

Question Number	Scheme	Marks
1	Volume = $\int_0^8 \frac{16\pi}{(x+2)^2} (dx)$ oe e.g. $\pi \int_0^8 \left(\frac{4}{x+2}\right)^2 (dx)$	B1
	$\int \frac{16}{(x+2)^2} dx = \frac{-16}{(x+2)}$ or e.g. $u = x+2 \Rightarrow \int \frac{16}{(x+2)^2} dx = \int \frac{16}{u^2} du = \frac{-16}{u}$	M1 A1
	$\left[\frac{-16\pi}{(x+2)} \right]_0^8 = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2} \right)$ or e.g. $\left[\frac{-16\pi}{u} \right]_2^{10} = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2} \right)$	M1
	$= \frac{32}{5} \pi$	A1
		(5)
		(5 marks)

Question	Scheme	Marks
7(a)	$V = (\pi) \int_{(0)}^{(4)} \left(\frac{1}{4} (4-x)e^x \right)^2 dx$	M1
	$\int (f(x))^2 dx = \frac{1}{16} \int (4-x)^2 e^{2x} dx = \dots$	M1
	$V = \frac{\pi}{16} \int_0^4 (x^2 - 8x + 16) e^{2x} dx$	A1
		(3)
(b)	$= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - \int (2x - 8) \times \frac{e^{2x}}{2} dx \right)$	M1 A1
	$= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - \left[(2x - 8) \frac{e^{2x}}{4} - \int 2 \times \frac{e^{2x}}{4} dx \right] \right)$ $= \left(\frac{\pi}{16} \right) \left((x^2 - 8x + 16) \frac{e^{2x}}{2} - (2x - 8) \frac{e^{2x}}{4} + \frac{e^{2x}}{4} \right)$ Depends on the first method mark.	dM1
	$V = \frac{\pi}{16} \left[(x^2 - 8x + 16) \frac{e^{2x}}{2} - (2x - 8) \frac{e^{2x}}{4} + \frac{e^{2x}}{4} \right]_0^4 = \frac{\pi}{16} \left(0 + 0 + \frac{e^8}{4} - \frac{16}{2} - \frac{8}{4} - \frac{1}{4} \right)$ Depends on both previous method marks.	ddM1
	$= \frac{\pi}{64} (e^8 - 41) \text{ (cm}^3\text{)}$	A1
		(5)

Question Number	Scheme	Marks
1	Volume = $\int_0^8 \frac{16\pi}{(x+2)^2} (dx)$ oe e.g. $\pi \int_0^8 \left(\frac{4}{x+2}\right)^2 (dx)$	B1
	$\int \frac{16}{(x+2)^2} dx = \frac{-16}{(x+2)}$ or e.g. $u = x+2 \Rightarrow \int \frac{16}{(x+2)^2} dx = \int \frac{16}{u^2} du = \frac{-16}{u}$	M1 A1
	$\left[\frac{-16\pi}{(x+2)} \right]_0^8 = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2} \right)$ or e.g. $\left[\frac{-16\pi}{u} \right]_2^{10} = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2} \right)$	M1
	$= \frac{32}{5} \pi$	A1
		(5)
		(5 marks)

Question	Scheme	Marks
1	$\int x \cos 3x dx = \frac{1}{3} x \sin 3x - \frac{1}{3} \int \sin 3x dx$	M1
	$= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x$	dM1 A1
	$\int_0^{\frac{\pi}{6}} x \cos 3x dx = \left[\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x \right]_0^{\frac{\pi}{6}} = \frac{1}{3} \frac{\pi}{6} \sin \frac{\pi}{2} + \frac{1}{9} \cos \frac{\pi}{2} - \left(0 + \frac{1}{9} \right)$	M1
	$= \frac{\pi}{18} - \frac{1}{9}$	A1
		(5)
		(5 marks)

Question Number	Scheme	Marks
4 (a)	$a = 3, b = 5$	B1
	E.g. $u = \sqrt{2x+1} \Rightarrow \frac{dx}{du} = u$ or $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ o.e.	B1
	$\int \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \int 2u e^u u du$	M1
	$= \int_3^5 2u^2 e^u du$	A1
		(4)
(b)	$\int \sqrt{8x+4} e^{2x+1} dx = \int 2u^2 e^u du$	
	$= 2u^2 e^u - \int 4u e^u du$	M1
	$= 2u^2 e^u - (4u e^u - 4e^u) = 2u^2 e^u - 4u e^u + 4e^u$	dM1 A1ft
	$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \left[2u^2 e^u - 4u e^u + 4e^u \right]_3^5 = (50e^5 - 20e^5 + 4e^5) - (18e^3 - 12e^3 + 4e^3)$	ddM1
	$= 34e^5 - 10e^3$	A1
		(5)
		(9 marks)

Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1} + \frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3) + B(2x-1) \Rightarrow A = \dots B = \dots$	M1
	$A = 1 \text{ or } B = 2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1} + \frac{2}{4x-3}$	A1
		(3)
(b)	$\int \left(\frac{1}{2x-1} + \frac{2}{4x-3} \right) dx = \frac{1}{2} \ln 2x-1 + \frac{1}{2} \ln 4x-3 (+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2} \ln(2x-1) + \frac{1}{2} \ln(4x-3) \right]_k^{3k} = \frac{1}{2} \ln(6k-1)(12k-3) - \frac{1}{2} \ln(2k-1)(4k-3)$ $= \frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2 - 170k + 57 = 0$	dM1A1
	$88k^2 - 170k + 57 = 0 \Rightarrow k = \dots$	ddM1
	$k = \frac{3}{2}$	A1
		(5)
		Total 11

Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4 + 10x^2 + 8 = 2x^2 - 7$ (and has real solutions).	B1
	$x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 5)(x^2 + 3) = 0 \Rightarrow x^2 = \dots$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow x^2 = \frac{-8 \pm \sqrt{8^2 - 4 \times 15}}{2}$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 4)^2 - 1 = 0 \Rightarrow x^2 = \dots$	M1
	$x^2 = -5, -3$	A1
	e.g. The two values of $x^2 < 0$ (so $x^4 + 8x^2 + 15 = 0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Question Number	Scheme	Notes	Marks
9(a)	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{d\theta} d\theta$ $= \pi \int (3 \sin \theta - \sin 2\theta)^2 (-5 \sin \theta) d\theta$	Applies $V = \pi \int y^2 \frac{dx}{d\theta} d\theta$ with or without the π	M1
	$= \pi \int (3 \sin \theta - 2 \sin \theta \cos \theta)^2 (-5 \sin \theta) d\theta$	Applies $\sin 2\theta = 2 \sin \theta \cos \theta$	M1
		Fully correct integral in terms of $\sin \theta$ and $\cos \theta$ only (π not needed)	A1
	$= \pi \int \sin^2 \theta (3 - 2 \cos \theta)^2 (-5 \sin \theta) d\theta$ $V = -5\pi \int \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = -5\pi \int_{\pi}^0 \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = 5\pi \int_0^{\pi} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta^*$	Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout.	A1*
			(4)
(b)	$u = \cos \theta \Rightarrow V = 5\pi \int \sin^3 \theta (3 - 2u)^2 \frac{du}{-\sin \theta}$	Applies the substitution correctly	M1
	$\theta = 0 \Rightarrow u = 1, \theta = \pi \Rightarrow u = -1$	Attempts to change θ limits to u limits	M1
	$V = -5\pi \int \sin^2 \theta (3 - 2u)^2 du = -5\pi \int (1 - u^2)(3 - 2u)^2 du$ Correct integral in terms of u only		A1
	$(1 - u^2)(3 - 2u)^2 = (1 - u^2)(9 - 12u + 4u^2)$ $= 9 - 12u - 5u^2 + 12u^3 - 4u^4$	Attempt to expand	M1
		Correct expansion	A1
	$V = 5\pi \int_{-1}^1 (9 - 12u - 5u^2 + 12u^3 - 4u^4) du$ $= 5\pi \left[9u - 6u^2 - \frac{5u^3}{3} + 3u^4 - \frac{4u^5}{5} \right]_{-1}^1 = \dots$ Integrates and applies their u limits		M1
	$= \frac{196}{3} \pi$	Cao	A1
			(7)
			Total 11

Question Number	Scheme	Notes	Marks
4(a)	$\frac{4-4x}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$	Correct form for the partial fractions	B1
	$4-4x = A(x-2)^2 + Bx(x-2) + Cx$ $\Rightarrow A = \dots$ or $B = \dots$ or $C = \dots$	Uses a correct strategy to find at least one of their constants	M1
	$\frac{4-4x}{x(x-2)^2} \equiv \frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2}$	2 correct constants	A1
		All correct	A1
			(4)
(b)	$\int \left(\frac{1}{x} - \frac{1}{x-2} - \frac{2}{(x-2)^2} \right) dx = \ln x - \ln(x-2) + \frac{2}{x-2} (+c)$		M1
	M1 for $\int \frac{\alpha}{x} dx = \beta \ln x$ or $\int \frac{\alpha}{x-2} dx = \beta \ln(x-2)$		M1
	M1 for $\int \frac{\alpha}{(x-2)^2} dx = \frac{\beta}{x-2}$		A1
	A1: All correct		
			(3)
(c)	$\left[\ln x - \ln(x-2) + \frac{2}{x-2} \right]_3^5 = \left(\ln 5 - \ln 3 + \frac{2}{3} \right) - (\ln 3 - \ln 1 + 2)$ $= \ln \frac{5}{9} - \frac{4}{3}$		M1
	M1: Correct use of limits and reaches the required form using log rules A1: Correct answer		A1
			(2)
			Total 9

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x dx = \frac{x^{-1}}{-1} \ln x - \int \frac{x^{-1}}{-1} \times \frac{1}{x} dx$ $= \frac{x^{-1}}{-1} \ln x + \int x^{-2} dx$ $= \frac{x^{-1}}{-1} \ln x + \frac{x^{-1}}{-1} (+c)$ $\int_1^e \frac{\ln x}{x^2} dx = \left[\frac{-1}{x} \ln x - \frac{1}{x} \right]_1^e = \left(\frac{-1}{e} \ln e - \frac{1}{e} \right) - \left(\frac{-1}{1} \ln 1 - \frac{1}{1} \right)$ $= 1 - \frac{2}{e}$	M1A1 dM1A1 ddM1 A1 (6)

Question	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{1}{2}(9-4t)^{-\frac{1}{2}} \times -4 = \left(\frac{-2}{\sqrt{9-4t}} \right)$	M1
	$A = \int_{(x=0)}^{(x=3)} y \frac{dx}{dt} (dt) = \int_{(t=\frac{9}{4})}^{(t=0)} \frac{t^3}{\sqrt{9+4t}} \times \frac{-2}{\sqrt{9-4t}} (dt)$	M1
	$= -2 \int_{(\frac{9}{4})}^{(0)} \frac{t^3}{\sqrt{81-16t^2}} (dt) \text{ or e.g. } -2 \int_{(\frac{9}{4})}^{(0)} \frac{t^3}{\sqrt{(9-4t)(9+4t)}} (dt)$ Depends on both previous M marks.	ddM1
	$= 2 \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81-16t^2}} dt$	A1
		(4)
(b)	$\frac{du}{dt} = -32t \text{ or } du = -32t dt \text{ oe}$ or $u = 81 - 16t^2 \Rightarrow t^2 = \frac{1}{16}(81-u) \Rightarrow t = \frac{1}{4}(81-u)^{\frac{1}{2}} \Rightarrow \frac{dt}{du} = -\frac{1}{8}(81-u)^{-\frac{1}{2}}$	B1
	$A = 2 \int_0^{\frac{9}{4}} \frac{t^2}{\sqrt{81-16t^2}} t dt = 2 \int_{(81)}^{(0)} \frac{(81-u)}{16\sqrt{u}} \frac{du}{-32}$ or $A = 2 \int_0^{\frac{9}{4}} \frac{t^2}{\sqrt{81-16t^2}} t dt = 2 \int_{(81)}^{(0)} \frac{\left(\frac{1}{4}(81-u)^{\frac{1}{2}}\right)^3}{\sqrt{u}} \times -\frac{1}{8}(81-u)^{-\frac{1}{2}} du$	M1
	$= k \int_{(81)}^{(0)} 81u^{\frac{1}{2}} - u^{\frac{1}{2}} du$	A1ft
	$= K \left[\frac{81u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_{(81)}^{(0)}$	M1
	$= -\frac{1}{256} \left(0 - \left(162\sqrt{81} - \frac{2}{3}\sqrt{81^3} \right) \right) = \dots$ Depends on all previous M marks.	ddM1
	$= \frac{243}{64}$	A1
		(6)
(10 marks)		

Question	Scheme	Marks
5(a)	$\int x^2 \cos 2x \, dx = x^2 \cdot \frac{1}{2} \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	M1 A1
	$= x^2 \cdot \frac{1}{2} \sin 2x - \left(-\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x \, dx \right)$	M1
	$= \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x (+c)$	A1
		(4)
(b)	$\frac{dy}{dt} = \frac{t^2 \cos^2 t}{y^2} \Rightarrow \text{e.g. } \int y^2 \frac{dy}{dt} \, dt \text{ or } \int y^2 \, dy = \int t^2 \cos^2 t \, dt$	B1
	$\Rightarrow \frac{y^3}{3} = \dots ; \dots = \frac{1}{2} \int t^2 + t^2 \cos 2t \, dt$	M1 ; M1
	$\dots = \frac{1}{6} t^3 + \frac{1}{2} \left(\frac{1}{2} t^2 \sin 2t + \frac{1}{2} t \cos 2t - \frac{1}{4} \sin 2t \right) (+c)$	M1
	$\Rightarrow y^3 = \frac{1}{2} t^3 + \frac{3}{4} t^2 \sin 2t + \frac{3}{4} t \cos 2t - \frac{3}{8} \sin 2t + c$	A1ft
		(5)
(9 marks)		

Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1} + \frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3) + B(2x-1) \Rightarrow A = \dots B = \dots$	M1
	$A=1 \text{ or } B=2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1} + \frac{2}{4x-3}$	A1
		(3)
(b)	$\int \left(\frac{1}{2x-1} + \frac{2}{4x-3} \right) dx = \frac{1}{2} \ln 2x-1 + \frac{1}{2} \ln 4x-3 (+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2} \ln(2x-1) + \frac{1}{2} \ln(4x-3) \right]_k^{3k} = \frac{1}{2} \ln(6k-1)(12k-3) - \frac{1}{2} \ln(2k-1)(4k-3)$ $= \frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2 - 170k + 57 = 0$	dM1A1
	$88k^2 - 170k + 57 = 0 \Rightarrow k = \dots$	ddM1
	$k = \frac{3}{2}$	A1
		(5)
		Total 11

Question Number	Scheme	Marks
7. (a)	Attempts $\overrightarrow{BA} \cdot \overrightarrow{BC} \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix} = 24 + 6 + 15 = 45$ Attempts to apply $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta \Rightarrow 45 = 7 \times \sqrt{50} \cos \theta$ $\cos \theta = \frac{9\sqrt{2}}{14} \text{ oe}$	M1 dM1 A1 (3)
(b)	Attempts $\sin \theta = \sqrt{1 - \left(\frac{9\sqrt{2}}{14}\right)^2}$ Attempts to apply $\mathbf{a} \mathbf{b} \sin \theta = 7 \times \sqrt{50} \times \frac{\sqrt{34}}{14}$ Area ABCD = $5\sqrt{17}$	M1 M1 A1 (3) (6 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
6(a)	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{(\theta - 15)^2} = \int -k dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = -(\theta - 15)^{-1}$	M1
	$-\frac{1}{\theta - 15} = -kt + c$	A1
	$t = 0, \theta = 85 \Rightarrow -\frac{1}{70} = c$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25} = 10k + \frac{1}{70} \Rightarrow k = \dots \left(\frac{9}{3500} \right)$	M1
	$\frac{1}{\theta - 15} = \frac{9t}{3500} + \frac{1}{70} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1
		(7)
(b)	$20 = \frac{135t + 4250}{9t + 50} \Rightarrow t = \dots$	M1
	$t = \text{awrt } 72$	A1
		(2)
		Total 9

Question	Scheme	Marks
7(a)	$u = 4x + 2 \sin 2x \Rightarrow \frac{du}{dx} = 4 + 4 \cos 2x$ oe	M1 A1
	$\frac{du}{dx} = 4 + 4 \cos 2x = 8 \cos^2 x \Rightarrow \int_{(0)}^{\left(\frac{\pi}{2}\right)} e^{4x+2\sin 2x} \cos^2 x \, dx = \int_{(0)}^{(2\pi)} \frac{e^u}{8} du$	M1
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	A1ft
	$\int e^{4x+2\sin 2x} \cos^2 x \, dx = \int e^{4x+2\sin 2x} \times \frac{1}{2} (1 \pm \cos 2x) \, dx$	(M1 A1)
	$= k e^{4x+2\sin 2x}, = \left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	(M1; A1ft)
Alt by inspection for first 4 marks	$= \left[\frac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}} = \frac{1}{8} (e^{2\pi} - 1) *$	A1*cso
		(5)
(b)	$V \left[= \pi \int_0^{\frac{\pi}{2}} y^2 \, dx = \pi \int_0^{\frac{\pi}{2}} \left(6e^{2x+\sin 2x} \cos x \right)^2 \, dx \right] = 36\pi \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx$	B1
	$K \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx = \frac{K}{8} (e^{2\pi} - 1)$	M1
	$= \frac{9\pi}{2} (e^{2\pi} - 1)$	A1
		(3)

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7. (a)	Attempts $\overrightarrow{BA} \cdot \overrightarrow{BC} \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix} = 24 + 6 + 15 = 45$ Attempts to apply $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta \Rightarrow 45 = 7 \times \sqrt{50} \cos \theta$ $\cos \theta = \frac{9\sqrt{2}}{14}$ oe	M1 dM1 A1 (3)
(b)	Attempts $\sin \theta = \sqrt{1 - \left(\frac{9\sqrt{2}}{14}\right)^2}$ Attempts to apply $ \mathbf{a} \mathbf{b} \sin \theta = 7 \times \sqrt{50} \times \frac{\sqrt{34}}{14}$ Area ABCD = $5\sqrt{17}$	M1 M1 A1 (3) (6 marks)

Question Number	Scheme	Marks
6	$u = 3 + 4 \sin x \Rightarrow \frac{du}{dx} = 4 \cos x$ $\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{32 \sin x \cos x}{(3 + 4 \sin x)^2} dx = \int \frac{2(u-3)}{u^2} du$ $= \int \frac{2}{u} - \frac{6}{u^2} du = 2 \ln u + \frac{6}{u}$ Uses limits of 5 and 7 $\Rightarrow 2 \ln 7 + \frac{6}{7} - 2 \ln 5 - \frac{6}{5} = -\frac{12}{35} + \ln \frac{49}{25}$	B1 M1 A1 dM1 A1 M1 A1 (7 marks)

Question Number	Scheme	Marks
7(a)	$\frac{du}{dx} = e^x$	B1
	$\int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)}$	M1
	$= \int \frac{24u + 12 - 2}{(2u)^2} \times \frac{2}{2u + 1} du = \int \frac{48u + 20}{4u^2(2u + 1)} du = \int \frac{12u + 5}{u^2(2u + 1)} du$	A1
	$x = \ln 3 \rightarrow a = \frac{5}{2}, x = \ln 8 \rightarrow b = \frac{15}{2}$	B1
(a)	Note that apart from the limits, this is a given answer so check the work carefully.	

(b)	$12u + 5 \equiv A(2u + 1) + Bu(2u + 1) + Cu^2 \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$ or e.g. $12u + 5 \equiv (2B + C)u^2 + (2A + B)u + A \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$	M1
	2 of: $A = 5, B = 2, C = -4$	A1
	$\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$	A1
		(3)

(b)

M1: Attempts to find at least one of the values for A , B and C by multiplying up and substituting values or by multiplying up and comparing coefficients.

When multiplying up must have at least one of the terms on the rhs correct for their lhs.

A1: Two of $A = 5, B = 2, C = -4$

A1: Correct fractions: $\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$ oe. Allow $\frac{5}{u^2} + \frac{2}{u} + \frac{4}{(2u+1)}$

If the correct fractions are not seen in part (b), allow this mark to score if the correct fractions are seen in part (c).

(c)	$\int \frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)} du = -5u^{-1} + 2 \ln u - 2 \ln(2u+1) (+c)$	M1A1ft
	$\left[-5u^{-1} + 2 \ln u - 2 \ln(2u+1) \right]_{2.5}^{7.5}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$ or $\left[-5 \left(e^x - \frac{1}{2} \right)^{-1} + 2 \ln \left(e^x - \frac{1}{2} \right) - 2 \ln \left(2 \left(e^x - \frac{1}{2} \right) + 1 \right) \right]_{\ln 3}^{\ln 8}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$	dM1
	$\frac{4}{3} + \ln \left(\frac{81}{64} \right)$	A1
		(4)
		(11 marks)

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx (+c)$		M1A1
	M1: For applying parts to obtain $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left\{ -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x \, dx \right\} (+c)$		M1
	Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \cos 3x \, dx$		
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx (+c)$		A1
	Fully correct application of parts twice		
Way 2	$\int e^{2x} \cos 3x \, dx + \frac{4}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c)$ $\Rightarrow \frac{13}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$		M1
	Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx (+c)$		M1A1
	M1: For applying parts to obtain $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx \right\} (+c)$		M1
	Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \cos 3x \, dx$		
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx (+c)$		A1
	Fully correct application of parts twice		
Way 2	$\int e^{2x} \cos 3x \, dx + \frac{9}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c)$ $\Rightarrow \frac{13}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$		M1
	Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			Total 6

Question Number	Scheme	Marks
5(i)	$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx (+c)$	M1A1
	$\int x e^x dx = x e^x - \int e^x dx (+c)$	M1
	$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x (+c)$	A1
	Also allow $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x)(+c)$	
		(4)
(ii)	$u = (1-3x)^{\frac{1}{2}} \Rightarrow u^2 = 1-3x \Rightarrow 2u \frac{du}{dx} = -3$ or $u = (1-3x)^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$	B1
	$\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{u} \left(-\frac{2u}{3}\right) du$ or $\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{\sqrt{1-3x}} \left(-\frac{2}{3}\sqrt{1-3x}\right) du$	M1
	$6 \int (u^2 - 1) du$ or $-6 \int (1 - u^2) du$	A1
	$6 \left(\frac{u^3}{3} - u \right) (+k)$	A1ft
	$6 \left(\frac{(1-3x)^{\frac{3}{2}}}{3} - (1-3x)^{\frac{1}{2}} \right) (+k) = 2(1-3x)^{\frac{1}{2}} (1-3x-3)(+k)$	M1
	$= -2(1-3x)^{\frac{1}{2}} (2+3x)(+k)$ or $= -2(1-3x)^{\frac{1}{2}} (3x+2)(+k)$	A1
		(6)
		Total 10

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int \frac{1}{k-3x} dx = \int dt \text{ oe}$	M1
	$\Rightarrow -\frac{1}{3} \ln(k-3x) = t(+c)$	M1 A1
	$t=0, x=0 \Rightarrow -\frac{1}{3} \ln k = c$	M1
	$-\ln(k-3x) = 3t - \ln k \Rightarrow \ln \frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k}x = e^{-3t} \Rightarrow x = \dots$	M1
	$x = \frac{k}{3}(1 - e^{-3t})$	A1
		(6)
(b)	$t \rightarrow \infty \Rightarrow e^{-3t} \Rightarrow 0 \Rightarrow x \rightarrow \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = \dots$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7(1 - e^{-3t}) = 5 \Rightarrow e^{-3t} = \dots$	M1
	$\Rightarrow t = -\frac{1}{3} \ln \frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
(11 marks)		

Question Number	Scheme	Marks
7	$x = 4 \sin \theta \Rightarrow \frac{dx}{d\theta} = 4 \cos \theta$	B1
	$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{(16-16 \sin^2 \theta)^{\frac{3}{2}}} d\theta = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$	M1 A1
	$= \frac{1}{16} \tan \theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60^\circ) - f(30^\circ)$	M1
	$= \frac{1}{16} \tan \frac{\pi}{3} - \frac{1}{16} \tan \frac{\pi}{6} = \frac{\sqrt{3}}{24}$	A1
(6 marks)		

Question Number	Scheme	Marks
6.(a)	$x = \frac{3}{2} \Rightarrow a = \frac{\pi}{6}$ $\int y^2 dx = \int y^2 \frac{dx}{dt} dt = \int (2 \sin 2t)^2 3 \cos t (dt)$ Uses $\sin 2t = 2 \sin t \cos t \Rightarrow \int (4 \sin t \cos t)^2 3 \cos t dt$ $\text{Volume} = \int \pi y^2 dx = 48\pi \int_0^{\left(\frac{\pi}{6}\right)} \sin^2 t \cos^3 t dt$	B1 M1 dM1 A1 (4)
(b)	$u = \sin t \Rightarrow \frac{du}{dt} = \cos t$ $k \int \sin^2 t \cos^3 t dt = k \int u^2 \cos^2 t du$ $= k \int u^2 (1 - \sin^2 t) du = k \int u^2 (1 - u^2) du$ $= k \left[\frac{u^3}{3} - \frac{u^5}{5} \right]$ $\text{Volume} = 48\pi \left[\frac{u^3}{3} - \frac{u^5}{5} \right]_0^{\frac{1}{2}} = \frac{17\pi}{10}$	B1 M1 dM1 ddM1A1 (5) (9 marks)

Question Number	Scheme	Marks
5(i)	$\int x^2 e^{4x} dx = \frac{1}{4} x^2 e^{4x} - \int \frac{1}{2} x e^{4x} dx$	M1 A1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \int \frac{1}{8} e^{4x} dx$	dM1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} + c$	A1
		(4)

(ii)	$\frac{2x+11}{(2x+1)(2-x)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2-x)} \Rightarrow A = \dots, B = \dots$	M1
	$= \frac{4}{(2x+1)} + \frac{3}{(2-x)}$	A1
	$\int \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 2x+1 - 3 \ln 2-x $	dM1 A1ft
	$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = (2 \ln 15 - 3 \ln 5) - (2 \ln 9 - 3 \ln 2)$	ddM1
	$= \ln \frac{8}{45}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{5t^2} \Rightarrow \int \frac{dA}{A^{\frac{3}{2}}} = \int \frac{dt}{5t^2} \quad \text{oe}$	M1
	$-2A^{-\frac{1}{2}} = -\frac{1}{5}t^{-1} (+c)$	M1 M1 A1
	Substitutes $t = 3, A = 2.25 \Rightarrow c = \dots \left(-\frac{19}{15} \right)$	M1
	Uses their $\frac{2}{\sqrt{A}} = \frac{1}{5t} + \frac{19}{15} \Rightarrow A = \left(\frac{30t}{19t+3} \right)^2$	M1, A1
		(7)
(b)	As $t \rightarrow \infty, A \rightarrow \left(\frac{30}{19} \right)^2 = \frac{900}{361}$ or awrt 2.49 cm ²	M1 A1
		(2)
		(9 marks)

Question Number	Scheme	Marks
4(a)	e.g. $5+17x-10x^2 = A(2x+1)(1-x) + Bx(2x+1) + Cx(1-x)$ $\Rightarrow A = \dots, B = \dots, C = \dots$ $A = 5, B = 4, C = 8 \Rightarrow f(x) = \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$	M1 A1A1
		(3)
(b)	$\int \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)} dx = 5 \ln x - 4 \ln 1-x + 4 \ln 2x+1 (+c)$ $\left[5 \ln x - 4 \ln 1-x + 4 \ln 2x+1 \right]_2^4$ $= (5 \ln 4 - 4 \ln 3 + 4 \ln 9) - (5 \ln 2 - 4 \ln 1 + 4 \ln 5) = 5 \ln 2 + 4 \ln \left(\frac{3}{5} \right)$	M1A1ftA1 dM1A1
		(5)
		(8 marks)

Question Number	Scheme	Marks
6	$u = \sqrt{x^3+1} \Rightarrow u^2 = x^3+1 \Rightarrow 2u \frac{du}{dx} = 3x^2$ or $u = \sqrt{x^3+1} \Rightarrow u^2 = x^3+1 \Rightarrow 2u = 3x^2 \frac{dx}{du}$ or $u = \sqrt{x^3+1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(x^3+1)^{-\frac{1}{2}} \times 3x^2$ or $x = (u^2-1)^{\frac{1}{3}} \Rightarrow \frac{dx}{du} = \frac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\int \frac{9x^5}{\sqrt{x^3+1}} dx = \int \frac{9x^5}{u} \frac{2u}{3x^2} du = \int 6x^3 du = 6 \int (u^2-1) du$ or e.g. $\int \frac{9x^5}{\sqrt{x^3+1}} dx = \int \frac{9x^5}{\sqrt{x^3+1}} \frac{2\sqrt{x^3+1}}{3x^2} du = \int 6x^3 du = 6 \int (u^2-1) du$	M1A1
	$6 \int (u^2-1) du = 2u^3 - 6u (+c) = 2(x^3+1)^{\frac{3}{2}} - 6(x^3+1)^{\frac{1}{2}} (+c)$	M1
	$= 2(x^3+1)^{\frac{1}{2}} [(x^3+1)-3] + c = 2(x^3+1)^{\frac{1}{2}} (x^3-2) + c$	A1
		(5)
		Total 5

Question Number	Scheme	Marks
7 (a)	$\frac{dV}{dr} = 4\pi r^2$	B1 (1)
(b) (i)	$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c \text{ (oe)}$	M1 A1
	$t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = \dots$	M1
	$V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3} \quad *$	A1 *
(ii)	150 cm^3	B1
		(5)
(c)	$t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$	M1
	$100 = \frac{4}{3} \pi r^3 \Rightarrow r = 2.88 \text{ cm}$	dM1 A1cao (3)
(d)	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$	M1
	$t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \dots$	dM1
	$\Rightarrow \frac{dr}{dt} = \text{awrt } 0.11 \text{ cm s}^{-1}$	A1 (3)
		(12 marks)

Question Number	Scheme	Notes	Marks
7(a)	$\frac{dV}{dt} = 300 - kV \Rightarrow \int \frac{dV}{300 - kV} = \int dt$	Correct separation of variables	B1
	$\int \frac{dV}{300 - kV} = -\frac{1}{k} \ln(300 - kV)$	$\int \frac{dV}{300 - kV} = \alpha \ln(300 - kV)$	M1
	$-\frac{1}{k} \ln(300 - kV) = t + c$	Correct equation including a constant of integration	A1
	$-\frac{1}{k} \ln(300 - kV) = t + c \Rightarrow \ln(300 - kV) = -kt + d$ $\Rightarrow 300 - kV = e^{-kt+d}$ Correct processing to remove the "ln"		M1
	$kV = 300 - e^{-kt+d} \Rightarrow V = \frac{300}{k} - Be^{-kt}$ $V = \frac{300}{k} + Ae^{-kt} *$	Correct proof	A1*
			(5)
(b)	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$V = \frac{300}{k} - \frac{300}{k} e^{-kt} \Rightarrow \frac{dV}{dt} = 300e^{-kt}$	$\frac{dV}{dt} = \alpha e^{-kt}$	M1
	$300e^{-10k} = 200 \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ and correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
			(4)
(b) Way 2	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$\frac{dV}{dt} = 200, t = 10 \Rightarrow 200 = 300 - kV$ $\Rightarrow kV = 100$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ to find a value for kV	M1
	$V = \frac{300}{k} + Ae^{-kt} \Rightarrow kV = 300 - 300e^{-10k}$ $\Rightarrow 100 = 300 - 300e^{-10k} \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Substitutes for kV, kA and $t = 10$ and uses correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
(c)	$6000 = \frac{3000}{\ln 1.5} - \frac{3000}{\ln 1.5} e^{-\frac{t}{10} \ln 1.5}$ $\Rightarrow e^{-\frac{t}{10} \ln 1.5} = 1 - 2 \ln 1.5$ $\Rightarrow -\frac{t}{10} \ln 1.5 = \ln(1 - 2 \ln 1.5)$	Correct strategy using $V = 6000$ to reach $at = \dots$	M1
	$t = 41$	Correct value	A1
			(2)
			Total 11

Question Number	Scheme	Marks
9 (a)	States or uses $V = \pi \times 4^2 \times h \Rightarrow \frac{dV}{dh} = 16\pi$ States or uses $\frac{dV}{dt} = 0.6\pi - 0.15\pi h$ Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 0.6\pi - 0.15\pi h = 16\pi \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{12-3h}{320} \quad *$	B1 B1 M1 A1* (4)
(b)	$\frac{dh}{dt} = \frac{12-3h}{320} \Rightarrow \int \frac{dh}{12-3h} = \int \frac{1}{320} dt$ $-\frac{1}{3} \ln(12-3h) = \frac{1}{320} t + c$ Uses $t = 0, h = 0.5 \quad -\frac{1}{3} \ln\left(\frac{21}{2}\right) = c \Rightarrow -\frac{1}{3} \ln(12-3h) = \frac{1}{320} t - \frac{1}{3} \ln\left(\frac{21}{2}\right)$ Substitutes $h = 3.5 \quad -\frac{1}{3} \ln\left(\frac{3}{2}\right) = \frac{1}{320} t - \frac{1}{3} \ln\left(\frac{21}{2}\right) \Rightarrow t = \dots$ 208 minutes cso	M1 A1 M1 A1 dM1 A1 (6)
		(10 marks)

Question Number	Scheme	Marks
7(a)	$\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)} \Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \dots \text{ or } B = \dots$	M1
	$A = -\frac{1}{2}, B = \frac{1}{2} \text{ giving } \frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$	A1
		[2]
(b)	$\frac{dx}{dt} = k(4-x)(2-x), t \geq 0 \Rightarrow \int \frac{1}{(4-x)(2-x)} dx = \int k dt$	
	$\frac{1}{2} \ln(4-x) - \frac{1}{2} \ln(2-x) = kt (+c)$	M1, A1
	$\{t=0, x=0 \Rightarrow\} \frac{1}{2} \ln 4 - \frac{1}{2} \ln 2 = 0 + c \Rightarrow c = \frac{1}{2} \ln 2$	M1
	$\frac{1}{2} \ln(4-x) - \frac{1}{2} \ln(2-x) = kt + \frac{1}{2} \ln 2 \Rightarrow \ln\left(\frac{4-x}{4-2x}\right) = 2kt$	
	$\frac{4-x}{4-2x} = e^{2kt}$	M1
	$4-x = 4e^{2kt} - 2xe^{2kt} \Rightarrow 4 - 4e^{2kt} = x - 2xe^{2kt}$	
	$\Rightarrow 4 - 4e^{2kt} = x(1 - 2e^{2kt}) \Rightarrow x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}} (*)$	dM1, A1*
		[6]
(c)	$1 = \frac{4 - 4e^{0.2t}}{1 - 2e^{0.2t}} \Rightarrow 2e^{0.2t} = 3$	M1
	$\Rightarrow 0.2t = \ln \frac{3}{2} \Rightarrow t = \text{awrt } 2.03$	A1
		[2]
10 marks		

Question Number	Scheme	Marks
2	$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \Rightarrow \int \frac{1}{y^2} dy = \int \frac{4}{\sqrt{4x+5}} dx$	B1
	$-\frac{1}{y} = 2\sqrt{4x+5} (+c)$	M1, A1
	Substitutes $y = \frac{1}{3}, x = -\frac{1}{4} \Rightarrow -3 = 4 + c \Rightarrow c = \dots$	dM1
	Rearranges $\frac{a}{y} = b\sqrt{4x+5} + c$ to $y = \dots$	ddM1
	$y = \frac{1}{7 - 2\sqrt{4x+5}}$	A1
		(6) (6 marks)

Question Number	Scheme		Marks
9(a)	$(V) = \pi \int_0^4 \left(\cos x + \frac{1}{5} e^x \right)^2 dx$		B1
	$\left(\cos x + \frac{1}{5} e^x \right)^2 = \cos^2 x + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \frac{\cos 2x + 1}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$		M1 dM1 A1
			(4)
(b)	Way I	Way Two	
	$\int e^x \cos x \, dx = \pm e^x \sin x \pm \int e^x \sin x \, dx$ $= e^x \sin x - \int e^x \sin x \, dx$	$\int e^x \cos x \, dx = e^x \cos x \pm \int e^x \sin x \, dx$ $= e^x \cos x + \int e^x \sin x \, dx$	M1 A1
	$\int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x - \int e^x \cos x \, dx$ $2 \int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x$		dM1
	$\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + (c)$		A1
			(4)
(c)	$\pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$ $= (\pi) \left[\frac{1}{2} x + \frac{\sin 2x}{4} + \frac{2}{5} \left(\frac{1}{2} e^x (\sin x + \cos x) \right) + \frac{1}{50} e^{2x} \right]_0^4$ $2\pi \left(2 + \frac{\sin 8}{4} + \frac{2}{5} \left(\frac{1}{2} e^4 (\sin(4) + \cos(4)) \right) + \frac{1}{50} e^8 - \frac{11}{50} \right)$ $= \text{awrt } 290 \text{ (cm}^3\text{)}$		M1A1ft dM1 A1
			(4)
			(12 marks)

(a)

Question Number	Scheme	Marks
6(a)	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{(\theta - 15)^2} = \int -k dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = -(\theta - 15)^{-1}$	M1
	$-\frac{1}{\theta - 15} = -kt + c$	A1
	$t = 0, \theta = 85 \Rightarrow -\frac{1}{70} = c$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25} = 10k + \frac{1}{70} \Rightarrow k = \dots \left(\frac{9}{3500} \right)$	M1
	$\frac{1}{\theta - 15} = \frac{9t}{3500} + \frac{1}{70} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1
		(7)
(b)	$20 = \frac{135t + 4250}{9t + 50} \Rightarrow t = \dots$	M1
	$t = \text{awrt } 72$	A1
		(2)
		Total 9

Question Number	Scheme	Marks
6.(a)	$x = \frac{3}{2} \Rightarrow a = \frac{\pi}{6}$ $\int y^2 dx = \int y^2 \frac{dx}{dt} dt = \int (2 \sin 2t)^2 3 \cos t (dt)$ Uses $\sin 2t = 2 \sin t \cos t \Rightarrow \int (4 \sin t \cos t)^2 3 \cos t dt$ $\text{Volume} = \int \pi y^2 dx = 48\pi \int_0^{\left(\frac{\pi}{6}\right)} \sin^2 t \cos^3 t dt$	B1 M1 dM1 A1 (4)
(b)	$u = \sin t \Rightarrow \frac{du}{dt} = \cos t$ $k \int \sin^2 t \cos^3 t dt = k \int u^2 \cos^2 t du$ $= k \int u^2 (1 - \sin^2 t) du = k \int u^2 (1 - u^2) du$ $= k \left[\frac{u^3}{3} - \frac{u^5}{5} \right]$ $\text{Volume} = 48\pi \left[\frac{u^3}{3} - \frac{u^5}{5} \right]_0^{\frac{1}{2}} = \frac{17\pi}{10}$	B1 M1 dM1 ddM1A1 (5) (9 marks)

Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$ $u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \frac{u + u^3}{(5 + u^2)^3} \times \frac{1}{1 + u^2} du$ $\int \frac{u}{(5 + u^2)^3} du *$	B1 M1dM1 A1* (4)
(b)	$\int \frac{u}{(5 + u^2)^3} du = -\frac{1}{4} (5 + u^2)^{-2} + c = -\frac{1}{4} (5 + \tan^2 x)^{-2} (+c)$	M1A1 (2) (6 marks)

Question Number	Scheme	Marks
5(a)	$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x dx = -x^{-1} \ln x + \int x^{-1} \times \frac{1}{x} dx$ $= -x^{-1} \ln x - x^{-1} (+c)$	M1 dM1 A1 (3)
(b)	$\frac{3+2x+\ln x}{x^2} = 3x^{-2} + 2x^{-1} + x^{-2} \ln x \quad \text{oe}$ $\text{Area} = \left[-\frac{3}{x} + 2 \ln x - \frac{\ln x}{x} - \frac{1}{x} \right]_2^4$ $= \left(-\frac{3}{4} + 2 \ln 4 - \frac{\ln 4}{4} - \frac{1}{4} \right) - \left(-\frac{3}{2} + 2 \ln 2 - \frac{\ln 2}{2} - \frac{1}{2} \right) = 1 + 2 \ln 2$	B1 M1 M1 A1 (4) (7 marks)

Question Number	Scheme	Marks
3(i)	$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$	M1
	$= \frac{1}{2} x^2 e^{2x} - \left\{ \frac{1}{2} x e^{2x} - \int \frac{e^{2x}}{2} dx \right\}$	<u>M1</u>
	$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4}$	A1
	$\int_0^4 x^2 e^{2x} dx = \left[\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4} \right]_0^4 = 8e^8 - 2e^8 + \frac{e^8}{4} - \frac{1}{4}$	M1
	$= \frac{25e^8}{4} - \frac{1}{4}$	A1
		(5)

(ii)	1. $u = 2x - 1 \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$ OR 2. $u = (2x - 1)^2 \Rightarrow \frac{du}{dx} = 4(2x - 1)$ or e.g. $du = 4(2x - 1)dx$ OR 3. $u = 2x \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$	B1
	1. $u = 2x - 1 \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2u + 2}{u^2} \times \frac{1}{2} du$ OR 2. $u = (2x - 1)^2 \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2\sqrt{u} + 2}{u} \times \frac{1}{4\sqrt{u}} du$ OR 3. $u = 2x \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2u}{(u - 1)^2} \times \frac{1}{2} du$	M1 A1
	1. $\int \left(\frac{1}{u} + \frac{1}{u^2} \right) du = \ln u - \frac{1}{u}$ OR 2. $\frac{1}{2} \int \left(\frac{1}{u} + \frac{1}{u^{\frac{3}{2}}} \right) du = \frac{1}{2} \ln u - \frac{1}{\sqrt{u}}$ OR 3. $\int \frac{u}{(u - 1)^2} du = -u(u - 1)^{-1} + \int \frac{1}{u - 1} du = -u(u - 1)^{-1} + \ln(u - 1)$	dM1 A1
	1. Uses limits $u = 5$ to $u = 20$: $\left(\ln 20 - \frac{1}{20} \right) - \left(\ln 5 - \frac{1}{5} \right)$ OR 2. Uses limits $u = 25$ to $u = 400$: $\left(\frac{1}{2} \ln 400 - \frac{1}{20} \right) - \left(\frac{1}{2} \ln 25 - \frac{1}{5} \right)$ OR 3. Uses limits $u = 6$ to $u = 21$: $\left(\ln 20 - \frac{21}{20} \right) - \left(\ln 5 - \frac{6}{5} \right)$	M1
	$= \frac{3}{20} + \ln 4$	A1
		(7)
		(12 marks)

Question Number	Scheme	Marks
3	Calculates the upper limit as $2 \ln 2$ or $\ln 4$ $\int \left(e^{0.5x} - 2 \right)^2 dx = \int \left(e^x - 4e^{0.5x} + 4 \right) dx = e^x - 8e^{0.5x} + 4x$ Volume = $\pi \left[e^x - 8e^{0.5x} + 4x \right]_0^{2 \ln 2} = \dots$ Volume = $8\pi \ln 2 - 5\pi$	B1 M1 A1 A1 dM1 A1 (6 marks)

Question Number	Scheme	Marks
7(a)	$\frac{du}{dx} = e^x$	B1
	$\int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)}$	M1
	$= \int \frac{24u + 12 - 2}{(2u)^2} \times \frac{2}{2u + 1} du = \int \frac{48u + 20}{4u^2(2u + 1)} du = \int \frac{12u + 5}{u^2(2u + 1)} du$	A1
	$x = \ln 3 \rightarrow a = \frac{5}{2}, \quad x = \ln 8 \rightarrow b = \frac{15}{2}$	B1
		(4)
(a) Note that apart from the limits, this is a given answer so check the work carefully.		

(b)	$12u + 5 \equiv A(2u + 1) + Bu(2u + 1) + Cu^2 \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$ or e.g. $12u + 5 \equiv (2B + C)u^2 + (2A + B)u + A \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$	M1
	2 of: $A = 5, B = 2, C = -4$	A1
	$\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u + 1)}$	A1
		(3)
	(b) M1: Attempts to find at least one of the values for A, B and C by multiplying up and substituting values or by multiplying up and comparing coefficients. When multiplying up must have at least one of the terms on the rhs correct for their lhs. A1: Two of $A = 5, B = 2, C = -4$ A1: Correct <u>fractions</u> : $\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u + 1)}$ oe. Allow $\frac{5}{u^2} + \frac{2}{u} + -\frac{4}{(2u + 1)}$ If the correct fractions are not seen in part (b), allow this mark to score if the correct fractions are seen in part (c).	

(c)	$\int \frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u + 1)} du = -5u^{-1} + 2 \ln u - 2 \ln(2u + 1) (+c)$	M1A1ft
	$\left[-5u^{-1} + 2 \ln u - 2 \ln(2u + 1) \right]_{2.5}^{7.5}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$ or $\left[-5 \left(e^x - \frac{1}{2} \right)^{-1} + 2 \ln \left(e^x - \frac{1}{2} \right) - 2 \ln \left(2 \left(e^x - \frac{1}{2} \right) + 1 \right) \right]_{\ln 3}^{\ln 8}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$	dM1
	$\frac{4}{3} + \ln \left(\frac{81}{64} \right)$	A1
		(4)
		(11 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15\right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9\right)$ $= \frac{2}{3} \ln\left(\frac{6}{5}\right) = \frac{1}{3} \ln\left(\frac{36}{25}\right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i} + 4\mathbf{j} - \mathbf{k} - (-10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}))$	M1
	e.g. $\mathbf{r} = -10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	A1
	e.g. $\mathbf{r} = -6\mathbf{i} + 4\mathbf{j} - \mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	
		(2)
(b)	$3 + 3\mu = -6 \Rightarrow \mu = -3$	M1
	$p + 12 = 4 \Rightarrow p = \dots$ or $q - 3 = -1 \Rightarrow q = \dots$	dM1
	$p = -8, q = 2$	A1
		(3)
(c)	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = 12 + 4 + 3$	M1
	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = \sqrt{26}\sqrt{26}\cos\theta = 19 \Rightarrow \cos\theta = \dots$	M1
	$(\cos\theta =) \frac{19}{26}$	A1
		(3)
(d)	$AC = AB\sin\theta = \sqrt{26}\sqrt{1 - \left(\frac{19}{26}\right)^2}$	M1
	$\frac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

Question Number	Scheme	Marks
7 (a)	$\frac{dV}{dr} = 4\pi r^2$	B1 (1)
(b) (i)	$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c$ (oe)	M1 A1
	$t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = \dots$	M1
	$V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3}$ *	A1 *
(ii)	150 cm ³	B1
		(5)
(c)	$t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$	M1
	$100 = \frac{4}{3}\pi r^3 \Rightarrow r = 2.88 \text{ cm}$	dM1 A1cao (3)
(d)	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$	M1
	$t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \dots$	dM1
	$\Rightarrow \frac{dr}{dt} = \text{awrt } 0.11 \text{ cm s}^{-1}$	A1
		(3)
		(12 marks)

Question	Scheme	Marks
7(a)	$u = 4x + 2 \sin 2x \Rightarrow \frac{du}{dx} = 4 + 4 \cos 2x$ oe	M1 A1
	$\frac{du}{dx} = 4 + 4 \cos 2x = 8 \cos^2 x \Rightarrow \int_{(0)}^{\left(\frac{\pi}{2}\right)} e^{4x+2\sin 2x} \cos^2 x \, dx = \int_{(0)}^{(2\pi)} \frac{e^u}{8} \, du$	M1
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	A1ft
	Alt by inspection for first 4 marks $\int e^{4x+2\sin 2x} \cos^2 x \, dx = \int e^{4x+2\sin 2x} \times \frac{1}{2} (1 \pm \cos 2x) \, dx$	(M1 A1)
	$= k e^{4x+2\sin 2x}; = \left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}}$	(M1; A1ft)
	$= \left[\frac{1}{8} e^u \right]_0^{2\pi}$ or $\left[\frac{1}{8} e^{4x+2\sin 2x} \right]_0^{\frac{\pi}{2}} = \frac{1}{8} (e^{2\pi} - 1) *$	A1*cso
		(5)
(b)	$V \left[= \pi \int_0^{\frac{\pi}{2}} y^2 \, dx = \pi \int_0^{\frac{\pi}{2}} \left(6e^{2x+\sin 2x} \cos x \right)^2 \, dx \right] = 36\pi \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx$	B1
	$K \int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx = \frac{K}{8} (e^{2\pi} - 1)$	M1
	$= \frac{9\pi}{2} (e^{2\pi} - 1)$	A1
		(3)

Question Number	Scheme	Marks
7(a)	Substitutes $x = 3$ and $t = 0$ into $x = \frac{k(9t+5)}{4t+3}$	M1
	$\Rightarrow 3 = \frac{5k}{3} \Rightarrow k = 1.8$ *	A1*
		(2)
(b)	4050	B1
		(1)
(c)	$\frac{3}{x(9-2x)} \equiv \frac{A}{x} + \frac{B}{9-2x}$	M1
	Either $A = \frac{1}{3}$ or $B = \frac{2}{3}$	A1
	$\frac{3}{9x-2x^2} \equiv \frac{1}{3x} + \frac{2}{3(9-2x)}$	A1
		(3)

(d)	$3 \frac{dx}{dt} = x(9-2x) \Rightarrow \int \frac{3}{x(9-2x)} dx = \int dt$ Or $3 \frac{dx}{dt} = x(9-2x) \Rightarrow \int \frac{1}{x(9-2x)} dx = \int \frac{1}{3} dt$ $\Rightarrow \frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t(+c) \text{ or } \frac{1}{9} \ln x - \frac{1}{9} \ln(9-2x) = \frac{1}{3} t(+c)$	M1 A1ft
	Substitutes $t = 0, x = 3 \Rightarrow c = 0$	M1
	$\frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t \Rightarrow \left(\frac{x}{9-2x} \right) = e^{3t}$	ddM1
	$\left(\frac{9-2x}{x} \right) = e^{-3t} \Rightarrow \frac{9}{x} - 2 = e^{-3t} \Rightarrow x = \frac{9}{2+e^{-3t}}^*$	A1*
		(5)
(e)	4500	B1
		(1)
		(12 marks)

Question Number	Scheme	Marks
2 (a)	$\frac{3x}{(2x-1)(x-2)} \equiv \frac{A}{2x-1} + \frac{B}{x-2} \Rightarrow \text{Values for } A \text{ and } B$	M1
	One correct value, either $A = -1$ or $B = 2$ Correct PF form $\frac{-1}{2x-1} + \frac{2}{x-2}$	A1 A1 (3)
(b)	$\int \frac{-1}{2x-1} + \frac{2}{x-2} dx = -\frac{1}{2} \ln(2x-1) + 2 \ln(x-2) + (c)$	M1, A1ft
	$\int_5^{25} \frac{3x}{(2x-1)(x-2)} dx = \left[-\frac{1}{2} \ln(2x-1) + 2 \ln(x-2) \right]_5^{25}$ $= \left(-\frac{1}{2} \ln 49 + 2 \ln 23 \right) - \left(-\frac{1}{2} \ln 9 + 2 \ln 3 \right)$ $\ln \frac{529}{21}$	dM1 A1 (4) (7 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = \frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)}$ $\int \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
8 (a)	$\int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \times \frac{1}{x} \, dx$ $= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 (+c)$	M1 dM1 A1 (3)
(b)	$\int x^2 \ln^2 x \, dx = \frac{1}{3} x^3 \ln^2 x - \int \frac{1}{3} x^3 \times 2 \ln x \times \frac{1}{x} \, dx$ $= \frac{1}{3} x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right)$ $\text{Volume} = \int_1^e \pi x^2 \ln^2 x \, dx = \pi \times \left[\frac{1}{3} x^3 \ln^2 x - \frac{2}{3} \left(\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 \right) \right]_1^e$ $= \frac{5}{27} \pi e^3 - \frac{2}{27} \pi$	M1 A1ft dM1 A1 (4) (7 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question	Scheme	Marks
5(a)	$\int x^2 \cos 2x \, dx = x^2 \cdot \frac{1}{2} \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	M1 A1
	$= x^2 \cdot \frac{1}{2} \sin 2x - \left(-\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x \, dx \right)$	M1
	$= \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x (+c)$	A1
		(4)
(b)	$\frac{dy}{dt} = \frac{t^2 \cos^2 t}{y^2} \Rightarrow \text{e.g. } \int y^2 \frac{dy}{dt} \, dt \text{ or } \int y^2 \, dy = \int t^2 \cos^2 t \, dt$	B1
	$\Rightarrow \frac{y^3}{3} = \dots ; \dots = \frac{1}{2} \int t^2 + t^2 \cos 2t \, dt$	M1 ; M1
	$\dots = \frac{1}{6} t^3 + \frac{1}{2} \left(\frac{1}{2} t^2 \sin 2t + \frac{1}{2} t \cos 2t - \frac{1}{4} \sin 2t \right) (+c)$	M1
	$\Rightarrow y^3 = \frac{1}{2} t^3 + \frac{3}{4} t^2 \sin 2t + \frac{3}{4} t \cos 2t - \frac{3}{8} \sin 2t + c$	A1ft
		(5)
(9 marks)		

Question	Scheme	Marks
9(a)	$\frac{d}{dy}(1+2\ln y)^{-2} = \alpha(1+2\ln y)^{-3} \times \frac{\beta}{y} = \frac{K}{y(1+2\ln y)^3}$	M1
	$= \frac{-4}{y(1+2\ln y)^3}$ oe	A1
		(2)
(b)	$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3$ $\Rightarrow \dots dy = \int k \sin(2x) dx$ or $\int \frac{k}{y(1+2\ln y)^3} dy = \int \dots dx$ and $\Rightarrow \dots dy = \int k \sin(2x) dx \Rightarrow \dots = A \cos 2x$ oe or $\Rightarrow \int \frac{k}{y(1+2\ln y)^3} dy = \int \dots dx \Rightarrow \frac{A}{(1+2\ln y)^2} = \dots$	M1
	$\Rightarrow \int \frac{A}{y(1+2\ln y)^3} dy = \int B \sin(2x) dx \Rightarrow \frac{C}{(1+2\ln y)^2} = D \cos 2x$ oe	M1
	One side integrated <u>correctly</u> $\int \frac{3}{y(1+2\ln y)^3} dy = -\frac{3}{4(1+2\ln y)^2}, \int \frac{1}{y(1+2\ln y)^3} dy = -\frac{1}{4(1+2\ln y)^2}$ or $\int \frac{1}{3} \sin(2x) dx = -\frac{1}{6} \cos(2x)$ oe, $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$ oe	A1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x)(+c)$ or $-\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6} \cos(2x)(+c)$	A1
		(4)
(c)	$y=1, x=\frac{\pi}{6} \Rightarrow -\frac{3}{4(1+2\ln 1)} = -\frac{1}{2} \cos\left(\frac{\pi}{3}\right) + c \Rightarrow c = \dots$	M1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x) - \frac{1}{2}$ or $-\frac{1}{4(1+2\ln y)^2} = -\frac{1}{6} \cos(2x) - \frac{1}{6}$	A1
	$-\frac{3}{4(1+2\ln y)^2} = -\frac{1}{2} \cos(2x) - \frac{1}{2} \Rightarrow \frac{3}{4(1+2\ln y)^2} = \frac{1}{2} + \frac{1}{2} \cos(2x) = \cos^2 x$	M1
	$\Rightarrow (1+2\ln y)^2 = \frac{3}{4} \sec^2 x \Rightarrow \ln y = \frac{1}{2} \left(\frac{\sqrt{3}}{2} \sec x - 1 \right)$ Depends on both previous method marks.	ddM1
	$\Rightarrow y = e^{\frac{\sqrt{3}}{4} \sec x - \frac{1}{2}}$	A1
		(5)

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dt} = 6t$ o.e.	B1
	$\text{Area} = \left(\int \sin t \sin 2t \times 6t \, dt = \right) \int \sin t \times 2 \sin t \cos t \times 6t \, dt$	M1
	$= 12 \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$	A1
		(3)
(b)	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - (12) \int_0^{\frac{\pi}{2}} \frac{1}{3} \sin^3 t \, dt$	M1
	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3} \right) \int_0^{\frac{\pi}{2}} \sin t (1 - \cos^2 t) \, dt$	M1
	$= \left[4t \sin^3 t + 4 \cos t - \frac{4}{3} \cos^3 t \right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3} \right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

Question	Scheme	Marks
2(a)	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$	
	$3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ E.g. $x=2 \rightarrow 10 = A \times 25 \Rightarrow A = \dots$ or $x = -\frac{1}{2} \rightarrow \frac{5}{2} = C \times -\frac{5}{2} \Rightarrow C = \dots$ or $x^2: 0 = 4A + 2B, x: 3 = 4A - 3B + C, x^0: 4 = A - 2B - 2C \rightarrow A = \dots$	M1
	One of $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
	$\Rightarrow A = \dots, B = \dots$ and $C = \dots$	M1
	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{2}{5(x-2)} - \frac{4}{5(2x+1)} - \frac{1}{(2x+1)^2}$ or $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
		(4)

(b)	$\int \frac{p}{x-2} dx = \alpha \ln(x-2) \text{ or } \int \frac{q}{2x+1} dx = \beta \ln(2x+1)$ Alt: $\int \frac{8x+9}{(2x+1)^2} dx = \int \frac{8x+4}{(2x+1)^2} + \frac{5}{(2x+1)^2} dx = \gamma \ln(2x+1)^2 + \dots$	M1
	$\int \frac{k}{(2x+1)^2} dx = \frac{K}{2x+1}$	M1
	$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} dx = \left[\frac{2}{5} \ln(x-2) - \frac{4}{5} \times \frac{1}{2} \ln(2x+1) + \frac{1}{4x+2} \right]_7^{12}$	A1ft
	$= \left(\frac{2}{5} \ln(10) - \frac{2}{5} \ln(25) + \frac{1}{50} \right) - \left(\frac{2}{5} \ln(5) - \frac{2}{5} \ln(15) + \frac{1}{30} \right) = \dots$	DM1
	$= \frac{2}{5} \ln \frac{10}{25} + \frac{1}{50} - \frac{2}{5} \ln \frac{5}{15} - \frac{1}{30} = \frac{2}{5} \ln \frac{10 \times 15}{25 \times 5} + \dots$	M1
	$\frac{2}{5} \ln \frac{6}{5} - \frac{1}{75} \quad (\text{oe})$	A1
		(6)
(10 marks)		

Question Number	Scheme	Marks
7(a) Way 1	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$	M1
	$= \dots - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	dM1
	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	A1
	$\frac{5}{4} \int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
7(a) Way 2	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + \int 2e^{2x} \cos x \, dx$	M1
	$= \dots + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	dM1
	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	A1
	$5 \int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
(b)	$\left(\frac{2}{5} e^{2\pi} \sin \pi - \frac{1}{5} e^{2\pi} \cos \pi \right) - \left(\frac{2}{5} e^0 \sin 0 - \frac{1}{5} e^0 \cos 0 \right) = \dots$	M1
	$= \frac{1}{5} e^{2\pi} + \frac{1}{5} = \frac{e^{2\pi} + 1}{5} *$	A1*
		(2)
		(7 marks)

Question Number	Scheme	Marks
7. (a)	Attempts $\overrightarrow{BA} \cdot \overrightarrow{BC} \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix} = 24 + 6 + 15 = 45$	M1
	Attempts to apply $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta \Rightarrow 45 = 7 \times \sqrt{50} \cos \theta$	dM1
	$\cos \theta = \frac{9\sqrt{2}}{14}$ oe	A1
		(3)
(b)	Attempts $\sin \theta = \sqrt{1 - \left(\frac{9\sqrt{2}}{14}\right)^2}$	M1
	Attempts to apply $ \mathbf{a} \mathbf{b} \sin \theta = 7 \times \sqrt{50} \times \frac{\sqrt{34}}{14}$	M1
	Area ABCD = $5\sqrt{17}$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
5(i)	$\int x^2 e^{4x} dx = \frac{1}{4} x^2 e^{4x} - \int \frac{1}{2} x e^{4x} dx$	M1 A1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \int \frac{1}{8} e^{4x} dx$	dM1
	$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} + c$	A1
		(4)

(ii)	$\frac{2x+11}{(2x+1)(2-x)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2-x)} \Rightarrow A = \dots, B = \dots$	M1
	$= \frac{4}{(2x+1)} + \frac{3}{(2-x)}$	A1
	$\int \frac{2x+11}{(2x+1)(2-x)} dx = 2 \ln 2x+1 - 3 \ln 2-x $	dM1 A1ft
	$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = (2 \ln 15 - 3 \ln 5) - (2 \ln 9 - 3 \ln 2)$	ddM1
	$= \ln \frac{8}{45}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x dx = \frac{x^{-1}}{-1} \ln x - \int \frac{x^{-1}}{-1} \times \frac{1}{x} dx$ $= \frac{x^{-1}}{-1} \ln x + \int x^{-2} dx$ $= \frac{x^{-1}}{-1} \ln x + \frac{x^{-1}}{-1} (+c)$ $\int_1^e \frac{\ln x}{x^2} dx = \left[\frac{-1}{x} \ln x - \frac{1}{x} \right]_1^e = \left(\frac{-1}{e} \ln e - \frac{1}{e} \right) - \left(\frac{-1}{1} \ln 1 - \frac{1}{1} \right)$ $= 1 - \frac{2}{e}$	M1A1 dM1A1 ddM1 A1 (6)

Question Number	Scheme	Marks
3(i)	$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$	M1
	$= \frac{1}{2} x^2 e^{2x} - \left\{ \frac{1}{2} x e^{2x} - \int \frac{e^{2x}}{2} dx \right\}$	<u>M1</u>
	$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4}$	A1
	$\int_0^4 x^2 e^{2x} dx = \left[\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{e^{2x}}{4} \right]_0^4 = 8e^8 - 2e^8 + \frac{e^8}{4} - \frac{1}{4}$	M1
	$= \frac{25e^8}{4} - \frac{1}{4}$	A1
		(5)

(ii)	1. $u = 2x - 1 \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$ OR 2. $u = (2x - 1)^2 \Rightarrow \frac{du}{dx} = 4(2x - 1)$ or e.g. $du = 4(2x - 1)dx$ OR 3. $u = 2x \Rightarrow \frac{du}{dx} = 2$ or e.g. $du = 2dx$	B1
	1. $u = 2x - 1 \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2u + 2}{u^2} \times \frac{1}{2} du$ OR 2. $u = (2x - 1)^2 \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2\sqrt{u} + 2}{u} \times \frac{1}{4\sqrt{u}} du$ OR 3. $u = 2x \Rightarrow \int \frac{4x}{(2x - 1)^2} dx = \int \frac{2u}{(u - 1)^2} \times \frac{1}{2} du$	M1 A1
	1. $\int \left(\frac{1}{u} + \frac{1}{u^2} \right) du = \ln u - \frac{1}{u}$ OR 2. $\frac{1}{2} \int \left(\frac{1}{u} + \frac{1}{u^{\frac{3}{2}}} \right) du = \frac{1}{2} \ln u - \frac{1}{\sqrt{u}}$ OR 3. $\int \frac{u}{(u - 1)^2} du = -u(u - 1)^{-1} + \int \frac{1}{u - 1} du = -u(u - 1)^{-1} + \ln(u - 1)$	dM1 A1
	1. Uses limits $u = 5$ to $u = 20$: $\left(\ln 20 - \frac{1}{20} \right) - \left(\ln 5 - \frac{1}{5} \right)$ OR 2. Uses limits $u = 25$ to $u = 400$: $\left(\frac{1}{2} \ln 400 - \frac{1}{20} \right) - \left(\frac{1}{2} \ln 25 - \frac{1}{5} \right)$ OR 3. Uses limits $u = 6$ to $u = 21$: $\left(\ln 20 - \frac{21}{20} \right) - \left(\ln 5 - \frac{6}{5} \right)$	M1
	$= \frac{3}{20} + \ln 4$	A1
		(7)
		(12 marks)

Question	Scheme	Marks
5(a)	$1 = 2 \sin u \Rightarrow p = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	B1
	$x = 2 \sin u \Rightarrow \frac{dx}{du} = 2 \cos u$ oe	M1
	$\int \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx = \int \frac{6 \sin u + 2}{(4-4 \sin^2 u)^{\frac{3}{2}}} 2 \cos u du = \int \frac{6 \sin u + 2}{(4 \cos^2 u)^{\frac{3}{2}}} 2 \cos u du$	M1
	$= \int \frac{12 \sin u}{8 \cos^2 u} + \frac{2}{4 \cos^2 u} du = \int_0^{\frac{\pi}{6}} \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du$ *	A1*
		(4)
(b)	$\int \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du = \frac{3}{2} \sec u + \frac{1}{2} \tan u$	M1A1
	$\left[\frac{3}{2} \sec u + \frac{1}{2} \tan u \right]_0^{\frac{\pi}{6}} = \left(\frac{3}{2} \sec\left(\frac{\pi}{6}\right) + \frac{1}{2} \tan\left(\frac{\pi}{6}\right) \right) - \left(\frac{3}{2} \sec 0 + \frac{1}{2} \tan 0 \right) = \dots$	M1
	$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2} = \frac{7\sqrt{3}}{6} - \frac{3}{2} \left(= \frac{7\sqrt{3}-9}{6} \right)$	A1
		(4)
		(8 marks)
Notes		

Question Number	Scheme	Marks
8 (a)	$\int y^{-\frac{1}{3}} dy = \int 6xe^{-2x} dx$	B1
	$\frac{3}{2} y^{\frac{2}{3}} = -3xe^{-2x} + \int 3e^{-2x} dx$	M1 M1
	$\frac{3}{2} y^{\frac{2}{3}} = -3xe^{-2x} - \frac{3}{2} e^{-2x} + c$	dM1 A1
	Substitutes (0,1) $\Rightarrow c = 3$	M1
	$y^2 = \left(-2xe^{-2x} - e^{-2x} + 2 \right)^3$	A1
		(7)
(b)	As $x \rightarrow \infty, e^{-2x} \rightarrow 0$ and $y^2 = (2)^3 \Rightarrow y = 2^{\frac{3}{2}}$	M1 A1
		(2)
		(9 marks)

Question Number	Scheme	Marks
3	$y \cos^2(2x) \frac{dy}{dx} = 3 \sin 2x \rightarrow \int y \, dy = \int 3 \frac{\sin 2x}{\cos^2 2x} \, dx$ $\int y \, dy = \int 3 \sec(2x) \tan(2x) \, dx$ $\frac{y^2}{2} = \frac{3}{2} \sec 2x (+c)$ $\frac{16}{2} = \frac{3}{2} \sec\left(2 \frac{\pi}{6}\right) + c \Rightarrow c = \dots (=5)$ $y^2 = 3 \sec 2x + 10$	M1 A1 M1A1 dM1 A1
		(6)

Question Number	Scheme	Marks
7(a)	$\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)} \Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \dots \text{ or } B = \dots$ $A = -\frac{1}{2}, B = \frac{1}{2} \text{ giving } \frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$	M1 A1 [2]
(b)	$\frac{dx}{dt} = k(4-x)(2-x), t \geq 0 \Rightarrow \int \frac{1}{(4-x)(2-x)} \, dx = \int k \, dt$ $\frac{1}{2} \ln(4-x) - \frac{1}{2} \ln(2-x) = kt (+c)$ $\{t=0, x=0 \Rightarrow\} \frac{1}{2} \ln 4 - \frac{1}{2} \ln 2 = 0 + c \left\{ \Rightarrow c = \frac{1}{2} \ln 2 \right\}$ $\frac{1}{2} \ln(4-x) - \frac{1}{2} \ln(2-x) = kt + \frac{1}{2} \ln 2 \Rightarrow \ln\left(\frac{4-x}{4-2x}\right) = 2kt$ $\frac{4-x}{4-2x} = e^{2kt}$ $4-x = 4e^{2kt} - 2xe^{2kt} \Rightarrow 4 - 4e^{2kt} = x - 2xe^{2kt}$ $\Rightarrow 4 - 4e^{2kt} = x(1 - 2e^{2kt}) \Rightarrow x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}} (*)$	M1, A1 M1 M1 dM1, A1* [6]
(c)	$1 = \frac{4 - 4e^{0.2t}}{1 - 2e^{0.2t}} \Rightarrow 2e^{0.2t} = 3$ $\Rightarrow 0.2t = \ln \frac{3}{2} \Rightarrow t = \text{awrt } 2.03$	M1 A1 [2]
		10 marks

Question Number	Scheme	Marks
2	Achieves a lower limit of 2 Attempts $\alpha \int y^2 dx = \alpha \int \frac{81}{(2x-3)^{2.5}} dx = \alpha \times \frac{-27}{(2x-3)^{1.5}}$ Correct attempt at volume of solid generated under curve = $= \pi \int_{-2}^6 y^2 dx = \left[\frac{k}{(2x-3)^{1.5}} \right]_{-2}^6 = \left(\left(-\frac{27}{27} \right) - \left(-\frac{27}{1} \right) \right)$ Volume = 26π Correct attempt at volume of solid = $\pi \times 9^2 \times (6 - (-2)) - 26\pi$ = 298π	B1 M1, A1 dM1 A1 ddM1 A1 (7 marks)

Question Number	Scheme	Marks
3.	States or implies Volume = $\int_{\sqrt{5}}^5 \pi \left(\frac{\sqrt{3x}}{\sqrt{3x^2+5}} \right)^2 dx$	B1
	$\int \left(\frac{\sqrt{3x}}{\sqrt{3x^2+5}} \right)^2 dx = \int \frac{3x}{(3x^2+5)} dx = \frac{1}{2} \ln(3x^2+5)$	M1A1
	Volume = $\left\{ \pi \right\} \left(\frac{1}{2} \ln(3 \times 25 + 5) - \frac{1}{2} \ln(3 \times 5 + 5) \right)$	M1
	= $\pi \ln 2$	A1
		(5 marks)

Question Number	Scheme	Marks
6	$u = \sqrt{x^3 + 1} \Rightarrow u^2 = x^3 + 1 \Rightarrow 2u \frac{du}{dx} = 3x^2$ or $u = \sqrt{x^3 + 1} \Rightarrow u^2 = x^3 + 1 \Rightarrow 2u = 3x^2 \frac{dx}{du}$ or $u = \sqrt{x^3 + 1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(x^3 + 1)^{-\frac{1}{2}} \times 3x^2$ or $x = (u^2 - 1)^{\frac{1}{3}} \Rightarrow \frac{dx}{du} = \frac{2}{3}u(u^2 - 1)^{-\frac{2}{3}}$	B1
	e.g. $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = \int \frac{9x^5}{u} \frac{2u}{3x^2} du = \int 6x^3 du = 6 \int (u^2 - 1) du$ or e.g. $\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = \int \frac{9x^5}{\sqrt{x^3 + 1}} \frac{2\sqrt{x^3 + 1}}{3x^2} du = \int 6x^3 du = 6 \int (u^2 - 1) du$	M1A1
	$6 \int (u^2 - 1) du = 2u^3 - 6u (+c) = 2(x^3 + 1)^{\frac{3}{2}} - 6(x^3 + 1)^{\frac{1}{2}} (+c)$	M1
	$= 2(x^3 + 1)^{\frac{1}{2}} [(x^3 + 1) - 3] + c = 2(x^3 + 1)^{\frac{1}{2}} (x^3 - 2) + c$	A1
		(5)
		Total 5

Question Number	Scheme	Marks
4(a)	e.g. $5 + 17x - 10x^2 = A(2x + 1)(1 - x) + Bx(2x + 1) + Cx(1 - x)$ $\Rightarrow A = \dots, B = \dots, C = \dots$	M1
	$A = 5, B = 4, C = 8 \Rightarrow f(x) = \frac{5}{x} + \frac{4}{(1 - x)} + \frac{8}{(2x + 1)}$	A1A1
(b)	$\int \frac{5}{x} + \frac{4}{(1 - x)} + \frac{8}{(2x + 1)} dx = 5 \ln x - 4 \ln 1 - x + 4 \ln 2x + 1 (+c)$ $\left[5 \ln x - 4 \ln 1 - x + 4 \ln 2x + 1 \right]_2^4$ $= (5 \ln 4 - 4 \ln 3 + 4 \ln 9) - (5 \ln 2 - 4 \ln 1 + 4 \ln 5) = 5 \ln 2 + 4 \ln \left(\frac{3}{5} \right)$	M1A1ftA1
		dM1A1
		(5)
		(8 marks)

Question	Scheme	Marks
4.	$\int_1^4 \frac{10}{5x+2x\sqrt{x}} dx$ $u = \sqrt{x} \Rightarrow x = u^2 \Rightarrow \frac{dx}{du} = 2u$ $\int \frac{10}{5x+2x\sqrt{x}} dx = \int \frac{10}{5u^2+2u^3} 2u du$ $= \int \frac{20}{5u+2u^2} du = \int \frac{4}{u} - \frac{8}{5+2u} du$ $= 4 \ln u - 4 \ln(5+2u)$ $\text{Limits} = [4 \ln u - 4 \ln(5+2u)]_1^2 = 4 \ln 2 - 4 \ln 9 + 4 \ln 7 = \dots$ $= 4 \ln \left(\frac{14}{9} \right)$	B1 M1 A1 dM1 A1 ddM1 M1 A1 (8 marks)

Question	Scheme	Marks
9(a)	$\text{Volume, } V = \pi \int_{\frac{1}{3}}^3 y^2 dx = \pi \int_{\frac{1}{3}}^3 \frac{x^{\frac{1}{2}}}{(1+x) \left(\arctan(\sqrt{x}) \right)^2} dx$	B1
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \text{ oe}$	M1 A1
	$\Rightarrow V = (\pi) \int \frac{2 \times \frac{1}{2} x^{-\frac{1}{2}}}{\left(\arctan(\sqrt{x}) \right)^2 (1+x)} dx = (\pi) \int \frac{2}{u^2 (1+\tan^2 u)} \sec^2 u du$	dM1
	$= \pi \int \frac{2}{u^2 \cancel{\sec^2 u}} \cancel{\sec^2 u} du = 2\pi \int \frac{1}{u^2} du$	A1
	$\left. \begin{array}{l} x=3 \Rightarrow u = \arctan \sqrt{3} = \frac{\pi}{3} \\ x=\frac{1}{3} \Rightarrow u = \arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{array} \right\} \Rightarrow V = 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{u^2} du$	B1
		(6)

(b)	$V = (2\pi) \left[-\frac{1}{u} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1
	$= (2\pi) \left[-\frac{1}{\pi/3} - \left(-\frac{1}{\pi/6} \right) \right]$	dM1
	Or $= (2\pi) \left[-\frac{1}{\arctan \sqrt{x}} \right]_{\frac{1}{3}}^3 = (2\pi) \left(-\frac{1}{\arctan \sqrt{3}} - \left(-\frac{1}{\arctan \sqrt{1/3}} \right) \right)$	
	$= 2\pi \left(-\frac{3}{\pi} + \frac{6}{\pi} \right) = 6$	A1
		(3)
(9 marks)		

Question Number	Scheme	Marks
1	$\int 2x \ln(3x) dx = x^2 \ln(3x) - \int \frac{1}{x} \times x^2 dx$	M1
	$= \dots + \dots x^2$	dM1
	$x^2 \ln(3x) - \frac{x^2}{2}$	A1
	$9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1
		(5 marks)

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dt} = 6t$ o.e.	B1
	$\text{Area} = \left(\int \sin t \sin 2t \times 6t dt = \right) \int \sin t \times 2 \sin t \cos t \times 6t dt$	M1
	$= 12 \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t dt$	A1
		(3)
(b)	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - (12) \int_0^{\frac{\pi}{2}} \frac{1}{3} \sin^3 t dt$	M1
	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3} \right) \int_0^{\frac{\pi}{2}} \sin t (1 - \cos^2 t) dt$	M1
	$= \left[4t \sin^3 t + 4 \cos t - \frac{4}{3} \cos^3 t \right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3} \right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

Question Number	Scheme	Marks
8(a)	$x = 2$	B1
		(1)
(b)	$(2.5, 1.5)$	B1
		(1)
(c)	$\frac{dy}{dx} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$	M1
	$t = 2 \Rightarrow \frac{dy}{dx} = \frac{5}{3} \Rightarrow y - 1.5 = -\frac{3}{5}(x - 2.5)$	dM1
	$3x + 5y = 15^*$	A1*
		(3)

(d)	l crosses x -axis at $x = 5$	B1
	$\frac{1}{3} \pi \times 1.5^2 (5 - 2.5) \left(= \frac{15}{8} \pi \right)$	M1
	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{dt} dt = \pi \int \left(t - \frac{1}{t} \right)^2 \left(1 - \frac{1}{t^2} \right) dt$	M1
	$\pi \int \left(t^2 - 2 + \frac{1}{t^2} \right) \left(1 - \frac{1}{t^2} \right) dt = \pi \int \left(t^2 + \frac{3}{t^2} - \frac{1}{t^4} - 3 \right) dt = \dots$	M1
	$\pi \left[\frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t \right]$	A1
	$\pi \left[\frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t \right]_1^2 = \pi \left(\left(\frac{8}{3} - \frac{3}{2} + \frac{1}{24} - 6 \right) - \left(\frac{1}{3} - 3 + \frac{1}{3} - 3 \right) \right)$	M1
	$V = \frac{13}{24} \pi + \frac{15}{8} \pi = \frac{29}{12} \pi$ Cao	A1
		(7)
		Total 12

Question Number	Scheme	Marks
5(i)	$\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx (+c)$	M1A1
	$\int x e^x dx = x e^x - \int e^x dx (+c)$	M1
	$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x (+c)$	A1
	Also allow $\int x^2 e^x dx = x^2 e^x - (2x e^x - 2e^x) (+c)$	
		(4)

(ii)	$u = (1-3x)^{\frac{1}{2}} \Rightarrow u^2 = 1-3x \Rightarrow 2u \frac{du}{dx} = -3$ or $u = (1-3x)^{\frac{1}{2}} \Rightarrow \frac{du}{dx} = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$	B1
	$\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{u} \left(-\frac{2u}{3}\right) du$ or $\int \frac{27x}{\sqrt{1-3x}} dx = 27 \int \frac{\frac{1-u^2}{3}}{\sqrt{1-3x}} \left(-\frac{2}{3}\sqrt{1-3x}\right) du$	M1
	$6 \int (u^2 - 1) du \text{ or } -6 \int (1 - u^2) du$	A1
	$6 \left(\frac{u^3}{3} - u \right) (+k)$	A1ft
	$6 \left(\frac{(1-3x)^{\frac{3}{2}}}{3} - (1-3x)^{\frac{1}{2}} \right) (+k) = 2(1-3x)^{\frac{1}{2}} (1-3x-3) (+k)$	M1
	$= -2(1-3x)^{\frac{1}{2}} (2+3x) (+k) \quad \text{or} \quad = -2(1-3x)^{\frac{1}{2}} (3x+2) (+k)$	A1
		(6)
		Total 10

Question Number	Scheme	Marks
5.	States or implies Volume = $\int_{\frac{1}{\sqrt{2}}}^k \pi \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx$ Attempts $\int \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx = \int \frac{144x}{(2x^2+3)^3} dx = -18(2x^2+3)^{-2}$ Sets $-\frac{18\pi}{(2k^2+3)^2} + \frac{18\pi}{(2 \times \frac{1}{2} + 3)^2} = \frac{713}{648} \pi$ $\Rightarrow -\frac{18}{(2k^2+3)^2} + \frac{9}{8} = \frac{713}{648} \Rightarrow (2k^2+3)^2 = 729 \Rightarrow k^2 = \dots$ $k = 2\sqrt{3}$	B1 M1A1 M1 ddM1 A1 (6 marks)

Question Number	Scheme	Marks
9(a)	Volume = $144\pi \int_0^{20} \frac{2x+3}{x^2+3x+5} dx$ or e.g. $\pi \int_0^{20} \left(12 \left(\frac{2x+3}{x^2+3x+5} \right)^{\frac{1}{2}} \right)^2 dx$	B1
	$144\pi \left[\ln(x^2+3x+5) \right]_0^{20}$	M1
	$= 144\pi (\ln 465 - \ln 5) = 144\pi \ln 93 \text{ (units}^3\text{)}$	dM1A1
		(4)
(a)		
(b)	$144\pi \left[\ln(x^2+3x+5) \right]_p^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln(p^2+3p+5)) = 360\pi$ or e.g. Let $p = 20 - a$: $144\pi \left[\ln(x^2+3x+5) \right]_{20-a}^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln((20-a)^2+3(20-a)+5)) = 360\pi$ or e.g. $144\pi \left[\ln(x^2+3x+5) \right]_0^{20-a} = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln((20-a)^2+3(20-a)+5) - \ln 5) = 144\pi \ln 93 - 360\pi$ or e.g. $144\pi \left[\ln(x^2+3x+5) \right]_0^p = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln(p^2+3p+5) - \ln 5) = 144\pi \ln 93 - 360\pi$	M1
	$\frac{5}{2} = \ln \left(\frac{465}{p^2+3p+5} \right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2+3p+5}$ $\Rightarrow p^2+3p+5 - 465e^{-\frac{5}{2}} = 0 \Rightarrow p = \dots$ or e.g. $\ln \left(\frac{a^2-43a+465}{5} \right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{a^2-43a+465}{5} = e^{\left(\frac{144\ln 93 - 360}{144} \right)} \Rightarrow a^2-43a+465 - 5e^{\left(\frac{144\ln 93 - 360}{144} \right)} = 0 \Rightarrow a = \dots$ or e.g. $\ln \left(\frac{p^2+3p+5}{5} \right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{p^2+3p+5}{5} = e^{\left(\frac{144\ln 93 - 360}{144} \right)} \Rightarrow p^2+3p+5 - 5e^{\left(\frac{144\ln 93 - 360}{144} \right)} = 0 \Rightarrow p = \dots$	dM1
	$a = \text{awrt } 15.5$	A1
		(3)
(b)		(7 marks)

Question Number	Scheme	Marks
7	$x = 4 \sin \theta \Rightarrow \frac{dx}{d\theta} = 4 \cos \theta$	B1
	$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{(16-16 \sin^2 \theta)^{\frac{3}{2}}} d\theta = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$	M1 A1
	$= \frac{1}{16} \tan \theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60^\circ) - f(30^\circ)$	M1
	$= \frac{1}{16} \tan \frac{\pi}{3} - \frac{1}{16} \tan \frac{\pi}{6} = \frac{\sqrt{3}}{24}$	A1
		(6 marks)

Question Number	Scheme	Marks
6	$u = \sqrt{x^3+1} \Rightarrow u^2 = x^3+1 \Rightarrow 2u \frac{du}{dx} = 3x^2$ or $u = \sqrt{x^3+1} \Rightarrow u^2 = x^3+1 \Rightarrow 2u = 3x^2 \frac{dx}{du}$ or $u = \sqrt{x^3+1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(x^3+1)^{-\frac{1}{2}} \times 3x^2$ or $x = (u^2-1)^{\frac{1}{3}} \Rightarrow \frac{dx}{du} = \frac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\int \frac{9x^5}{\sqrt{x^3+1}} dx = \int \frac{9x^5}{u} \frac{2u}{3x^2} du = \int 6x^3 du = 6 \int (u^2-1) du$ or e.g. $\int \frac{9x^5}{\sqrt{x^3+1}} dx = \int \frac{9x^5}{\sqrt{x^3+1}} \frac{2\sqrt{x^3+1}}{3x^2} du = \int 6x^3 du = 6 \int (u^2-1) du$	M1A1
	$6 \int (u^2-1) du = 2u^3 - 6u (+c) = 2(x^3+1)^{\frac{3}{2}} - 6(x^3+1)^{\frac{1}{2}} (+c)$	M1
	$= 2(x^3+1)^{\frac{1}{2}} [(x^3+1)-3] + c = 2(x^3+1)^{\frac{1}{2}} (x^3-2) + c$	A1
		(5)
		Total 5

Question Number	Scheme	Marks
6 (a)(i)	$\text{Area } R = \int y \frac{dx}{dt} dt = \int 4 \sin t \times -4 \sin 2t dt$ $= -\int 32 \sin^2 t \cos t dt$ $x=0 \Rightarrow t = \frac{\pi}{4} \text{ and } y=0 \Rightarrow t=0 \Rightarrow \text{Area} = -\int_{\frac{\pi}{4}}^0 32 \sin^2 t \cos t dt = \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t dt \quad *$	M1 A1 dM1 A1* (4)
(ii)	$\text{Area} = \left[\frac{32}{3} \sin^3 t \right]_0^{\frac{\pi}{4}} = \frac{32}{3} \times \frac{2\sqrt{2}}{8} = \frac{8\sqrt{2}}{3}$	M1 A1 (2)
(b)	Attempts to use $\cos 2t = 1 - 2 \sin^2 t \Rightarrow \frac{x}{2} = 1 - 2 \left(\frac{y}{4} \right)^2$ $y = \sqrt{8 - 4x}$	M1A1 A1 (3)
(c)	Range is $0 \leq x \leq 4$	B1 (1)
		(10 marks)

Question Number	Scheme	Marks
9(a)	$\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3} \text{ (or } 60^\circ) \text{ (Allow } x = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \text{ (or } 60^\circ))$	B1
	$V = (\pi) \int y^2 dx = (\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta \text{ oe}$	M1A1
	$4(\pi) \int \sin^2 2\theta \sec^2 \theta d\theta = 4(\pi) \int 4 \sin^2 \theta \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}} d\theta$	dM1
	$= 16(\pi) \int \sin^2 \theta d\theta \text{ oe e.g. } 16(\pi) \int (1 - \cos^2 \theta) d\theta$	A1
	$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \Rightarrow 16(\pi) \int \sin^2 \theta d\theta = 16(\pi) \int \frac{1 - \cos 2\theta}{2} d\theta$	dM1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi (1 - \cos 2\theta) d\theta$	A1 Cso
		(7)
(b)	$\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$	B1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi (1 - \cos 2\theta) d\theta = [8\pi\theta - 4\pi \sin 2\theta]_0^{\frac{\pi}{3}} = \frac{8}{3}\pi^2 - 2\sqrt{3}\pi$	M1 A1
		(3)
		(10 marks)

Question	Scheme	Marks
2(a)	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$	
	$3x+4 = A(2x+1)^2 + B(x-2)(2x+1) + C(x-2)$ E.g. $x=2 \rightarrow 10 = A \times 25 \Rightarrow A = \dots$ or $x = -\frac{1}{2} \rightarrow \frac{5}{2} = C \times -\frac{5}{2} \Rightarrow C = \dots$ or $x^2: 0 = 4A + 2B, x: 3 = 4A - 3B + C, x^0: 4 = A - 2B - 2C \rightarrow A = \dots$	M1
	One of $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
	$\Rightarrow A = \dots, B = \dots$ and $C = \dots$	M1
	$\frac{3x+4}{(x-2)(2x+1)^2} = \frac{2}{5(x-2)} - \frac{4}{5(2x+1)} - \frac{1}{(2x+1)^2}$ or $A = \frac{2}{5}, B = -\frac{4}{5}, C = -1$	A1
		(4)

(b)	$\int \frac{p}{x-2} dx = \alpha \ln(x-2) \text{ or } \int \frac{q}{2x+1} dx = \beta \ln(2x+1)$ Alt: $\int \frac{8x+9}{(2x+1)^2} dx = \int \frac{8x+4}{(2x+1)^2} + \frac{5}{(2x+1)^2} dx = \gamma \ln(2x+1)^2 + \dots$	M1
	$\int \frac{k}{(2x+1)^2} dx = \frac{K}{2x+1}$	M1
	$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} dx = \left[\frac{2}{5} \ln(x-2) - \frac{4}{5} \times \frac{1}{2} \ln(2x+1) + \frac{1}{4x+2} \right]_7^{12}$	A1ft
	$= \left(\frac{2}{5} \ln(10) - \frac{2}{5} \ln(25) + \frac{1}{50} \right) - \left(\frac{2}{5} \ln(5) - \frac{2}{5} \ln(15) + \frac{1}{30} \right) = \dots$	DM1
	$= \frac{2}{5} \ln \frac{10}{25} + \frac{1}{50} - \frac{2}{5} \ln \frac{5}{15} - \frac{1}{30} = \frac{2}{5} \ln \frac{10 \times 15}{25 \times 5} + \dots$	M1
	$\frac{2}{5} \ln \frac{6}{5} - \frac{1}{75} \quad (\text{oe})$	A1
		(6)
(10 marks)		

Question Number	Scheme	Marks
5	$u = 3 + \sqrt{2x-1} \Rightarrow x = \frac{(u-3)^2 + 1}{2} \Rightarrow \frac{dx}{du} = u-3$ or $u = 3 + \sqrt{2x-1} \Rightarrow \frac{du}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}} \times 2 = \frac{1}{\sqrt{2x-1}} = \frac{1}{u-3}$	M1 A1
	$\int \frac{4}{3 + \sqrt{2x-1}} dx = \int \frac{4}{u} \times (u-3) du$	M1
	$\int \frac{4}{u} \times (u-3) du = \int \left(4 - \frac{12}{u}\right) du$	dM1
	$\int \left(4 - \frac{12}{u}\right) du = 4u - 12 \ln u \quad \text{or} \quad k(4u - 12 \ln u)$	ddM1 A1ft
	$\int_1^{13} \frac{4}{3 + \sqrt{2x-1}} dx = [4u - 12 \ln u]_4^8 = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$ or $\int_1^{13} \frac{4}{3 + \sqrt{2x-1}} dx = \left[4(3 + \sqrt{2x-1}) - 12 \ln(3 + \sqrt{2x-1})\right]_1^{13} = (4 \times 8 - 12 \ln 8) - (4 \times 4 - 12 \ln 4)$	M1
	$= 16 - 12 \ln 2$	A1
		(8 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question	Scheme	Marks
7 (i)	Chooses a suitable substitution for $\int_1^5 \frac{3x}{\sqrt{2x-1}} dx$ E.g. $u = \sqrt{2x-1}$ E.g. $u = 2x-1$ $u = \sqrt{2x-1} \Rightarrow \frac{dx}{du} = u$ $u = 2x-1 \Rightarrow \frac{dx}{du} = \frac{1}{2}$ $\int \frac{3x}{\sqrt{2x-1}} dx$ $\int \frac{3x}{\sqrt{2x-1}} dx$ $= \int \frac{3}{2}u^2 + \frac{3}{2} du$ $= \int \frac{3}{4}u^{\frac{1}{2}} + \frac{3}{4}u^{-\frac{1}{2}} du$ $= \frac{1}{2}u^3 + \frac{3}{2}u$ $= \frac{1}{2}u^{\frac{3}{2}} + \frac{3}{2}u^{\frac{1}{2}}$ Limits $= \left[\frac{1}{2}u^3 + \frac{3}{2}u \right]_1^3 = \dots$ $= \left[\frac{1}{2}u^{\frac{3}{2}} + \frac{3}{2}u^{\frac{1}{2}} \right]_1^9 = \dots$ $\left(\frac{1}{2} \times 3^3 + \frac{3}{2} \times 3 \right) - \left(\frac{1}{2} \times 1^3 + \frac{3}{2} \times 1 \right) = 16$ $\left(\frac{1}{2} \times 9^{\frac{3}{2}} + \frac{3}{2} \times 9^{\frac{1}{2}} \right) - \left(\frac{1}{2} \times 1 + \frac{3}{2} \times 1 \right) = 16$	B1 M1 dM1 A1 M1 A1 (6)
7 (ii)	$\frac{6x^2-16}{(x+1)(2x-3)} = 3 + \dots$ $\frac{6x^2-16}{(x+1)(2x-3)} = 3 + \frac{2}{x+1} - \frac{1}{2x-3}$ $\int \frac{6x^2-16}{(x+1)(2x-3)} dx = \int 3 + \frac{2}{x+1} - \frac{1}{2x-3} dx$ $= 3x + 2 \ln x+1 - \frac{1}{2} \ln 2x-3 + c$	B1 M1 A1 M1 A1ft A1 (6) (12 marks)

Question	Scheme	Marks
1	$\int x \cos 3x dx = \frac{1}{3} x \sin 3x - \frac{1}{3} \int \sin 3x dx$ $= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x$ $\int_0^{\frac{\pi}{6}} x \cos 3x dx = \left[\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x \right]_0^{\frac{\pi}{6}} = \frac{1}{3} \frac{\pi}{6} \sin \frac{\pi}{2} + \frac{1}{9} \cos \frac{\pi}{2} - \left(0 + \frac{1}{9} \right)$ $= \frac{\pi}{18} - \frac{1}{9}$	M1 dM1 A1 M1 A1 (5)
		(5 marks)

Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$ $u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \frac{u+u^3}{(5+u^2)^3} \times \frac{1}{1+u^2} du$ $\int \frac{u}{(5+u^2)^3} du *$	B1 M1dM1 A1*
		(4)
(b)	$\int \frac{u}{(5+u^2)^3} du = -\frac{1}{4}(5+u^2)^{-2} + c = -\frac{1}{4}(5+\tan^2 x)^{-2} (+c)$	M1A1
		(2)
		(6 marks)

Question	Scheme	Marks
5(a)	$y = \frac{2}{t(3-t)} = 1 \Rightarrow 2 = 3t - t^2 \Rightarrow t = \dots$	M1
	$(t^2 - 3t + 2 = (t-1)(t-2) = 0 \Rightarrow) a=1, b=2$	A1
		(2)
(b)	Area under curve $= \int_{t=1}^{t=2} y dx = \int_1^2 y \frac{dx}{dt} dt = \int_1^2 \frac{2}{t(3-t)} \times (2t+2) dt$	M1 A1
	$t=1 \Rightarrow x=3, \quad t=2 \Rightarrow x=8$	B1
	So area of R $= 1 \times (8-3) - \int_1^2 \frac{2}{t(3-t)} \times (2t+2) dt$	M1
	$= 5 - 4 \int_1^2 \frac{t+1}{t(3-t)} dt$	A1
		(5)

(c)(i)	$\frac{t+1}{t(3-t)} = \frac{A}{t} + \frac{B}{3-t}$	M1
	$\Rightarrow t+1 = A(3-t) + Bt$	
	E.g. $t=0 \Rightarrow 1=3A \Rightarrow A=\frac{1}{3}$; $t=3 \Rightarrow 4=3B \Rightarrow B=\frac{4}{3}$	M1
	$\frac{t+1}{t(3-t)} = \frac{1}{3t} + \frac{4}{3(3-t)}$	A1
	$\int \frac{t+1}{t(3-t)} dt = \int \frac{1}{3t} + \frac{4}{3(3-t)} dt = \frac{1}{3} \ln t - \frac{4}{3} \ln(3-t)$	M1
	Area = $5 - 4 \left[\frac{1}{3} \ln t - \frac{4}{3} \ln(3-t) \right]_1^2 = 5 - 4 \left(\frac{1}{3} \ln 2 - \frac{4}{3} \ln 1 - \frac{1}{3} \ln 1 + \frac{4}{3} \ln 2 \right)$ $= 5 - 4 \left(\frac{1}{3} \ln 2 + \frac{4}{3} \ln 2 \right)$	dM1
(ii)	$= 5 - \frac{20}{3} \ln 2$	A1
		(6)
	(13 marks)	

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dt} = 6t$ o.e.	B1
	Area = $\left(\int \sin t \sin 2t \times 6t dt = \right) \int \sin t \times 2 \sin t \cos t \times 6t dt$	M1
	$= 12 \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t dt$	A1
		(3)
(b)	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - (12) \int_0^{\frac{\pi}{2}} \frac{1}{3} \sin^3 t dt$	M1
	$= (12) \left[\frac{t}{3} \sin^3 t \right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3} \right) \int_0^{\frac{\pi}{2}} \sin t (1 - \cos^2 t) dt$	M1
	$= \left[4t \sin^3 t + 4 \cos t - \frac{4}{3} \cos^3 t \right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3} \right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8