	M1
	$y = 9$	A1
		(2)
(b)	$2^x \rightarrow 2^x \ln 2$	B1
	$-4xy \rightarrow -4x \frac{dy}{dx} - 4y$ OR $y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$2^x \ln 2 - 4x \frac{dy}{dx} - 4y + 2y \frac{dy}{dx} = 0$	A1
	$\frac{dy}{dx} (2y - 4x) = 4y - 2^x \ln 2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{4y - 2^x \ln 2}{2y - 4x}$ or $\frac{dy}{dx} = \frac{2^x \ln 2 - 4y}{4x - 2y}$ or $\frac{dy}{dx} = \frac{\ln 2 e^{x \ln 2} - 4y}{4x - 2y}$	A1
		(5)
(c)	$(2, 9) \rightarrow \frac{dy}{dx} = \frac{4(9) - 2^2 \ln 2}{2(9) - 4(2)}$ $\Rightarrow y - "9" = \frac{36 - 4 \ln 2}{10} (x - 2)$	M1
	$y = 0 \Rightarrow 0 - "9" = \frac{36 - 4 \ln 2}{10} (x - 2) \Rightarrow x = \dots$	dM1
	$x = \frac{4 \ln 2 + 9}{2 \ln 2 - 18}$ o.e. e.g. $x = \frac{-8 \ln 2 - 18}{-4 \ln 2 + 36}$	A1
		(3)
		Total 10

Question	Scheme	Marks
3	$8x^3 - 3y^2 + 2xy = 9$	
	Either $3y^2 \rightarrow ky \frac{dy}{dx}$ or $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	M1
	Both $3y^2 \rightarrow ky \frac{dy}{dx}$ and $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	dM1
	$8x^3 - 3y^2 + 2xy = 9 \rightarrow 24x^2 - 6y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$	A1
	$(2, 5) : 96 - 30 \frac{dy}{dx} + 10 + 4 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots \left(= \frac{53}{13} \right)$	M1
	$\frac{dy}{dx} = \frac{53}{13}$ or $\frac{dx}{dy} = \frac{13}{53}$ o.e.	A1
	Normal: $y - 5 = -\frac{13}{53} (x - 2)$	M1
	$13x + 53y - 291 = 0$	A1
		(7)
		(7 marks)

Question Number	Scheme	Marks
3	$8y \frac{dy}{dx} = xe^{-y^2} \rightarrow \int 8ye^{y^2} dy = \int x dx$	B1
	$4e^{y^2} = \frac{x^2}{2} (+c)$	M1A1
	$4 = \frac{5^2}{2} + c \Rightarrow c = \dots$	dM1
	$y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$	A1
		(5 marks)

B1: Separates the variables correctly with some indication of integration

Question Number	Scheme	Marks
2(a)	$2^x \rightarrow 2^x \ln 2$	B1
	$y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	$4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$(10y + 4x^2) \frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

2(b)	At $x = 0$ $5y^2 = 45 \Rightarrow y = 3$	M1 A1
	$x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10 \times 2^0 \ln 2 - 3 - 8 \times 0 \times 3}{10 \times 3 + 4 \times 0^2} = \frac{10 \ln 2 - 3}{30}$	
		(2)

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = \cos t + 6 \cos t \sin t \quad \frac{dy}{dt} = 3 \cos t - 2 \sin t$	B1B1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3 \cos t - 2 \sin t}{\cos t + 6 \cos t \sin t} = \frac{3 \cos \pi - 2 \sin \pi}{\cos \pi + 6 \cos \pi \sin \pi} = 3 \quad *$	M1A1*
(b)		(4)
	When $t = \pi, x = -3, y = -2$	B1
	$y - "-2" = 3(x - "-3")$	M1
	$y = 3x + 7$	A1
		(3)
(c)	$y = 3x + 7 \Rightarrow 3 \sin t + 2 \cos t = 3(\sin t - 3 \cos^2 t) + 7$ or	M1
	$y = 3(x + 3 \cos^2 t) + 2 \cos t \Rightarrow 3x + 7 = 3x + 9 \cos^2 t + 2 \cos t$	
	$\Rightarrow 9 \cos^2 t + 2 \cos t - 7 = 0 \quad *$	A1*
		(2)
(d)	$\cos t = \frac{7}{9}$	B1
	$y = 3 \times \frac{\sqrt{32}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$	M1A1
		(3)
		(12 marks)

Question	Scheme	Marks
3	$8x^3 - 3y^2 + 2xy = 9$	
	Either $3y^2 \rightarrow ky \frac{dy}{dx}$ or $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	M1
	Both $3y^2 \rightarrow ky \frac{dy}{dx}$ and $2xy \rightarrow \alpha y + \beta x \frac{dy}{dx}$	dM1
	$8x^3 - 3y^2 + 2xy = 9 \rightarrow 24x^2 - 6y \frac{dy}{dx} + 2y + 2x \frac{dy}{dx} = 0$	A1
	$(2, 5) : 96 - 30 \frac{dy}{dx} + 10 + 4 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \dots \left(= \frac{53}{13} \right)$	M1
	$\frac{dy}{dx} = \frac{53}{13}$ or $\frac{dx}{dy} = \frac{13}{53}$ o.e.	A1
	Normal: $y - 5 = "-\frac{13}{53}"(x - 2)$	M1
	$13x + 53y - 291 = 0$	A1
		(7)
		(7 marks)

Question Number	Scheme	Marks
5 (a)	$y^2 = 2x^2 + 15x + 10y \Rightarrow 2y \frac{dy}{dx} = 4x + 15 + 10 \frac{dy}{dx}$	M1 A1
	$(2y - 10) \frac{dy}{dx} = 4x + 15 \Rightarrow \frac{dy}{dx} = \frac{4x + 15}{2y - 10}$ oe	M1, A1
		(4)
(b)	Deduces that $2y - 10 = 0 \Rightarrow y = 5$	B1ft
	Substitutes $y = 5$ into $y^2 = 2x^2 + 15x + 10y \Rightarrow 2x^2 + 15x + 25 = 0$ and solves for x	M1
	$(p =) -5, (q =) -\frac{5}{2}$	A1
		(3)
		(7 marks)

Question Number	Scheme	Marks
9(a)(i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3 \sin^2 t \cos t}{-4 \sin 2t}$	M1A1
	$\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{3 \sin^2 t \cos t}{-8 \sin t \cos t} = -\frac{3}{8} \sin t$ or e.g. $\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{\frac{3}{2} \sin 2t \sin t}{-4 \sin 2t} = -\frac{3}{8} \sin t$	A1
(ii)	$t = \frac{\pi}{6} \Rightarrow x = 1, y = \frac{1}{8}$	B1
	$m = -\frac{3}{8} \sin\left(\frac{\pi}{6}\right) = -\frac{3}{16} \Rightarrow y - \frac{1}{8} = -\frac{3}{16}(x - 1)$	M1
	$3x + 16y - 5 = 0$ *	A1*
		(6)
(b)	$3(2 \cos 2t) + 16(\sin^3 t) - 5 = 0$	M1
	$6(1 - 2 \sin^2 t) + 16 \sin^3 t - 5 = 0$	dM1
	$16 \sin^3 t - 12 \sin^2 t + 1 = 0$	A1
	$(2 \sin t - 1)^2 (4 \sin t + 1) = 0 \Rightarrow \sin t = -\frac{1}{4}$	ddM1
	$\sin t = -\frac{1}{4} \Rightarrow x = 2 \cos 2t = \dots$ and $y = \sin^3 t = \dots$	dddM1
	$Q\left(\frac{7}{4}, -\frac{1}{64}\right)$	A1
		(6)
		(12 marks)

Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2} \quad \text{and} \quad y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6\left(\frac{1}{2}\right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = " \frac{38}{13} " \left(x - \frac{1}{2} \right)$	M1
	$38x - 13y + 33 = 0$	A1
		(6)
		(8 marks)

Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$	B1
	$V = \frac{4}{3} \pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$	B1
	$\pm k = \lambda r^2 \times \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2} \checkmark *$	A1*
		(4 marks)

Question Number	Scheme	Marks
8(a)	A and B are where $y = 0$ so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3$ (0 and -3) Substitutes $t = 3$ in $x = 3^2 + 2 \times 3 = 15$ B(15, 0) $A = (3, 0)$	M1 A1* B1 (3)
8(a) ALT	Uses answer $x = 15$: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ B(15, 0) $A = (3, 0)$	M1 A1* B1 (3)
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 9}{2t + 2}$ Substitutes $t = 3$ into $\frac{dy}{dx} = \frac{3t^2 - 9}{2t + 2} \Rightarrow \frac{dy}{dx} = \frac{9}{4}$ Uses their $\frac{9}{4}$ with (15, 0) to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 (4)
(c)	Substitutes $x = t^2 + 2t$, $y = t^3 - 9t$, into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\frac{15}{4}$ Coordinates of X are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$	M1 dM1 A1 dM1 A1 (5) (12 marks)

Question Number	Scheme	Marks
5(a)	$y^3 - x^2 + 4x^2y = k$	
	$y^3 \rightarrow 3y^2 \frac{dy}{dx}$	M1
	$4x^2y \rightarrow 8xy + 4x^2 \frac{dy}{dx}$	M1
	$y^3 - x^2 + 4x^2y = k \Rightarrow 3y^2 \frac{dy}{dx} - 2x + 8xy + 4x^2 \frac{dy}{dx} = 0$	A1
	$\Rightarrow \left(3y^2 + 4x^2\right) \frac{dy}{dx} = 2x - 8xy$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{2x - 8xy}{3y^2 + 4x^2}$	A1
		(5)
(b)	$\frac{dy}{dx} = -1$ at P	M1
	Uses $\frac{dy}{dx} = \pm 1$ and $y = x$ to set up and solve equation in x , y or ' p ' $\Rightarrow -1 = \frac{2p - 8p^2}{3p^2 + 4p^2} \Rightarrow p = 2$	M1 A1
	e.g. $k = 2^3 - 2^2 + 4 \times 2^3$	ddM1
	$k = 36$	A1
		(5)
		(10 marks)

Question Number	Scheme	Marks
4 (a)	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $2xy \rightarrow \alpha x \frac{dy}{dx} + \beta y$	M1
	Correct differentiation $8x + 2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 24$ oe	A1
	Collects terms in $(2y - 2x) \frac{dy}{dx} = 24 - 8x + 2y$	M1
	$\left(\frac{dy}{dx} = \right) \frac{12 - 4x + y}{y - x}$ oe	A1
		(5)
(b)	Sets $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$	M1
	Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$	dM1
	$x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$	A1
	Solves $x = 6, y = 0$ or $x = 2, y = 8$	ddM1
	Chooses $a(x) = 2, b(y) = 8$ or states $(2, 8)$ only	A1
		(5) (10 marks)

Question Number	Scheme	Marks
3 (a)	Attempts a correct identity $3x^3 + 8x^2 - 3x - 6 \equiv (Ax + B)x(x + 3) + C(x + 3) + Dx$ o.e. Correct method for two constants e.g. $x = 0, x = -3 \Rightarrow C = \dots, D = \dots$ Two correct constants e.g. $C = -2$ and $D = 2$ Correct method to find all four constants (e.g. by comparing coefficients) $A = 3, B = -1, C = -2, D = 2$	M1 dM1 A1 ddM1 A1 (5)
(b)	$g(x) = 3x - 1 - \frac{2}{x} + \frac{2}{x+3}$ $\Rightarrow g'(x) = 3 + \frac{2}{x^2} - \frac{2}{(x+3)^2}$	M1 A1ft (2)
(c)	Explains that (if $x > 0$) $\frac{2}{x^2} > \frac{2}{(x+3)^2}$ so $\frac{2}{x^2} - \frac{2}{(x+3)^2} > 0$ and $\Rightarrow g'(x) > 3$	B1 (1)
3 (a) ALT	Via division Attempts to divide $3x^3 + 8x^2 - 3x - 6$ by $x^2 + 3x$ forming a linear quotient Correct method for two constants implied by quotient of $3x + \dots$ Correct quotient $3x - 1$ Correct method to find all four constants (e.g. uses PF on $\frac{\text{rem}}{x(x+3)}$) $A = 3, B = -1, C = -2, D = 2$	M1 dM1 A1 ddM1 A1 (5)

Question Number	Scheme	Marks
4 (a)	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $2xy \rightarrow \alpha x \frac{dy}{dx} + \beta y$	M1
	Correct differentiation $8x + 2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 24$ oe	A1
	Collects terms in $(2y - 2x) \frac{dy}{dx} = 24 - 8x + 2y$	M1
	$\left(\frac{dy}{dx} = \right) \frac{12 - 4x + y}{y - x}$ oe	A1
		(5)
(b)	Sets $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$	M1
	Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$	dM1
	$x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$	A1
	Solves $x = 6, y = 0$ or $x = 2, y = 8$	ddM1
	Chooses $a(x) = 2, b(y) = 8$ or states $(2, 8)$ only	A1
		(5) (10 marks)

Question	Scheme	Marks
1	E.g $xy^2 \rightarrow (\dots+)2xy \frac{dy}{dx}$ or $x^2y \rightarrow (\dots+)x^2 \frac{dy}{dx}$ but see notes.	M1
	$xy^2 \rightarrow y^2 + kxy \frac{dy}{dx}$ or $x^2y \rightarrow kxy + x^2 \frac{dy}{dx}$	dM1
	$y^2 + 2xy \frac{dy}{dx} = 2xy + x^2 \frac{dy}{dx}$ oe	A1
	$(x^2 - 2xy) \frac{dy}{dx} = y^2 - 2xy \Rightarrow \frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy} \Rightarrow \frac{dy}{dx} \Big _p = \frac{3^2 - 12}{2^2 - 12} = \dots$ or $3^2 + 2 \times 2 \times 3 \frac{dy}{dx} \Big _p = 2 \times 2 \times 3 + 2^2 \frac{dy}{dx} \Big _p \Rightarrow \frac{dy}{dx} \Big _p = \frac{9 - 12}{4 - 12} = \dots$	M1
	Tangent is $y - 3 = \frac{3}{8}(x - 2)$	dM1
	$\Rightarrow 3x - 8y + 18 = 0$	A1
		(6)
		(6 marks)

Question	Scheme	Marks
3(a)	$\frac{dA}{dt} = -0.5$	B1
	$A = \pi x^2 \Rightarrow \frac{dA}{dx} = 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \frac{-0.5}{2\pi x} \quad \left(= \frac{-1}{4\pi x} \right)$	M1
	$\frac{dx}{dt} = -0.011368...$	A1cso
		(4)
(b)	$V = \pi x^2(3x) = 3\pi x^3$	B1
	$\frac{dV}{dx} = 9\pi x^2$	B1ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 9\pi x^2 \times -\frac{1}{4\pi x} \quad (= -2.25x)$	M1
	$\left(\frac{dV}{dt} = \right) -9 \Rightarrow (\text{Rate of decrease} =) 9 \text{ (mm}^3 \text{ s}^{-1})$	A1
		(4)
		(8 marks)

Question	Scheme	Marks
4(a)	$(l =) \sqrt{25 + r^2}$	B1
		(1)
(b)	$(S = \pi r^2 +) \pi r \sqrt{25 + r^2} \Rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \sqrt{25 + r^2} + \pi r \cdot \frac{1}{2} (25 + r^2)^{-\frac{1}{2}} \cdot 2r$	M1
	Or $(S = \pi r^2 +) \pi \sqrt{25r^2 + r^4} \rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \cdot \frac{1}{2} (25r^2 + r^4)^{-\frac{1}{2}} \times (50r + 4r^3)$	A1
	$(S = \pi(l^2 - 25)) + \pi l \sqrt{l^2 - 25} \Rightarrow \left(\frac{dS}{dl} = 2\pi l + \right) \pi \sqrt{l^2 - 25} + \pi l \cdot \frac{1}{2} (l^2 - 25)^{-\frac{1}{2}} \cdot 2l$	(M1 A1)
	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = .. \times 3$	M1
	$= 81.5 \text{ (cm}^2 \text{ / min)}$	A1
		(4)
		(5 marks)

Question Number	Scheme	Marks
6 (a)	$y = x^{\sin x} \Rightarrow \ln y = \sin x \ln x$ Differentiates $\frac{1}{y} \frac{dy}{dx} = \frac{\sin x}{x} + \ln x \cos x$ $\frac{dy}{dx} = \frac{y \sin x}{x} + y \ln x \cos x$ oe	B1 M1 M1 A1 A1 (5)
(b)	Puts $\frac{dy}{dx} = 0 \Rightarrow \frac{\sin x}{x} + \ln x \cos x = 0$ $\frac{\sin x}{\cos x} + x \ln x = 0 \Rightarrow \tan x + x \ln x = 0^*$	M1 A1* (2) (7 marks)

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	dM1A1
		(3)
Alternative 1		
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3} V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	dM1A1
Alternative 2		
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5 \frac{dt}{dx}$	dM1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)

Note this is now being marked as M1dM1A1 not B1M1A1

Question Number	Scheme	Marks
6 (a)	For correct parameter $t = \frac{\pi}{4}$ or x coordinate at P $x = 4$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4 \sin 2t}{3 \sec^2 t}$ Equation of tangent $y - 0 = -\frac{2}{3}(x - 4) \Rightarrow y = -\frac{2}{3}x + \frac{8}{3}$	B1 M1 A1 dM1 A1 (5)
(b)	$k = 1 - \sqrt{3}$	B1 (1)
(c)	-1,, f,, 2	M1 A1 (2)
		(8 marks)

Question Number	Scheme	Marks
5.	One of $\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2$ Two of $\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2$ All three of $\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2$	B1 B1 B1
Main	Full attempt to find $\frac{dV}{dt}$ E.g. $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times \frac{dA}{dt} = 18\pi x^2 \times \frac{1}{2\pi x} \times \frac{\pi}{20}$ Finds $\frac{dV}{dt}$ at $x = 2 \Rightarrow \frac{dV}{dt} = 18\pi 2^2 \times \frac{1}{80} = \frac{9}{10}\pi$	M1 dM1A1 (6 marks)
	Note that the M1 may be found in two steps: E.g. I Finds $\frac{dx}{dt}$ from $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ followed by $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ E.g. II Finds $\frac{dV}{dA}$ from $\frac{dV}{dx} = \frac{dV}{dA} \times \frac{dA}{dx}$ followed by $\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$	
Alt	$\frac{dA}{dt} = \frac{\pi}{20}$ $V = 6\pi x^3, A = \pi x^2 \Rightarrow V = 6\pi \left(\frac{A}{\pi}\right)^{\frac{3}{2}} \quad \frac{dV}{dA} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$ $\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} \Rightarrow \frac{dV}{dt} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}} \times \frac{\pi}{20}$ $x = 2 \Rightarrow A = 4\pi \quad \Rightarrow \frac{dV}{dt} \Big _{A=4\pi} = \frac{9}{10}\pi$	B1 B1, B1 M1 dM1A1 (6 marks)

Question Number	Scheme	Marks
5(a)	$y^3 - x^2 + 4x^2y = k$	
	$y^3 \rightarrow 3y^2 \frac{dy}{dx}$	M1
	$4x^2y \rightarrow 8xy + 4x^2 \frac{dy}{dx}$	M1
	$y^3 - x^2 + 4x^2y = k \Rightarrow 3y^2 \frac{dy}{dx} - 2x + 8xy + 4x^2 \frac{dy}{dx} = 0$	A1
	$\Rightarrow \left(3y^2 + 4x^2\right) \frac{dy}{dx} = 2x - 8xy$	M1
	$\Rightarrow \frac{dy}{dx} = \frac{2x - 8xy}{3y^2 + 4x^2}$	A1
		(5)
(b)	$\frac{dy}{dx} = -1$ at P	M1
	Uses $\frac{dy}{dx} = \pm 1$ and $y = x$ to set up and solve equation in x , y or ' p ' $\Rightarrow -1 = \frac{2p - 8p^2}{3p^2 + 4p^2} \Rightarrow p = 2$	M1 A1
	e.g. $k = 2^3 - 2^2 + 4 \times 2^3$	ddM1
	$k = 36$	A1
		(5)
		(10 marks)

Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h} = \frac{12}{30} \Rightarrow r = \frac{12}{30}h \Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2}{5}h\right)^2 h$	M1
	$V = \frac{4\pi h^3}{75} *$	A1*
		(2)
(b)	$V = \frac{4\pi h^3}{75} \Rightarrow \frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$	B1
	$V = 1.5 \times 60 \times 2\pi$ $180\pi = \frac{4\pi h^3}{75} \Rightarrow h^3 = 3375 \Rightarrow h = \dots(15)$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{75}{12\pi \times 15^2} \times 2\pi = \dots$	M1
	$\frac{1}{18} \text{ (cm s}^{-1}\text{)}$	A1
		(4)
		Total 6

Question	Scheme	Marks
4(a)	$(l =) \sqrt{25 + r^2}$	B1
		(1)
(b)	$(S = \pi r^2 +) \pi r \sqrt{25 + r^2} \Rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \sqrt{25 + r^2} + \pi r \cdot \frac{1}{2} (25 + r^2)^{-\frac{1}{2}} \cdot 2r$ Or $(S = \pi r^2 +) \pi \sqrt{25r^2 + r^4} \rightarrow \left(\frac{dS}{dr} = 2\pi r + \right) \pi \cdot \frac{1}{2} (25r^2 + r^4)^{-\frac{1}{2}} \times (50r + 4r^3)$	M1 A1
	$(S = \pi(l^2 - 25)) + \pi l \sqrt{l^2 - 25} \Rightarrow \left(\frac{dS}{dl} = 2\pi l + \right) \pi \sqrt{l^2 - 25} + \pi l \cdot \frac{1}{2} (l^2 - 25)^{-\frac{1}{2}} \cdot 2l$	(M1 A1)
	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = \dots \times 3$	M1
	$= 81.5 \text{ (cm}^2 / \text{min)}$	A1
		(4)
(5 marks)		

Question Number	Scheme	Marks
4 (a)	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$ Attempt at product rule $2xy \rightarrow \alpha x \frac{dy}{dx} + \beta y$ Correct differentiation $8x + 2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 24$ oe Collects terms in $(2y - 2x) \frac{dy}{dx} = 24 - 8x + 2y$ $\left(\frac{dy}{dx} = \right) \frac{12 - 4x + y}{y - x}$ oe	M1 M1 A1 M1 A1 (5)
(b)	Sets $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$ $x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$ Solves $x = 6, y = 0$ or $x = 2, y = 8$ Chooses $a(x) = 2, b(y) = 8$ or states $(2, 8)$ only	M1 dM1 A1 ddM1 A1 (5) (10 marks)

Question Number	Scheme	Marks
4(a)	A and B are where $t^3 - 4t = 0 \Rightarrow t(t^2 - 4) = 0 \Rightarrow t = 2$ or -2 Substitutes $t = 2, x = 2 \times 4 - 6 \times 2 = -4$ Hence $A = (-4, 0)$ When $t = -2, x = 2 \times 4 - 6 \times -2 = 20, (y = 0)$ Hence $B = (20, 0)$ *	M1 A1 B1* (3)
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 4}{4t - 6}$ Sub $t = -2$ into $\frac{dy}{dx} = \frac{3t^2 - 4}{4t - 6} \Rightarrow \text{gradient} = \left(-\frac{4}{7}\right)$ Uses their $\left(-\frac{4}{7}\right)$ and $(20, 0)$ to produce eqn of tangent $\Rightarrow 7y + 4x - 80 = 0$ *	M1A1 M1 M1 A1* (5)
(c)	Substitutes $x = 2t^2 - 6t, y = t^3 - 4t$, into $7y + 4x - 80 = 0$ $\Rightarrow 7(t^3 - 4t) + 4(2t^2 - 6t) - 80 = 0$ $\Rightarrow 7t^3 + 8t^2 - 52t - 80 = 0$ $\Rightarrow (t + 2)^2(7t - 20) = 0$ $t = -\frac{20}{7} \Rightarrow x = \dots$ $x = -\frac{40}{49}$	M1 A1 dM1 A1 (4) (12 marks)

Question	Scheme	Marks
4(a)	$A = \frac{1}{2}5^2(\theta - \sin \theta) \Rightarrow \frac{dA}{d\theta} = \frac{25}{2}(1 \pm \cos \theta)$	M1
	$\frac{dA}{d\theta} = \frac{25}{2}(1 - \cos \theta)$	A1
		(2)
(b)	$\frac{d\theta}{dt} = 0.1$	B1
	$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt} \text{ (oe)}$	B1
	$\frac{dA}{dt} = \frac{25}{2} \left(1 - \cos \frac{\pi}{3}\right) \times 0.1$	M1
	$= \frac{5}{8} \text{ (cm}^2 \text{ per second)}$	A1
		(4)
		(6 marks)

Question	Scheme	Marks
4(a)	$A = \frac{1}{2}5^2(\theta - \sin \theta) \Rightarrow \frac{dA}{d\theta} = \frac{25}{2}(1 \pm \cos \theta)$	M1
	$\frac{dA}{d\theta} = \frac{25}{2}(1 - \cos \theta)$	A1
		(2)
(b)	$\frac{d\theta}{dt} = 0.1$	B1
	$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt} \text{ (oe)}$	B1
	$\frac{dA}{dt} = \frac{25}{2} \left(1 - \cos \frac{\pi}{3}\right) \times 0.1$	M1
	$= \frac{5}{8} \text{ (cm}^2 \text{ per second)}$	A1
		(4)
(6 marks)		

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question	Scheme	Marks
4(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \rightarrow (8 \times) 3y^2 \frac{dy}{dx}$	B1
	$-9kx^2y \rightarrow \dots kxy \pm \dots - 9kx^2 \frac{dy}{dx}$	M1
	$48x^2 - 18kxy - 9kx^2 \frac{dy}{dx} + 24y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} (24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2} \quad *$	A1*
		(4)
(b)	$\frac{dy}{dx} = 0, x = \frac{5}{2} \Rightarrow \frac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0 \quad \text{or}$	M1
	$x = \frac{5}{2} \Rightarrow 16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2 y + 8y^3 = 875$	
	$15ky - 100 = 0 \quad \text{or} \quad 250 - \frac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2 \left(\frac{20}{3k}\right) + 8\left(\frac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = \dots \left(= \frac{64}{27} \right) \Rightarrow k = \dots$	M1
	$k = \frac{4}{3}$	A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
3 (a)	Attempts to find $\frac{1/3 \times 20^2 \times 24}{160} = 20$ seconds	M1 A1
		(2)
(b)	Attempts $\frac{dV}{dh} = h^2 + \frac{8}{3}h$	M1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$	M1 A1
	Substitutes $h = 5 \Rightarrow 160 = \left(5^2 + \frac{40}{3}\right) \times \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{96}{23} (4.2) \text{ cm s}^{-1}$	dM1 A1
		(5)
		(7 marks)

Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h} = \frac{12}{30} \Rightarrow r = \frac{12}{30}h \Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2}{5}h\right)^2 h$	M1
	$V = \frac{4\pi h^3}{75}^*$	A1*
		(2)
(b)	$V = \frac{4\pi h^3}{75} \Rightarrow \frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$	B1
	$V = 1.5 \times 60 \times 2\pi$	M1
	$180\pi = \frac{4\pi h^3}{75} \Rightarrow h^3 = 3375 \Rightarrow h = \dots(15)$	
	$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{75}{12\pi \times 15^2} \times 2\pi = \dots$	M1
	$\frac{1}{18} \text{ (cm s}^{-1}\text{)}$	A1
		(4)
		Total 6

Question Number	Scheme	Notes	Marks
3(a)	$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$ $\Rightarrow 6y \frac{dy}{dx} - 22x + 11x \frac{dy}{dx} + 11y = 20 \frac{dy}{dx} - 36$		M1M1A1
	M1: $y^2 \rightarrow Ay \frac{dy}{dx}$		
	M1: $11xy \rightarrow px \frac{dy}{dx} + qy$		M1
	A1: All correct		
	$(6y + 11x - 20) \frac{dy}{dx} = 22x - 11y - 36 \Rightarrow \frac{dy}{dx} = \dots$ Collects terms in $\frac{dy}{dx}$ (must be 3 and from the appropriate terms) and makes $\frac{dy}{dx}$ the subject		
(b)	$\frac{dy}{dx} = \frac{22x - 11y - 36}{6y + 11x - 20}$	Correct expression or correct equivalent	A1
			(5)
	$x = 4 \Rightarrow 3y^2 - 176 + 44y = 20y - 144 + 28$	Substitutes $x = 4$ into C to obtain a 3TQ in y	M1
	$3y^2 + 24y - 60 = 0 \Rightarrow y = \dots$	Solves for y	M1
	$y = -10 \text{ (}, 2\text{)}$	Correct value	A1
	$(4, -10) \rightarrow \frac{dy}{dx} = \frac{88 + 110 - 36}{-60 + 44 - 20}$	Substitutes $x = 4$ and their negative y into their $\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = -\frac{9}{2}$	Correct value	A1
			(5)
			Total 10

Question Number	Scheme	Marks
5.	One of $\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2$	B1
	Two of $\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2$	B1
Main	All three of $\frac{dA}{dt} = \frac{\pi}{20}, \frac{dA}{dx} = 2\pi x, \frac{dV}{dx} = 18\pi x^2$	B1
	Full attempt to find $\frac{dV}{dt}$	M1
	E.g. $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times \frac{dA}{dt} = 18\pi x^2 \times \frac{1}{2\pi x} \times \frac{\pi}{20}$	
	Finds $\frac{dV}{dt}$ at $x = 2 \Rightarrow \frac{dV}{dt} = 18\pi 2^2 \times \frac{1}{80} = \frac{9}{10}\pi$	dM1A1
	Note that the M1 may be found in two steps: E.g. I Finds $\frac{dx}{dt}$ from $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ followed by $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ E.g. II Finds $\frac{dV}{dA}$ from $\frac{dV}{dx} = \frac{dV}{dA} \times \frac{dA}{dx}$ followed by $\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$	(6 marks)
Alt	$\frac{dA}{dt} = \frac{\pi}{20}$ $V = 6\pi x^3, A = \pi x^2 \Rightarrow V = 6\pi \left(\frac{A}{\pi}\right)^{\frac{3}{2}} \frac{dV}{dA} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$ $\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} \Rightarrow \frac{dV}{dt} = 9\left(\frac{A}{\pi}\right)^{\frac{1}{2}} \times \frac{\pi}{20}$ $x = 2 \Rightarrow A = 4\pi \Rightarrow \frac{dV}{dt} \Big _{A=4\pi} = \frac{9}{10}\pi$	B1 B1, B1 M1 dM1A1 (6 marks)

Question Number	Scheme	Marks
5(a)	e.g. $\frac{r}{h} = \frac{12}{30} \Rightarrow r = \frac{12}{30}h \Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2}{5}h\right)^2 h$	M1
	$V = \frac{4\pi h^3}{75}^*$	A1*
		(2)
(b)	$V = \frac{4\pi h^3}{75} \Rightarrow \frac{dV}{dh} = \frac{12\pi h^2}{75} \left(= \frac{4\pi h^2}{25} \right)$	B1
	$V = 1.5 \times 60 \times 2\pi$ $180\pi = \frac{4\pi h^3}{75} \Rightarrow h^3 = 3375 \Rightarrow h = \dots(15)$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{75}{12\pi \times 15^2} \times 2\pi = \dots$	M1
	$\frac{1}{18} \text{ (cm s}^{-1}\text{)}$	A1
		(4)
		Total 6

Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$ $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$ $\pm k = \lambda r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2} \checkmark *$	B1 B1 M1 A1*
		(4 marks)

Question Number	Scheme	Marks
2(a)	States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$	B1
	Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} \Rightarrow 4 = 12x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \dots$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
		(4)

(b)	States or uses $\frac{dV}{dx} = 3x^2$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{1}{3x} = x$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(3)

Question Number	Scheme	Marks
3	Differentiates wrt x $3^x \ln 3 + 6 \frac{dy}{dx} = \frac{3}{2} y^2 + 3xy \frac{dy}{dx}$ Substitutes (2, 3) AND rearranges to get $\frac{dy}{dx}$ or vice versa $\Rightarrow 9 \ln 3 + 6 \frac{dy}{dx} = \frac{27}{2} + 18 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{9 \ln 3 - \frac{27}{2}}{12} = \frac{6 \ln 3 - 9}{8} = \frac{-9 + \ln 729}{8}$	B1 <u>B1</u> , <u>M1</u> , A1 M1 A1, A1 (7) (7 marks)

Question Number	Scheme	Marks
2(a)	$2^x \rightarrow 2^x \ln 2$	B1
	$y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	$4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$(10y + 4x^2) \frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

2(b)	At $x = 0$ $5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10 \times 2^0 \ln 2 - 3 - 8 \times 0 \times 3}{10 \times 3 + 4 \times 0^2} = \frac{10 \ln 2 - 3}{30}$	M1 A1
		(2)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15\right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9\right)$ $= \frac{2}{3} \ln\left(\frac{6}{5}\right) = \frac{1}{3} \ln\left(\frac{36}{25}\right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question	Scheme	Marks
9(a)	$\frac{d}{dy}(1+2\ln y)^{-2} = \alpha(1+2\ln y)^{-3} \times \frac{\beta}{y} = \frac{K}{y(1+2\ln y)^3}$	M1
	$= \frac{-4}{y(1+2\ln y)^3}$ oe	A1
		(2)
(b)	$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3$ $\Rightarrow \dots dy = \int k \sin(2x) dx$ or $\int \frac{k}{y(1+2\ln y)^3} dy = \int \dots dx$ and $\Rightarrow \dots dy = \int k \sin(2x) dx \Rightarrow \dots = A \cos 2x$ oe or $\Rightarrow \int \frac{k}{y(1+2\ln y)^3} dy = \int \dots dx \Rightarrow \frac{A}{(1+2\ln y)^2} = \dots$	M1
	$\Rightarrow \int \frac{A}{y(1+2\ln y)^3} dy = \int B \sin(2x) dx \Rightarrow \frac{C}{(1+2\ln y)^2} = D \cos 2x$ oe	M1
	One side integrated <u>correctly</u> $\int \frac{3}{y(1+2\ln y)^3} dy = -\frac{3}{4(1+2\ln y)^2}, \int \frac{1}{y(1+2\ln y)^3} dy = -\frac{1}{4(1+2\ln y)^2}$ or $\int \frac{1}{3} \sin(2x) dx = -\frac{1}{6} \cos(2x)$ oe, $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$ oe	A1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x)(+c)$ or $-\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6} \cos(2x)(+c)$	A1
		(4)
(c)	$y=1, x=\frac{\pi}{6} \Rightarrow -\frac{3}{4(1+2\ln 1)} = -\frac{1}{2} \cos\left(\frac{\pi}{3}\right) + c \Rightarrow c = \dots$	M1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x) - \frac{1}{2}$ or $-\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6} \cos(2x) - \frac{1}{6}$	A1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x) - \frac{1}{2} \Rightarrow \frac{3}{4(1+2\ln y)^2} = \frac{1}{2} + \frac{1}{2} \cos(2x) = \cos^2 x$	M1
	$\Rightarrow (1+2\ln y)^2 = \frac{3}{4} \sec^2 x \Rightarrow \ln y = \frac{1}{2} \left(\frac{\sqrt{3}}{2} \sec x - 1 \right)$ Depends on both previous method marks.	ddM1
	$\Rightarrow y = e^{\frac{\sqrt{3}}{4} \sec x - \frac{1}{2}}$	A1
		(5)

Question	Scheme	Marks
4(a)	Area of one face = $\frac{1}{2}x \times x \times \sin 60^\circ = \frac{1}{2}x^2 \frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20 \times \frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
	(2)	
(b)	$\frac{dA}{dx} = 10\sqrt{3}x$ and $\frac{dV}{dx} = \frac{15}{12}(3+\sqrt{5})x^2$	M1
	$\frac{dV}{dA} = \frac{dV}{dx} \times \frac{dx}{dA} = \frac{dV}{dx} \div \frac{dA}{dx} = \dots$ (oe)	M1
	$= \frac{15(3+\sqrt{5})x^2}{12 \times 10\sqrt{3}x} = \frac{(3+\sqrt{5})x}{8\sqrt{3}}$ *	A1*
	(3)	
(c)	$\frac{dA}{dt} = 0.025$	B1
	$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt} = \frac{(3+\sqrt{5})x}{8\sqrt{3}} \times 0.025 = \dots$	M1
	When $x = 2$, $\frac{dV}{dt} = \frac{2(3+\sqrt{5})}{8\sqrt{3}} \times 0.025 = \dots$	
	awrt 0.019 (cm ³ s ⁻¹)	A1
	(3)	
		(8 marks)

Question	Scheme	Marks
3(a)	$y^2x + 3y = 4x^2 + k$	
	$y^2x \rightarrow \dots \times y \frac{dy}{dx} (+\dots)$ or $3y \rightarrow 3 \frac{dy}{dx}$	M1
	$y^2x \rightarrow 2xy \frac{dy}{dx} + y^2$	A1
	$\dots + 3y = 4x^2 + k \rightarrow \dots + 3 \frac{dy}{dx} = 8x$	A1
	$2xy \frac{dy}{dx} + y^2 + 3 \frac{dy}{dx} = 8x \Rightarrow \frac{dy}{dx} = \dots$	dM1
	$\frac{dy}{dx} = \frac{8x - y^2}{2xy + 3}$ oe	A1
	(5)	
(b)	$\frac{dy}{dx} = 0 \Rightarrow 8x - 4 = 0 \Rightarrow x = \dots$	M1
	$p = \frac{1}{2}$	A1
	$k = 2^2 \times \frac{1}{2} + 3 \times 2 - 4 \times \frac{1}{4} = \dots$	M1
	$k = 7$	A1
	(4)	



Question Number	Scheme	Marks
4(i)	$\frac{dV}{dt} = 70\pi$	B1
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 70\pi = 4\pi \times 5^2 \times \frac{dr}{dt}$ or e.g. $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \Rightarrow \frac{dr}{dt} = 70\pi \times \frac{1}{4\pi \times 5^2}$	M1
	$\left(\frac{dr}{dt}\right) = 0.7 \text{ (cm s}^{-1}\text{) oe e.g. } \frac{7}{10}$	A1
		(4)

(ii)	$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int h^3 dh = \int k dt \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0, h = 4 \Rightarrow \frac{1}{4} \times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5, h = 6 \Rightarrow \frac{1}{4} \times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = \dots$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
9(a)(i)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3 \sin^2 t \cos t}{-4 \sin 2t}$	M1A1
	$\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{3 \sin^2 t \cos t}{-8 \sin t \cos t} = -\frac{3}{8} \sin t$ or e.g. $\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{\frac{3}{2} \sin 2t \sin t}{-4 \sin 2t} = -\frac{3}{8} \sin t$	A1
(ii)	$t = \frac{\pi}{6} \Rightarrow x = 1, y = \frac{1}{8}$	B1
	$m = -\frac{3}{8} \sin\left(\frac{\pi}{6}\right) = -\frac{3}{16} \Rightarrow y - \frac{1}{8} = -\frac{3}{16}(x - 1)$	M1
	$3x + 16y - 5 = 0$ *	A1*
		(6)
(b)	$3(2 \cos 2t) + 16(\sin^3 t) - 5 = 0$	M1
	$6(1 - 2 \sin^2 t) + 16 \sin^3 t - 5 = 0$	dM1
	$16 \sin^3 t - 12 \sin^2 t + 1 = 0$	A1
	$(2 \sin t - 1)^2 (4 \sin t + 1) = 0 \Rightarrow \sin t = -\frac{1}{4}$	ddM1
	$\sin t = -\frac{1}{4} \Rightarrow x = 2 \cos 2t = \dots$ and $y = \sin^3 t = \dots$	dddM1
	$\mathcal{Q}\left(\frac{7}{4}, -\frac{1}{64}\right)$	A1
		(6)
		(12 marks)

Question Number	Scheme	Marks
3	$8y \frac{dy}{dx} = x e^{-y^2} \rightarrow \int 8y e^{y^2} dy = \int x dx$	B1
	$4e^{y^2} = \frac{x^2}{2} (+c)$	M1A1
	$4 = \frac{5^2}{2} + c \Rightarrow c = \dots$	dM1
	$y^2 = \ln\left(\frac{x^2 - 17}{8}\right)$	A1
		(5 marks)

B1: Separates the variables correctly with some indication of integration

Question Number	Scheme	Marks
11 (a)	$(x+y)^3 + 10y^2 = 108x$ Differentiates $10y^2$ to $20y \frac{dy}{dx}$ Differentiates $(x+y)^3$ to $3(x+y)^2 \left(1 + \frac{dy}{dx}\right)$ $3(x+y)^2 \left(1 + \frac{dy}{dx}\right) + 20y \frac{dy}{dx} = 108$ $3(x+y)^2 \frac{dy}{dx} + 20y \frac{dy}{dx} = 108 - 3(x+y)^2$ $\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2} *$	B1 M1 A1 dM1 A1* (5)
(b)	Deduces that P and Q are where $108 - 3(x+y)^2 = 0 \Rightarrow (x+y)^2 = 36$ Attempts to substitute $(x+y) = \pm 6$ into $(x+y)^3 + 10y^2 = 108x$ to form an equation in y (or x) Solves $216 + 10y^2 = 108(6-y)$ and finds the negative root Awrt 13900 metres	M1 dM1 ddM1 A1 (4) (9 marks)

Question Number	Scheme	Marks
2(a)	$x = 2 \Rightarrow 4 - 8y + y^2 = 13 \Rightarrow (y^2 - 8y - 9 = 0 \Rightarrow) y = \dots$ $y = 9$	M1 A1 (2)
(b)	$2^x \rightarrow 2^x \ln 2$ $-4xy \rightarrow -4x \frac{dy}{dx} - 4y$ OR $y^2 \rightarrow 2y \frac{dy}{dx}$ $2^x \ln 2 - 4x \frac{dy}{dx} - 4y + 2y \frac{dy}{dx} = 0$ $\frac{dy}{dx} (2y - 4x) = 4y - 2^x \ln 2 \Rightarrow \frac{dy}{dx} = \dots$ $\frac{dy}{dx} = \frac{4y - 2^x \ln 2}{2y - 4x}$ or $\frac{dy}{dx} = \frac{2^x \ln 2 - 4y}{4x - 2y}$ or $\frac{dy}{dx} = \frac{\ln 2 e^{x \ln 2} - 4y}{4x - 2y}$	B1 M1 A1 M1 A1 (5)
(c)	$(2, 9) \rightarrow \frac{dy}{dx} = \frac{4(9) - 2^2 \ln 2}{2(9) - 4(2)}$ $\Rightarrow y - "9" = \frac{36 - 4 \ln 2}{10} (x - 2)$ $y = 0 \Rightarrow 0 - "9" = \frac{36 - 4 \ln 2}{10} (x - 2) \Rightarrow x = \dots$ $x = \frac{4 \ln 2 + 9}{2 \ln 2 - 18}$ o.e. e.g. $x = \frac{-8 \ln 2 - 18}{-4 \ln 2 + 36}$	M1 dM1 A1 (3)
		Total 10

Question Number	Scheme	Marks
3	Differentiates wrt x $3^x \ln 3 + 6 \frac{dy}{dx} = \frac{3}{2} y^2 + 3xy \frac{dy}{dx}$ Substitutes (2, 3) AND rearranges to get $\frac{dy}{dx}$ or vice versa $\Rightarrow 9 \ln 3 + 6 \frac{dy}{dx} = \frac{27}{2} + 18 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{9 \ln 3 - \frac{27}{2}}{12} = \frac{6 \ln 3 - 9}{8} = \frac{-9 + \ln 729}{8}$	B1 <u>B1</u>, <u>M1</u>, A1 M1 A1, A1 (7) (7 marks)