

Question Number	Scheme	Marks
6(a)	$4y^2 + 3x = 6ye^{-2x}$	
	$4y^2 + 3x \rightarrow 8y \frac{dy}{dx} + 3$	B1
	$6ye^{-2x} \rightarrow -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx}$	M1 A1
	$8y \frac{dy}{dx} + 3 = -12ye^{-2x} + 6e^{-2x} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y}$ oe	M1 A1
		(5)
(b)	Sets $x = 0$ in $4y^2 + 3x = 6ye^{-2x} \Rightarrow y = \frac{3}{2}$ oe	B1
	Substitutes $\left(0, \frac{3}{2}\right)$ in their $\frac{dy}{dx} = \frac{12ye^{-2x} + 3}{6e^{-2x} - 8y} = \left(\frac{7}{-2}\right)$	M1
	$m_N = -1 \div \frac{7}{-2} \Rightarrow y = \frac{2}{7}x + \frac{3}{2}$	dM1
	$y = \frac{2}{7}x + \frac{3}{2}$ oe e.g. $y = \frac{6}{21}x + \frac{3}{2}$	A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
7(a)	$\frac{du}{dx} = e^x$	B1
	$\int \frac{24e^x - 2}{(2e^x - 1)^2} dx = \int \frac{24\left(u + \frac{1}{2}\right) - 2}{(2u)^2} \times \frac{du}{\left(u + \frac{1}{2}\right)}$	M1
	$= \int \frac{24u + 12 - 2}{(2u)^2} \times \frac{2}{2u + 1} du = \int \frac{48u + 20}{4u^2(2u + 1)} du = \int \frac{12u + 5}{u^2(2u + 1)} du$	A1
	$x = \ln 3 \rightarrow a = \frac{5}{2}, \quad x = \ln 8 \rightarrow b = \frac{15}{2}$	B1
		(4)

(a) Note that apart from the limits, this is a given answer so check the work carefully.

(b)	$12u + 5 \equiv A(2u+1) + Bu(2u+1) + Cu^2 \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$ or e.g. $12u + 5 \equiv (2B+C)u^2 + (2A+B)u + A \Rightarrow A = \dots \text{ or } B = \dots \text{ or } C = \dots$	M1
	2 of: $A = 5, B = 2, C = -4$	A1
	$\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$	A1
		(3)
(b) M1: Attempts to find at least one of the values for A, B and C by multiplying up and substituting values or by multiplying up and comparing coefficients. When multiplying up must have at least one of the terms on the rhs correct for their lhs. A1: Two of $A = 5, B = 2, C = -4$ A1: Correct <u>fractions</u>: $\frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)}$ oe. Allow $\frac{5}{u^2} + \frac{2}{u} + -\frac{4}{(2u+1)}$ If the correct fractions are not seen in part (b), allow this mark to score if the correct fractions are seen in part (c).		

(c)	$\int \frac{5}{u^2} + \frac{2}{u} - \frac{4}{(2u+1)} du = -5u^{-1} + 2 \ln u - 2 \ln(2u+1) (+c)$	M1A1ft
	$\left[-5u^{-1} + 2 \ln u - 2 \ln(2u+1) \right]_{2.5}^{7.5}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$ or $\left[-5 \left(e^x - \frac{1}{2} \right)^{-1} + 2 \ln \left(e^x - \frac{1}{2} \right) - 2 \ln \left(2 \left(e^x - \frac{1}{2} \right) + 1 \right) \right]_{\ln 3}^{\ln 8}$ $= (-5 \times 7.5^{-1} + 2 \ln 7.5 - 2 \ln(16)) - (-5 \times 2.5^{-1} + 2 \ln 2.5 - 2 \ln(6))$	dM1
	$\frac{4}{3} + \ln \left(\frac{81}{64} \right)$	A1
		(4)
		(11 marks)

Question Number	Scheme	Marks
2(a)	$2^x \rightarrow 2^x \ln 2$	B1
	$y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	$4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$(10y + 4x^2) \frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

2(b)	At $x = 0$ $5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10 \times 2^0 \ln 2 - 3 - 8 \times 0 \times 3}{10 \times 3 + 4 \times 0^2} = \frac{10 \ln 2 - 3}{30}$	M1 A1
		(2)

Question Number	Scheme	Marks
9(a)	Volume = $144\pi \int_0^{20} \frac{2x+3}{x^2+3x+5} dx$ or e.g. $\pi \int_0^{20} \left(12 \left(\frac{2x+3}{x^2+3x+5} \right)^{\frac{1}{2}} \right)^2 dx$	B1
	$144\pi \left[\ln(x^2+3x+5) \right]_0^{20}$	M1
	$= 144\pi (\ln 465 - \ln 5) = 144\pi \ln 93 \text{ (units}^3\text{)}$	dM1A1
		(4)

(a)

(b)	$144\pi \left[\ln(x^2+3x+5) \right]_p^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln(p^2+3p+5)) = 360\pi$ or e.g. $\text{Let } p = 20 - a: 144\pi \left[\ln(x^2+3x+5) \right]_{20-a}^{20} = 360\pi$ $\Rightarrow 144\pi (\ln 465 - \ln((20-a)^2+3(20-a)+5)) = 360\pi$ or e.g. $144\pi \left[\ln(x^2+3x+5) \right]_0^{20-a} = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln((20-a)^2+3(20-a)+5) - \ln 5) = 144\pi \ln 93 - 360\pi$ or e.g. $144\pi \left[\ln(x^2+3x+5) \right]_0^p = 144\pi \ln 93 - 360\pi$ $\Rightarrow 144\pi (\ln(p^2+3p+5) - \ln 5) = 144\pi \ln 93 - 360\pi$	M1
	$\frac{5}{2} = \ln \left(\frac{465}{p^2+3p+5} \right) \Rightarrow e^{\frac{5}{2}} = \frac{465}{p^2+3p+5}$ $\Rightarrow p^2+3p+5 - 465e^{\frac{5}{2}} = 0 \Rightarrow p = \dots$ or e.g. $\ln \left(\frac{a^2-43a+465}{5} \right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{a^2-43a+465}{5} = e^{\left(\frac{144\ln 93 - 360}{144} \right)} \Rightarrow a^2-43a+465 - 5e^{\left(\frac{144\ln 93 - 360}{144} \right)} = 0 \Rightarrow a = \dots$ or e.g. $\ln \left(\frac{p^2+3p+5}{5} \right) = \frac{144\pi \ln 93 - 360\pi}{144\pi}$ $\Rightarrow \frac{p^2+3p+5}{5} = e^{\left(\frac{144\ln 93 - 360}{144} \right)} \Rightarrow p^2+3p+5 - 5e^{\left(\frac{144\ln 93 - 360}{144} \right)} = 0 \Rightarrow p = \dots$	dM1
	$a = \text{awrt } 15.5$	A1
		(3)
		(7 marks)

(h)

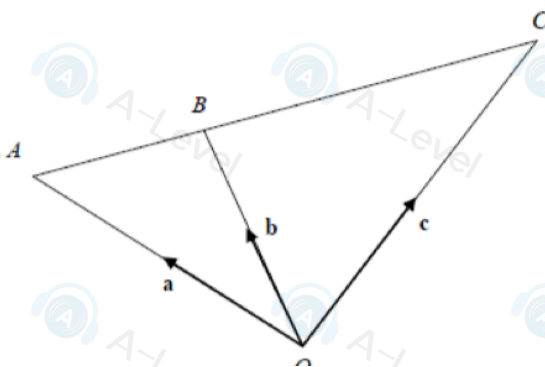
Question Number	Scheme	Marks
1 (a)	$\frac{5x+10}{(1-x)(2+3x)} \equiv \frac{A}{1-x} + \frac{B}{2+3x} \Rightarrow$ Value for A or B	M1
	One correct value, either $A = 3$ or $B = 4$	A1
	Correct PF form $\frac{3}{1-x} + \frac{4}{2+3x}$	A1
		(3)
(b)(i)	$\frac{A}{1-x} = A(1-x)^{-1} = A(1+x+x^2+\dots)$	B1
	$\left\{\frac{B}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1} = \left\{\frac{B}{2}\right\}\left(1+(-1)\frac{3x}{2} + \frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2 + \dots\right); = \frac{B}{2}\left(1-\frac{3x}{2} + \frac{9x^2}{4} + \dots\right)$	M1; A1
	$f(x) = 3 \times \left(1+x+x^2+\dots\right) + \frac{4}{2}\left(1-\frac{3x}{2} + \frac{9x^2}{4} + \dots\right)$	M1
	$= 5 + \frac{15}{2}x^2 + \dots$	A1
		(5)
(b)(ii)	$ x < \frac{2}{3}$	B1
		(1)
		(9 marks)

Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) " $n^2 - 4n + 5$ is even and n is even"	B1
	Sets e.g. $n = 2k$ and attempts $(2k)^2 - 4(2k) + 5$	M1
	$4k^2 - 8k + 5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4(k^2 - 2k) + 5$ or e.g. $4(k^2 - 2k + 1) + 1$ or e.g. $2(2k^2 - 4k + 2) + 1$ or e.g. $(2k - 2)^2 + 1$ Condone $4k^2 - 8k + 5 = \text{even} - \text{even} + \text{odd} = \text{odd}$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Question Number	Scheme	Marks
6(a)	$(2, 3, -7)$	B1
		(1)
(b)	Attempts $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix} = 1 \times 4 + 2 \times -1 + 2 \times 8 = (18)$	M1
	Attempts $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta: 18 = \sqrt{1^2 + 2^2 + 2^2} \times \sqrt{4^2 + (-1)^2 + 8^2} \cos \theta$	dM1
	$\cos \theta = \frac{2}{3}$	A1
		(3)
(c)	Uses $\lambda = 6$ to find length PQ E.g. $\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k} \Rightarrow PQ = \sqrt{6^2 + 12^2 + 12^2} = (18)$ Or $PQ = 6 \times \sqrt{1^2 + 2^2 + 2^2} = (18)$	M1
	Area $QPR = \frac{1}{2} ab \sin C = \frac{1}{2} \times 18^2 \times \sqrt{1 - \left(\frac{2}{3}\right)^2} = 54\sqrt{5}$	M1 A1
		(3)
(d)	Attempts a correct method of finding at least one value for μ e.g. $(4\mu)^2 + \mu^2 + (8\mu)^2 = 6^2 + 12^2 + 12^2$ $\Rightarrow \mu = \frac{"18"}{\sqrt{4^2 + (-1)^2 + 8^2}} = (\pm)2$	M1
	Attempts one correct position $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} \pm 2 \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$	dM1
	Possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$	A1
		(3)
		(10 marks)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15\right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9\right)$ $= \frac{2}{3} \ln\left(\frac{6}{5}\right) = \frac{1}{3} \ln\left(\frac{36}{25}\right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
9 (i)	Attempts two of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$ either way around Attempts $\mathbf{c} - \mathbf{b} = 2 \times (\mathbf{b} - \mathbf{a})$ or such as $\mathbf{c} - \mathbf{a} = 3 \times (\mathbf{b} - \mathbf{a})$ $\Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ *	M1 dM1 A1 * (3)
(ii)	Assume that there exists a number n that isn't a multiple of 3 yet n^2 is a multiple of 3 If n is not a multiple of 3 then $m = 3p + 1$ or $m = 3p + 2$ ($p \in \mathbb{N}$) giving $m^2 = (3p + 1)^2 = 9p^2 + 6p + 1$ Or $m^2 = (3p + 2)^2 = 9p^2 + 12p + 4 = 3(3p^2 + 4p + 1) + 1$ $(3p + 1)^2 = 9p^2 + 6p + 1 = 3(3p^2 + 2p) + 1$ is one more than a multiple of 3 $(3p + 2)^2 = 9p^2 + 12p + 4$ is not a multiple of 3 as 3 does not divide into 4 (exactly) Hence if n is a multiple of 3 then n^2 is a multiple of 3	B1 M1 M1 A1 A1 (5) (8 marks)



Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = \frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)}$ $\int \frac{2/3}{(t+1)} - \frac{4/3}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15\right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9\right)$ $= \frac{2}{3} \ln\left(\frac{6}{5}\right) = \frac{1}{3} \ln\left(\frac{36}{25}\right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
1(a)	$(8-3x)^{-\frac{1}{3}} = \frac{1}{2} \left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}}$	B1
	$\left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}} = 1 + \left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right) + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{3}{8}x\right)^2 + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{3}{8}x\right)^3 + \dots$	M1
	$(8-3x)^{-\frac{1}{3}} = \frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2 + \frac{7}{1536}x^3 + \dots$	A1 A1
		(4)
(b)	$\frac{1}{2} + \frac{1}{16}\left(\frac{2}{3}\right) + \frac{1}{64}\left(\frac{2}{3}\right)^2 + \frac{7}{1536}\left(\frac{2}{3}\right)^3 + \dots = \frac{2851}{5184} \Rightarrow \sqrt[3]{6} = \left(\frac{2851}{5184}\right)^{-1} = \dots$	M1
	$= \frac{5184}{2851} \text{ or } 1 \frac{2333}{2851}$	A1
		(2)
		Total 6

Question	Scheme	Marks
5(a)	$\int x^2 \cos 2x \, dx = x^2 \cdot \frac{1}{2} \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	M1 A1
	$= x^2 \cdot \frac{1}{2} \sin 2x - \left(-\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x \, dx \right)$	M1
	$= \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x - \frac{1}{4} \sin 2x (+c)$	A1
		(4)
(b)	$\frac{dy}{dt} = \frac{t^2 \cos^2 t}{y^2} \Rightarrow \text{e.g. } \int y^2 \frac{dy}{dt} \, dt \text{ or } \int y^2 \, dy = \int t^2 \cos^2 t \, dt$	B1
	$\Rightarrow \frac{y^3}{3} = \dots ; \dots = \frac{1}{2} \int t^2 + t^2 \cos 2t \, dt$	M1 ; M1
	$\dots = \frac{1}{6} t^3 + \frac{1}{2} \left(\frac{1}{2} t^2 \sin 2t + \frac{1}{2} t \cos 2t - \frac{1}{4} \sin 2t \right) (+c)$	M1
	$\Rightarrow y^3 = \frac{1}{2} t^3 + \frac{3}{4} t^2 \sin 2t + \frac{3}{4} t \cos 2t - \frac{3}{8} \sin 2t + c$	A1ft
		(5)
(9 marks)		

Question Number	Scheme	Marks
1(a)	$2y^2 - 3y = 7 + 13 \Rightarrow 2y^2 - 3y - 20 = 0 \Rightarrow y = \dots$	M1
	$y = -\frac{5}{2} \text{ and } y = 4$	A1
		(2)
(b)	$2y^2 \rightarrow 4y \frac{dy}{dx}$	B1
	$-6xy \rightarrow \pm \dots x \frac{dy}{dx} \pm \dots y$	M1
	$4y \frac{dy}{dx} - 6x \frac{dy}{dx} - 6y = 14e^{2x-1}$	A1
	$4(4) \frac{dy}{dx} - 6 \left(\frac{1}{2} \right) \frac{dy}{dx} - 6(4) = 14e^{2\left(\frac{1}{2}\right)-1} \Rightarrow \frac{dy}{dx} = \dots$	M1
	$y - "4" = " \frac{38}{13} " \left(x - \frac{1}{2} \right)$	M1
	$38x - 13y + 33 = 0$	A1
		(6)
(8 marks)		

Question Number	Scheme	Marks
3(a)	$B = 6 \times 4^{\frac{1}{2}} = 3$	B1
	$3 \times \left(-\frac{1}{2}\right) \left(\frac{A}{4}\right) = -\frac{1}{4} \Rightarrow A = \frac{2}{3}$	M1 A1
	$C = 3 \times \frac{\left(-\frac{1}{2}\right) \left(-\frac{1}{2} - 1\right) \left(\frac{A}{4}\right)^2}{2} \Rightarrow C = \frac{1}{32}$	M1 A1
		(5)

3(b)	$-6 < x \leq 6$	B1ft
		(1)
(c)	coefficient of x^3 is $3 \times \frac{\left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \left(-\frac{5}{2}\right) \left(\frac{A}{4}\right)^3}{3!} = \dots$	M1
	$-\frac{5}{1152}$	A1
		(2)

Question Number	Scheme	Marks
7. (a)	Attempts $\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ -5 \end{pmatrix} = 24 + 6 + 15 = 45$	M1
	Attempts to apply $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta \Rightarrow 45 = 7 \times \sqrt{50} \cos \theta$	dM1
	$\cos \theta = \frac{9\sqrt{2}}{14}$ oe	A1
		(3)
(b)	Attempts $\sin \theta = \sqrt{1 - \left(\frac{9\sqrt{2}}{14}\right)^2}$	M1
	Attempts to apply $ \mathbf{a} \mathbf{b} \sin \theta = 7 \times \sqrt{50} \times \frac{\sqrt{34}}{14}$	M1
	Area ABCD = $5\sqrt{17}$	A1
		(3) (6 marks)

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i} + 4\mathbf{j} - \mathbf{k} - (-10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}))$	M1
	e.g. $\mathbf{r} = -10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	A1
	e.g. $\mathbf{r} = -6\mathbf{i} + 4\mathbf{j} - \mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	
		(2)
(b)	$3 + 3\mu = -6 \Rightarrow \mu = -3$	M1
	$p + 12 = 4 \Rightarrow p = \dots$ or $q - 3 = -1 \Rightarrow q = \dots$	dM1
	$p = -8, q = 2$	A1
		(3)
(c)	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = 12 + 4 + 3$	M1
	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = \sqrt{26}\sqrt{26}\cos\theta = 19 \Rightarrow \cos\theta = \dots$	M1
	$(\cos\theta =) \frac{19}{26}$	A1
		(3)
(d)	$AC = AB\sin\theta = \sqrt{26}\sqrt{1 - \left(\frac{19}{26}\right)^2}$	M1
	$\frac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

Question Number	Scheme	Marks
4 (a)	Attempt at chain rule $y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	Attempt at product rule $2xy \rightarrow \alpha x \frac{dy}{dx} + \beta y$	M1
	Correct differentiation $8x + 2y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 24$ oe	A1
	Collects terms in $(2y - 2x) \frac{dy}{dx} = 24 - 8x + 2y$	M1
	$\left(\frac{dy}{dx} =\right) \frac{12 - 4x + y}{y - x}$ oe	A1
		(5)
(b)	Sets $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$	M1
	Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$	dM1
	$x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$	A1
	Solves $x = 6, y = 0$ or $x = 2, y = 8$	ddM1
	Chooses $a(x) = 2, b(y) = 8$ or states (2, 8) only	A1
		(5)
		(10 marks)

Question Number	Scheme	Marks
6 (a)(i)	$\overrightarrow{AB} = (\pm) \left[(8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}) - (2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) \right] = \dots$	M1
	$\overrightarrow{AB} = 6\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$	A1
(ii)	$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ o.e. such as $\mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 6 \\ -12 \end{pmatrix}$	B1ft
		(3)
(b)	Attempts $\pm \overrightarrow{CP} = \pm \begin{pmatrix} 2 + \lambda - 3 \\ -3 + \lambda - 5 \\ 5 - 2\lambda - 2 \end{pmatrix}$	M1
	$\overrightarrow{CP} \cdot \mathbf{k} = 0 \Rightarrow \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} \lambda - 1 \\ \lambda - 8 \\ -2\lambda + 3 \end{pmatrix} = 0 \Rightarrow 1(\lambda - 1) + 1(\lambda - 8) - 2(-2\lambda + 3) = 0$ Alt: $(\lambda - 1)^2 + (\lambda - 8)^2 + (-2\lambda + 3)^2 = 6\lambda^2 - 30\lambda + 74 = 6\left(\lambda - \frac{5}{2}\right)^2 + \frac{73}{2}$	dM1
	$\Rightarrow \lambda = \frac{5}{2}$ [use of $\overrightarrow{AB}$ in $\overrightarrow{CP}$ gives $\lambda = \frac{5}{12}$, use of $\overrightarrow{OB}$ $\lambda = \frac{-7}{2}$ or $\frac{-7}{12}$]	A1
	$\overrightarrow{OP} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$	ddM1, A1
		(5)
		(8 marks)

Question Number	Scheme	Marks
4(a)	$\sqrt{1 - 4x^2} = 1 - \frac{1}{2} \times 4x^2$	B1
	$\frac{\frac{1}{2} \times -\frac{1}{2} \times (-4x^2)^2}{2}$ or $\frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2} \times (-4x^2)^3}{3!}$ $= 1 - 2x^2 - 2x^4 - 4x^6 + \dots$	M1 A1, A1 (4)
(b)	Substitutes $x = \frac{1}{4}$ into both sides of (a) e.g. $\sqrt{\frac{3}{4}} \approx 1 - 2 \times \left(\frac{1}{4}\right)^2 - 2 \times \left(\frac{1}{4}\right)^4 - 4 \times \left(\frac{1}{4}\right)^6$ $\sqrt{3} \approx 1.7324$ cao	M1 A1 (2) (6 marks)

Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$ $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$ $\pm k = \lambda r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2} \checkmark *$	B1 B1 M1 A1*
		(4 marks)

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = \cos t + 6 \cos t \sin t \quad \frac{dy}{dt} = 3 \cos t - 2 \sin t$	B1B1
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{3 \cos t - 2 \sin t}{\cos t + 6 \cos t \sin t} = \frac{3 \cos \pi - 2 \sin \pi}{\cos \pi + 6 \cos \pi \sin \pi} = 3 \quad *$	M1A1*
		(4)
(b)	When $t = \pi, x = -3, y = -2$	B1
	$y - "-2" = 3(x - "-3")$	M1
	$y = 3x + 7$	A1
		(3)
(c)	$y = 3x + 7 \Rightarrow 3 \sin t + 2 \cos t = 3(\sin t - 3 \cos^2 t) + 7 \text{ or}$ $y = 3(x + 3 \cos^2 t) + 2 \cos t \Rightarrow 3x + 7 = 3x + 9 \cos^2 t + 2 \cos t$ $\Rightarrow 9 \cos^2 t + 2 \cos t - 7 = 0 \quad *$	M1
		A1*
		(2)
(d)	$\cos t = \frac{7}{9}$ $y = 3 \times \frac{\sqrt{32}}{9} + 2 \times \frac{7}{9} = \frac{4\sqrt{2}}{3} + \frac{14}{9}$	B1
		M1A1
		(3)
		(12 marks)

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{2(1-t) - (-1)(3+2t)}{(1-t)^2} = \frac{5}{(1-t)^2} \quad \text{and} \quad \frac{dy}{dt} = -2t$ $\text{At } t=2 \Rightarrow \frac{dy}{dx} = \frac{(-1)^2}{5} \times (-2(2)) = -\frac{4}{5}$ $(-7, -3)$ $y+3 = \frac{5}{4}(x+7)$ $5x-4y+23=0$	M1A1 B1 M1 A1 (5)
(b)	$x = \frac{3+2t}{1-t} \rightarrow t = \frac{x-3}{x+2}$ $y = 1 - \left(\frac{x-3}{x+2} \right)^2 = \frac{(x+2)^2 - (x-3)^2}{(x+2)^2} = \frac{10x-5}{(x+2)^2}$ $x \neq -2$	M1 dM1A1 B1 (4)
		(9 marks)

Question Number	Scheme	Marks
7(a)	Substitutes $x=3$ and $t=0$ into $x = \frac{k(9t+5)}{4t+3}$	M1
	$\Rightarrow 3 = \frac{5k}{3} \Rightarrow k = 1.8$ *	A1*
		(2)
(b)	4050	B1
		(1)
(c)	$\frac{3}{x(9-2x)} \equiv \frac{A}{x} + \frac{B}{9-2x}$	M1
	Either $A = \frac{1}{3}$ or $B = \frac{2}{3}$	A1
	$\frac{3}{9x-2x^2} \equiv \frac{1}{3x} + \frac{2}{3(9-2x)}$	A1
		(3)

(d)	$3 \frac{dx}{dt} = x(9-2x) \Rightarrow \int \frac{3}{x(9-2x)} dx = \int dt$ Or $3 \frac{dx}{dt} = x(9-2x) \Rightarrow \int \frac{1}{x(9-2x)} dx = \int \frac{1}{3} dt$ $\Rightarrow \frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t(+c) \text{ or } \frac{1}{9} \ln x - \frac{1}{9} \ln(9-2x) = \frac{1}{3} t(+c)$	M1 A1ft
	Substitutes $t=0, x=3 \Rightarrow c=0$	M1
	$\frac{1}{3} \ln x - \frac{1}{3} \ln(9-2x) = t \Rightarrow \left(\frac{x}{9-2x} \right) = e^{3t}$	ddM1
	$\left(\frac{9-2x}{x} \right) = e^{-3t} \Rightarrow \frac{9}{x} - 2 = e^{-3t} \Rightarrow x = \frac{9}{2+e^{-3t}}^*$	A1*
		(5)
(e)	4500	B1
		(1)
		(12 marks)

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4} - \frac{1}{2}x \right)^{-\frac{3}{2}} = 8(1-2x)^{-\frac{3}{2}}$	B1
	$(1-2x)^{-\frac{3}{2}} = 1 + \left(-\frac{3}{2}\right)(-2x) + \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2 + \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)\left(-\frac{3}{2}-2\right)}{3!}(-2x)^3 + \dots$	M1 A1
	$\left(\frac{1}{4} - \frac{1}{2}x \right)^{-\frac{3}{2}} = 8 + 24x + 60x^2 + 140x^3 + \dots$	A1, A1
		(5)
(b)	$n = 2$	B1
		(1)
(c)	$\left(\frac{1}{4} - \frac{1}{2}x \right)^2 = \frac{1}{16} - \frac{1}{4}x + \frac{1}{4}x^2$	B1
	$\left(\frac{1}{4} - \frac{1}{2}x \right)^{\frac{1}{2}} = \left(\frac{1}{16} - \frac{1}{4}x + \frac{1}{4}x^2 \right) (8 + 24x + 60x^2 + 140x^3 + \dots)$ $= 8 \times \frac{1}{16} + 24x \times \frac{1}{16} + 60x^2 \times \frac{1}{16} - 8 \times \frac{1}{4}x - 24x \times \frac{1}{4}x + 8 \times \frac{1}{4}x^2$	M1
	$= \frac{1}{2} - \frac{1}{2}x - \frac{1}{4}x^2 + \dots$	A1
		(3)
		Total 9

Question Number	Scheme	Marks
1(a)	$(8-3x)^{-\frac{1}{3}} = \frac{1}{2} \left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}}$	B1
	$\left(1 - \frac{3}{8}x\right)^{-\frac{1}{3}} = 1 + \left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right) + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)}{2!}\left(-\frac{3}{8}x\right)^2 + \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3!}\left(-\frac{3}{8}x\right)^3 + \dots$	M1
	$(8-3x)^{-\frac{1}{3}} = \frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2 + \frac{7}{1536}x^3 + \dots$	A1 A1
		(4)
(b)	$\frac{1}{2} + \frac{1}{16}\left(\frac{2}{3}\right) + \frac{1}{64}\left(\frac{2}{3}\right)^2 + \frac{7}{1536}\left(\frac{2}{3}\right)^3 + \dots = \frac{2851}{5184} \Rightarrow \sqrt[3]{6} = \left(\frac{2851}{5184}\right)^{-1} = \dots$	M1
	$= \frac{5184}{2851} \text{ or } 1 \frac{2333}{2851}$	A1
		(2)
		Total 6

Question Number	Scheme	Marks
4 (a)	$a=3, b=5$	B1
	E.g $u = \sqrt{2x+1} \Rightarrow \frac{dx}{du} = u \text{ or } \frac{du}{dx} = (2x+1)^{-\frac{1}{2}} \text{ o.e.}$	B1
	$\int \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \int 2u e^u u du$	M1
	$= \int_3^5 2u^2 e^u du$	A1
		(4)
(b)	$\int \sqrt{8x+4} e^{2x+1} dx = \int 2u^2 e^u du$	
	$= 2u^2 e^u - \int 4u e^u du$	M1
	$= 2u^2 e^u - (4ue^u - 4e^u) = 2u^2 e^u - 4ue^u + 4e^u$	dM1 A1ft
	$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \left[2u^2 e^u - 4ue^u + 4e^u \right]_3^5 = (50e^5 - 20e^5 + 4e^5) - (18e^3 - 12e^3 + 4e^3)$	ddM1
	$= 34e^5 - 10e^3$	A1
		(5)
		(9 marks)

Question	Scheme	Marks
9	For question 9 many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader. M1: Will be scored for setting up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage. A1: Scored for a correct statement from which it is possible to draw a contradiction. dM1: For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions. A1: Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technically a correct assumption as long as a relevant assumption has been made. E.g. Accept "Assume $n^2 - 2$ is divisible by 4 for all n"	
9	(Assume that there is an n with $n^2 - 2$ is divisible by 4 so) $n^2 - 2 = 4k$	M1
	then $n^2 = 4k + 2 = 2(2k + 1)$ (so is even)	A1
	Hence n^2 is even so $n (=2m)$ is even hence n^2 is a multiple of 4 As n^2 is a multiple of 4 then $n^2 - 2 = 4m^2 - 2 = 2(2m^2 - 1)$ cannot be a multiple of 4 (as $2m - 1$ is odd) so there is a contradiction.	dM1
	So the original assumption has been shown false. Hence " $n^2 - 2$ is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Question Number	Scheme	Marks
6(a)	$\frac{d\theta}{dt} = -k(\theta - 15)^2 \Rightarrow \int \frac{d\theta}{(\theta - 15)^2} = \int -k dt$	B1
	$\int \frac{d\theta}{(\theta - 15)^2} = -(\theta - 15)^{-1}$	M1
	$-\frac{1}{\theta - 15} = -kt + c$	A1
	$t = 0, \theta = 85 \Rightarrow -\frac{1}{70} = c$	M1
	$t = 10, \theta = 40 \Rightarrow \frac{1}{25} = 10k + \frac{1}{70} \Rightarrow k = \dots \left(\frac{9}{3500} \right)$	M1
	$\frac{1}{\theta - 15} = \frac{9t}{3500} + \frac{1}{70} \Rightarrow \theta = \dots$	M1
	$\theta = \frac{135t + 4250}{9t + 50}$	A1
		(7)
(b)	$20 = \frac{135t + 4250}{9t + 50} \Rightarrow t = \dots$	M1
	$t = \text{awrt } 72$	A1
		(2)
		Total 9

Question Number	Scheme	Marks
2	$\frac{dV}{dt} = \pm k$ $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dr} = 4\pi r^2$ $\pm k = \lambda r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{\pm k}{4\pi r^2} \Rightarrow \frac{dr}{dt} \propto \frac{1}{r^2} \checkmark *$	B1 B1 M1 A1*
		(4 marks)

Question Number	Scheme	Marks
2(a)	$4^{-\frac{1}{2}} \text{ or } \frac{1}{4^{\frac{1}{2}}} \text{ or } \frac{1}{2}$ $(4-5x)^{-\frac{1}{2}} = \frac{1}{2} \left(1 - \frac{5x}{4}\right)^{-\frac{1}{2}}$ $= \dots \left[1 + \left(-\frac{1}{2}\right) \times \left(-\frac{5x}{4}\right) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2!} \times \left(-\frac{5x}{4}\right)^2 + \dots \right]$ $= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$ $\frac{2+kx}{(2-3x)^3} = (2+kx) \left(\frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots \right)$	B1 M1A1 A1 (4)
(b)	Compares x terms leading to $k = \dots$ E.g. $\frac{10}{16} + \frac{k}{2} = \frac{3}{10} \Rightarrow k = -\frac{13}{20}$	M1 A1 (2)
(c)	Compares x^2 terms leading to $m = \dots$ E.g. $m = \frac{75}{128} + \frac{5}{16} \times -\frac{13}{20} \Rightarrow m = \frac{49}{128}$	M1 A1 (2)
		(8 marks)
Alt (a)	$4^{-\frac{1}{2}} \text{ or } \frac{1}{4^{\frac{1}{2}}}$ $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) 4^{-\frac{3}{2}} (-5x) + \frac{-\frac{1}{2} \times -\frac{3}{2}}{2} 4^{-\frac{5}{2}} (-5x)^2$ $= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots$	B1 M1A1 A1 (4)

Question Number	Scheme	Marks
2	$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \Rightarrow \int \frac{1}{y^2} dy = \int \frac{4}{\sqrt{4x+5}} dx$ $-\frac{1}{y} = 2\sqrt{4x+5} (+c)$ Substitutes $y = \frac{1}{3}, x = -\frac{1}{4} \Rightarrow -3 = 4 + c \Rightarrow c = \dots$ Rearranges $\frac{a}{y} = b\sqrt{4x+5} + c$ to $y = \dots$ $y = \frac{1}{7 - 2\sqrt{4x+5}}$	B1 M1, A1 dM1 ddM1 A1 (6) (6 marks)

Question Number	Scheme	Marks
9(a)	$\tan \theta = \sqrt{3} \Rightarrow k = \frac{\pi}{3} \text{ (or } 60^\circ \text{)} \text{ (Allow } x = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \text{ (or } 60^\circ \text{))}$	B1
	$V = (\pi) \int y^2 dx = (\pi) \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta \text{ oe}$	M1A1
	$4(\pi) \int \sin^2 2\theta \sec^2 \theta d\theta = 4(\pi) \int 4 \sin^2 \theta \cancel{\cos^2 \theta} \times \frac{1}{\cancel{\cos^2 \theta}} d\theta$	dM1
	$= 16(\pi) \int \sin^2 \theta d\theta \text{ oe e.g. } 16(\pi) \int (1 - \cos^2 \theta) d\theta$	A1
	$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \Rightarrow 16(\pi) \int \sin^2 \theta d\theta = 16(\pi) \int \frac{1 - \cos 2\theta}{2} d\theta$	dM1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi (1 - \cos 2\theta) d\theta$	A1 Cso
		(7)
(b)	$\int (1 - \cos 2\theta) d\theta \rightarrow \theta - \frac{\sin 2\theta}{2}$	B1
	$\text{Volume} = \int_0^{\frac{\pi}{3}} 8\pi (1 - \cos 2\theta) d\theta = [8\pi\theta - 4\pi \sin 2\theta]_0^{\frac{\pi}{3}} = \frac{8}{3}\pi^2 - 2\sqrt{3}\pi$	M1 A1
		(3)
		(10 marks)

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i} + 4\mathbf{j} - \mathbf{k} - (-10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}))$	M1
	e.g. $\mathbf{r} = -10\mathbf{i} + 5\mathbf{j} - 4\mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	A1
	e.g. $\mathbf{r} = -6\mathbf{i} + 4\mathbf{j} - \mathbf{k} \pm \lambda(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$	
		(2)
(b)	$3 + 3\mu = -6 \Rightarrow \mu = -3$	M1
	$p + 12 = 4 \Rightarrow p = \dots$ or $q - 3 = -1 \Rightarrow q = \dots$	dM1
	$p = -8, q = 2$	A1
		(3)
(c)	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = 12 + 4 + 3$	M1
	$(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (3\mathbf{i} - 4\mathbf{j} + \mathbf{k}) = \sqrt{26}\sqrt{26}\cos\theta = 19 \Rightarrow \cos\theta = \dots$	M1
	$(\cos\theta =) \frac{19}{26}$	A1
		(3)
(d)	$AC = AB\sin\theta = \sqrt{26}\sqrt{1 - \left(\frac{19}{26}\right)^2}$	M1
	$\frac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

Question	Scheme	Marks
9(a)	Volume, $V = \pi \int_{\frac{1}{3}}^3 y^2 dx = \pi \int_{\frac{1}{3}}^3 \frac{x^{-\frac{1}{2}}}{(1+x)\left(\arctan(\sqrt{x})\right)^2} dx$	B1
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ oe	M1 A1
	$\Rightarrow V = (\pi) \int \frac{2 \times \frac{1}{2} x^{-\frac{1}{2}}}{\left(\arctan(\sqrt{x})\right)^2 (1+x)} dx = (\pi) \int \frac{2}{u^2 (1 + \tan^2 u)} \sec^2 u du$	dM1
	$= \pi \int \frac{2}{u^2 \sec^2 u} \sec^2 u du = 2\pi \int \frac{1}{u^2} du$	A1
	$\left. \begin{array}{l} x = 3 \Rightarrow u = \arctan \sqrt{3} = \frac{\pi}{3} \\ x = \frac{1}{3} \Rightarrow u = \arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{array} \right\} \Rightarrow V = 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{u^2} du$	B1
		(6)

(b)	$V = (2\pi) \left[-\frac{1}{u} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1
	$= (2\pi) \left[-\frac{1}{\pi/3} - \left(-\frac{1}{\pi/6} \right) \right]$ Or $= (2\pi) \left[-\frac{1}{\arctan \sqrt{x}} \right]_{\frac{1}{3}}^3 = (2\pi) \left(-\frac{1}{\arctan \sqrt{3}} - \left(-\frac{1}{\arctan \sqrt{1/3}} \right) \right)$	dM1
	$= 2\pi \left(-\frac{3}{\pi} + \frac{6}{\pi} \right) = 6$	A1
		(3)
(9 marks)		

Question Number	Scheme	Marks
5(a)	$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x dx = -x^{-1} \ln x + \int x^{-1} \times \frac{1}{x} dx$ $= -x^{-1} \ln x - x^{-1} (+c)$	M1 dM1 A1 (3)
(b)	$\frac{3+2x+\ln x}{x^2} = 3x^{-2} + 2x^{-1} + x^{-2} \ln x \text{ oe}$ $\text{Area} = \left[-\frac{3}{x} + 2 \ln x - \frac{\ln x}{x} - \frac{1}{x} \right]_2^4$ $= \left(-\frac{3}{4} + 2 \ln 4 - \frac{\ln 4}{4} - \frac{1}{4} \right) - \left(-\frac{3}{2} + 2 \ln 2 - \frac{\ln 2}{2} - \frac{1}{2} \right) = 1 + 2 \ln 2$	B1 M1 M1 A1 (4) (7 marks)

Question Number	Scheme	Marks
7(a) Way 1	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \int \frac{1}{2} e^{2x} \cos x \, dx$	M1
	$= \dots - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	dm1
	$\int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x - \int \frac{1}{4} e^{2x} \sin x \, dx$	A1
	$\frac{5}{4} \int e^{2x} \sin x \, dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
7(a) Way 2	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + \int 2e^{2x} \cos x \, dx$	M1
	$= \dots + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	dm1
	$\int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x - \int 4e^{2x} \sin x \, dx$	A1
	$5 \int e^{2x} \sin x \, dx = -e^{2x} \cos x + 2e^{2x} \sin x \Rightarrow \int e^{2x} \sin x \, dx = \dots$	ddM1
	$= \frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x + c$	A1
		(5)
(b)	$\left(\frac{2}{5} e^{2\pi} \sin \pi - \frac{1}{5} e^{2\pi} \cos \pi \right) - \left(\frac{2}{5} e^0 \sin 0 - \frac{1}{5} e^0 \cos 0 \right) = \dots$	M1
	$= \frac{1}{5} e^{2\pi} + \frac{1}{5} = \frac{e^{2\pi} + 1}{5} *$	A1*
		(2)
		(7 marks)

Question Number	Scheme	Marks
7(a)	$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$	B1
	$u^2 = \tan^2 x = \sec^2 x - 1 \rightarrow \int \frac{u+u^3}{(5+u^2)^3} \times \frac{1}{1+u^2} du$	M1dM1
	$\int \frac{u}{(5+u^2)^3} du *$	A1*
		(4)
(b)	$\int \frac{u}{(5+u^2)^3} du = -\frac{1}{4} (5+u^2)^{-2} + c = -\frac{1}{4} (5+\tan^2 x)^{-2} (+c)$	M1A1
		(2)
		(6 marks)

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int \frac{1}{k-3x} dx = \int dt \text{ oe}$	M1
	$\Rightarrow -\frac{1}{3} \ln(k-3x) = t(+c)$	M1 A1
	$t=0, x=0 \Rightarrow -\frac{1}{3} \ln k = c$	M1
	$-\ln(k-3x) = 3t - \ln k \Rightarrow \ln \frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k}x = e^{-3t} \Rightarrow x = \dots$	M1
	$x = \frac{k}{3}(1 - e^{-3t})$	A1
		(6)
(b)	$t \rightarrow \infty \Rightarrow e^{-3t} \Rightarrow 0 \Rightarrow x \rightarrow \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = \dots$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7(1 - e^{-3t}) = 5 \Rightarrow e^{-3t} = \dots$	M1
	$\Rightarrow t = -\frac{1}{3} \ln \frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
(11 marks)		

Question	Scheme	Marks
8(a)	$(8-3x)^{\frac{4}{3}} = 16 \left(1 - \frac{3}{8}x\right)^{\frac{4}{3}}$ but condone $16(1-kx)^{\frac{4}{3}}, k \neq 3$	M1
	"Correct" term 3 or term 4 in	
	$(1-kx)^{\frac{4}{3}} = 1 \pm \frac{4}{3}kx + \frac{\frac{4}{3} \times \frac{1}{3}}{2} (\pm kx)^2 + \frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6} (\pm kx)^3 + \dots$	M1
	Two terms correct of $16 - 8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$ following M1, M1 and $k = \pm \frac{3}{8}$	A1
	$= 16 - 8x + \frac{x^2}{2} + \frac{x^3}{24} + \dots$	A1
		(4)

(b)	Assume the curves meet so $8x - \frac{15x^2}{2} + 8 = "16 - 8x + \frac{x^2}{2} + \frac{x^3}{24}"$ (for some x)	M1
	$\Rightarrow 0 = 8 - 16x + 8x^2 + \frac{x^3}{24} + \dots \Rightarrow 0 = 8(x^2 - 2x) + 8 + \frac{x^3}{24} + \dots \Rightarrow 8(x-1)^2 \pm \dots$	M1
	$\Rightarrow 0 = 8(x-1)^2 + \frac{x^3}{24}$	A1
	Requires - Argument that $8(x-1)^2 + \frac{x^3}{24} > 0$. E.g. $8(x-1)^2 \geq 0$ and $\frac{x^3}{24} > 0$ (for $0 < x < \frac{8}{3}$) so $8(x-1)^2 + \frac{x^3}{24} > 0$ - Minimal statement (contradiction) and conclusion	A1
		(4)
(8 marks)		

Question Number	Scheme	Marks
4(a)	Attempts direction vector by subtracting $(5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$ and $(4\mathbf{i} + 8\mathbf{j} + \mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r} = 4\mathbf{i} + 8\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}), \quad \mathbf{r} = 5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	A1
		(2)
(b)	E.g. $\overrightarrow{PC} = \begin{pmatrix} 4 + \lambda \\ 8 - 2\lambda \\ 1 + 2\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 + \lambda \\ 10 - 2\lambda \\ 2\lambda \end{pmatrix}$	M1
	Uses $\overrightarrow{PC} \cdot (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) = 0$ E.g. $\Rightarrow 1(2 + \lambda) - 2(10 - 2\lambda) + 2 \times 2\lambda = 0 \Rightarrow \lambda = \dots$	dM1
	E.g. I $\lambda = 2 \Rightarrow \mathbf{c} = (4 + 2)\mathbf{i} + (8 - 4)\mathbf{j} + (1 + 4)\mathbf{k}$ E.g. II $\mu = 1 \Rightarrow \mathbf{c} = (5 + 1)\mathbf{i} + (6 - 2)\mathbf{j} + (3 + 2)\mathbf{k}$	ddM1
	$(6, 4, 5)$	A1
		(4)
(c)	$\overrightarrow{OP'} = \mathbf{p} + 2\overrightarrow{PC} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} + 2(4\mathbf{i} + 6\mathbf{j} + 4\mathbf{k})$	M1
	$(10, 10, 9)$	A1
		(2)
(d)	$ \overrightarrow{PP'} = 2 \overrightarrow{PC} = 2\sqrt{4^2 + 6^2 + 4^2} = \dots$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

Question Number	Scheme	Marks
5.	States or implies $\text{Volume} = \int_{\frac{1}{\sqrt{2}}}^k \pi \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx$ Attempts $\int \left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}} \right)^2 dx = \int \frac{144x}{(2x^2+3)^3} dx = -18(2x^2+3)^{-2}$ Sets $-\frac{18\pi}{(2k^2+3)^2} + \frac{18\pi}{(2 \times \frac{1}{2} + 3)^2} = \frac{713}{648}\pi$ $\Rightarrow -\frac{18}{(2k^2+3)^2} + \frac{9}{8} = \frac{713}{648} \Rightarrow (2k^2+3)^2 = 729 \Rightarrow k^2 = \dots$ $k = 2\sqrt{3}$	B1 M1A1 M1 ddM1 A1 (6 marks)

Question Number	Scheme	Marks
1	Assume there are positive numbers a and b such that $\frac{9a}{b} + \frac{4b}{a} < 12$ Multiplies by $ab \Rightarrow 9a^2 + 4b^2 < 12ab$ $\Rightarrow 9a^2 - 12ab + 4b^2 < 0 \Rightarrow (3a - 2b)^2 < 0$ Square numbers cannot be negative, so we have a contradiction and hence, if a and b are positive real numbers, then $\frac{9a}{b} + \frac{4b}{a} \geq 12$	B1 M1 A1 A1* (4 marks)

Question Number	Scheme	Marks
8(a)	3π	B1
		(1)
(b)	$\frac{dy}{dx} = \frac{-2 \sin t}{6 - 6 \cos 2t}$	M1 A1
	$= \frac{-2 \sin t}{6 - 6(1 - 2 \sin^2 t)} = \frac{-2 \sin t}{12 \sin^2 t} = -\frac{1}{6} \operatorname{cosec} t$	M1 A1*
		(4)
(c)	$P = \left(\frac{3\pi}{2} - 3, \sqrt{2} \right)$	B1
	E.g. $y - \sqrt{2} = -\frac{\sqrt{2}}{6} \left(0 - \left(\frac{3\pi}{2} - 3 \right) \right) \Rightarrow y = \dots$ or $\sqrt{2} = -\frac{\sqrt{2}}{6} \left(\frac{3\pi}{2} - 3 \right) + c \Rightarrow c = \dots$	M1
	$\frac{\pi\sqrt{2}}{4} + \frac{\sqrt{2}}{2} \text{ o.e}$	A1
		(3)

(d)(i)	Attempts $y^2 \frac{dx}{dt} = 4 \cos^2 t (6 - 6 \cos 2t)$	M1 A1
	$= (2 \cos 2t + 2)(6 - 6 \cos 2t) = 6(2 - 2 \cos^2 2t) = 6(2 - (1 + \cos 4t))$	dM1
	Volume $= \int_0^{\frac{\pi}{2}} \pi y^2 \frac{dx}{dt} dt = \int_0^{\frac{\pi}{2}} 6\pi(1 - \cos 4t) dt$	A1
(ii)	$\left[6\pi t - \frac{6\pi \sin 4t}{4} \right]_0^{\frac{\pi}{2}}, = 3\pi^2$	M1, A1
		(6)
		(14 marks)

Question Number	Scheme		Marks
9(a)	$(V) = \pi \int_0^4 \left(\cos x + \frac{1}{5} e^x \right)^2 dx$ $\left(\cos x + \frac{1}{5} e^x \right)^2 = \cos^2 x + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \frac{\cos 2x + 1}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x}$ $= \pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$		B1 M1 dM1 A1
			(4)
(b)	Way I $\int e^x \cos x \, dx = \pm e^x \sin x \pm \int e^x \sin x \, dx$ $= e^x \sin x - \int e^x \sin x \, dx$	Way Two $\int e^x \cos x \, dx = e^x \cos x \pm \int e^x \sin x \, dx$ $= e^x \cos x + \int e^x \sin x \, dx$	M1 A1
	$\int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x - \int e^x \cos x \, dx$ $2 \int e^x \cos x \, dx = e^x \cos x \pm e^x \sin x$		dM1
	$\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + (c)$		A1
			(4)
(c)	$\pi \int_0^4 \left(\frac{1}{2} + \frac{\cos 2x}{2} + \frac{2}{5} e^x \cos x + \frac{1}{25} e^{2x} \right) dx$ $= (\pi) \left[\frac{1}{2} x + \frac{\sin 2x}{4} + \frac{2}{5} \left(\frac{1}{2} e^x (\sin x + \cos x) \right) + \frac{1}{50} e^{2x} \right]_0^4$ $2\pi \left(2 + \frac{\sin 8}{4} + \frac{2}{5} \left(\frac{1}{2} e^4 (\sin(4) + \cos(4)) \right) + \frac{1}{50} e^8 - \frac{11}{50} \right)$ $= \text{awrt } 290 \text{ (cm}^3\text{)}$		M1A1ft dM1 A1
			(4)
			(12 marks)

(a)

Question Number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = \frac{-1}{(t+1)^2}, \quad \frac{dx}{dt} = \frac{2}{2t+5} \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{2t+5}{2(t+1)^2}$ $\text{Sets } -\frac{2t+5}{2(t+1)^2} = -4 \Rightarrow 8t^2 + 14t + 3 = 0$ $\Rightarrow (4t+1)(2t+3) = 0 \Rightarrow t = -\frac{1}{4} \Rightarrow (y =) \frac{4}{3}$	M1A1 M1A1 dM1A1 (6)
(b) (i)	$a = 2, b = 5$ $\text{Area} = \int_2^5 y \frac{dx}{dt} dt = \int_2^5 \frac{1}{t+1} \times \frac{2}{2t+5} dt = \int_2^5 \frac{2}{(t+1)(2t+5)} dt$	B1 M1A1
(ii)	$\frac{2}{(t+1)(2t+5)} \equiv \frac{A}{(t+1)} + \frac{B}{(2t+5)} \Rightarrow A = \dots, B = \dots$ $= \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)}$ $\int \frac{\frac{2}{3}}{(t+1)} - \frac{\frac{4}{3}}{(2t+5)} dt = \frac{2}{3} \ln t+1 - \frac{2}{3} \ln 2t+5 $ $\int_2^5 \frac{2}{(t+1)(2t+5)} dt = \left(\frac{2}{3} \ln 6 - \frac{2}{3} \ln 15 \right) - \left(\frac{2}{3} \ln 3 - \frac{2}{3} \ln 9 \right)$ $= \frac{2}{3} \ln \left(\frac{6}{5} \right) = \frac{1}{3} \ln \left(\frac{36}{25} \right)$	M1 A1 M1A1ft dM1A1 (9) (15 marks)

Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) " $n^2 - 4n + 5$ is even and n is even"	B1
	Sets e.g. $n = 2k$ and attempts $(2k)^2 - 4(2k) + 5$	M1
	$4k^2 - 8k + 5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4(k^2 - 2k) + 5$ or e.g. $4(k^2 - 2k + 1) + 1$ or e.g. $2(2k^2 - 4k + 2) + 1$ or e.g. $(2k - 2)^2 + 1$ Condone $4k^2 - 8k + 5 = \text{even} - \text{even} + \text{odd} = \text{odd}$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Question Number	Scheme	Marks
8(a)	$x = 2$	B1
		(1)
(b)	$(2.5, 1.5)$	B1
		(1)
(c)	$\frac{dy}{dx} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$	M1
	$t = 2 \Rightarrow \frac{dy}{dx} = \frac{5}{3} \Rightarrow y - 1.5 = -\frac{3}{5}(x - 2.5)$	dM1
	$3x + 5y = 15^*$	A1*
		(3)

(d)	l crosses x -axis at $x = 5$	B1
	$\frac{1}{3} \pi \times 1.5^2 (5 - 2.5) \left(= \frac{15}{8} \pi \right)$	M1
	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{dt} dt = \pi \int \left(t - \frac{1}{t} \right)^2 \left(1 - \frac{1}{t^2} \right) dt$	M1
	$\pi \int \left(t^2 - 2 + \frac{1}{t^2} \right) \left(1 - \frac{1}{t^2} \right) dt = \pi \int \left(t^2 + \frac{3}{t^2} - \frac{1}{t^4} - 3 \right) dt = \dots$	M1
	$\pi \left[\frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t \right]$	A1
	$\pi \left[\frac{t^3}{3} - \frac{3}{t} + \frac{1}{3t^3} - 3t \right]_1^2 = \pi \left(\left(\frac{8}{3} - \frac{3}{2} + \frac{1}{24} - 6 \right) - \left(\frac{1}{3} - 3 + \frac{1}{3} - 3 \right) \right)$	M1
	$V = \frac{13}{24} \pi + \frac{15}{8} \pi = \frac{29}{12} \pi$ Cao	A1
		(7)
		Total 12

Question Number	Scheme	Marks
8 (a)	$\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \Rightarrow \begin{array}{l} 4+3\lambda=2+2\mu \\ -3-2\lambda=0-1\mu \\ 2-1\lambda=-9-3\mu \end{array} \quad \text{any two of these}$ Full method to find either λ or μ eg $(1)+3(3) \Rightarrow \mu = \dots$ Either $\lambda = -4$ or $\mu = -5$ Position vector of intersection is $\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - 4 \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = \text{OR} \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} - 5 \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} =$ $= \begin{pmatrix} -8 \\ 5 \\ 6 \end{pmatrix}$	M1 M1 A1 dM1 A1 (5)
(b)	Co-ordinates or position vector of point $Q = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\mu-8 \\ 7-1\mu \\ -9-3\mu \end{pmatrix}$ Uses $\overrightarrow{PQ} \cdot \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = 0 \Rightarrow 4\mu - 16 - 7 + \mu + 27 + 9\mu = 0 \Rightarrow \mu = -\frac{2}{7}$ Substitutes their μ into $\begin{pmatrix} 2+2\mu \\ 0-1\mu \\ -9-3\mu \end{pmatrix} \Rightarrow Q = \left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7} \right)$	M1 dM1 A1 ddM1 A1 (5) (10 marks)

Question Number	Scheme	Marks
10	Assume there is an angle x, $90^\circ < x < 180^\circ$, $\left \frac{\cos 2x}{\cos x - \sin x} \right \dots 1$ $\frac{\cos 2x}{\cos x - \sin x} \equiv \frac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x - \sin x}$ $\cos x + \sin x \dots 1 \Rightarrow (\cos x + \sin x)^2 \dots 1 \Rightarrow 1 + 2 \sin x \cos x \dots 1 \Rightarrow \sin 2x \dots 0$ This is a contradiction, as $\sin \alpha$ is negative for all angles $180^\circ < \alpha < 360^\circ$, hence true *	B1 M1 dM1 A1* (4 marks)

Question Number	Scheme	Marks
2	Assume that C_1 and C_2 do intersect so that $x^4 + 10x^2 + 8 = 2x^2 - 7$ (and has real solutions).	B1
	$x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 5)(x^2 + 3) = 0 \Rightarrow x^2 = \dots$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow x^2 = \frac{-8 \pm \sqrt{8^2 - 4 \times 15}}{2}$ or $x^4 + 8x^2 + 15 = 0 \Rightarrow (x^2 + 4)^2 - 1 = 0 \Rightarrow x^2 = \dots$	M1
	$x^2 = -5, -3$	A1
	e.g. The two values of $x^2 < 0$ (so $x^4 + 8x^2 + 15 = 0$ has no real roots) which is not possible so C_1 and C_2 do not intersect	A1
		(4)
		Total 4

Question Number	Scheme	Marks
4 (a)	$a = 3, b = 5$	B1
	E.g. $u = \sqrt{2x+1} \Rightarrow \frac{dx}{du} = u$ or $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ o.e.	B1
	$\int \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \int 2u e'' u du$	M1
	$= \int_3^5 2u^2 e'' du$	A1
		(4)
(b)	$\int \sqrt{8x+4} e^{2x+1} dx = \int 2u^2 e'' du$	
	$= 2u^2 e'' - \int 4u e'' du$	M1
	$= 2u^2 e'' - (4ue'' - 4e'') = 2u^2 e'' - 4ue'' + 4e''$	dM1 A1ft
	$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx = \left[2u^2 e'' - 4ue'' + 4e'' \right]_3^5 = (50e^5 - 20e^5 + 4e^5) - (18e^3 - 12e^3 + 4e^3)$	ddM1
	$= 34e^5 - 10e^3$	A1
		(5)
		(9 marks)

Question Number	Scheme	Marks
1(a)	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}} = \frac{1}{2}(\dots)$	B1
	$= (1-20x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (-20x) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (-20x)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (-20x)^3 \dots$	M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1 A1
	Special case: If the final answer is left as $\frac{1}{2}(1-10x-50x^2-500x^3+\dots)$ Award SC B1M1A1A1A0	
		(5)
	Alternative by direct expansion	
	$\left(\frac{1}{4}-5x\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-5x)^1 + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-5x)^2 + \frac{\frac{1}{2} \times -\frac{1}{2} \times -\frac{3}{2}}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-5x)^3$	B1M1A1
	$= \frac{1}{2} - 5x - 25x^2 - 250x^3 + \dots$	A1A1
(b)	$\left(\frac{1}{4}-\frac{5}{100}\right)^{\frac{1}{2}} = \left(\frac{1}{5}\right)^{\frac{1}{2}} = \frac{1}{2} - 5 \times \frac{1}{100} - 25\left(\frac{1}{100}\right)^2 - 250\left(\frac{1}{100}\right)^3 + \dots$ $\frac{\sqrt{5}}{5} \approx \frac{1789}{4000} \text{ or } \frac{1}{\sqrt{5}} \approx \frac{1789}{4000}$ $\Rightarrow \sqrt{5} \approx 5 \times \frac{1789}{4000} \text{ or } \sqrt{5} \approx 1 \div \frac{1789}{4000}$	M1
	$\sqrt{5} \approx \frac{1789}{800} \text{ or } \frac{4000}{1789}$	A1
		(2)
		(7 marks)

Question Number	Scheme	Notes	Marks
8	Assume that there exist positive real numbers x and y such $\frac{9x}{y} + \frac{y}{x} < 6$	Starts the proof by contradicting the given statement	B1
	$\frac{9x}{y} + \frac{y}{x} < 6 \Rightarrow 9x^2 + y^2 < 6xy$ as x and y are both positive	Multiplies through by xy	M1
	$\Rightarrow 9x^2 + y^2 - 6xy < 0$ $\Rightarrow (3x - y)^2 < 0$	Reaches a correct contradictory statement	A1
	As x and y are positive real numbers, this is a contradiction and so $\frac{9x}{y} + \frac{y}{x} < 6$ must be incorrect and so $\frac{9x}{y} + \frac{y}{x} \dots 6^*$	Makes a suitable conclusion	A1*
			(4)
			Total 4

Question Number	Scheme	Marks
2(a)	States or uses $S = 6x^2$ or $\frac{dS}{dt} = 4$	B1
	States or uses $S = 6x^2$ and $\frac{dS}{dt} = 4$	B1
	Attempts to use $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} \Rightarrow 4 = 12x \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \dots$	M1
	$\Rightarrow \frac{dx}{dt} = \frac{1}{3x}$ oe	A1
		(4)

(b)	States or uses $\frac{dV}{dx} = 3x^2$	B1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{1}{3x} = x$	M1
	$\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$	A1
		(3)

Question Number	Scheme	Marks
7	Assume that $\sqrt{7}$ is rational so that $\sqrt{7} = \frac{a}{b} \Rightarrow 7 = \frac{a^2}{b^2}$ (where a and b have no factors in common)	M1
	$7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2$ So a^2 is a multiple of 7 which means a is a multiple of 7	A1
	$a = 7k \Rightarrow 7b^2 = 49k^2 \Rightarrow b^2 = 7k^2$	M1
	So b^2 is a multiple of 7 which means b is a multiple of 7. As a and b are both multiples of 7, this contradicts the fact that a and b have no factors in common. Hence $\sqrt{7}$ must be irrational.	A1
		(4)
		Total 4

Question Number	Scheme	Marks
7	$x = 4 \sin \theta \Rightarrow \frac{dx}{d\theta} = 4 \cos \theta$	B1
	$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx = \int \frac{4 \cos \theta}{(16-16 \sin^2 \theta)^{\frac{3}{2}}} d\theta = \int \frac{4 \cos \theta}{64 \cos^3 \theta} d\theta$	M1 A1
	$= \frac{1}{16} \tan \theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60^\circ) - f(30^\circ)$	M1
	$= \frac{1}{16} \tan \frac{\pi}{3} - \frac{1}{16} \tan \frac{\pi}{6} = \frac{\sqrt{3}}{24}$	A1
		(6 marks)

Question Number	Scheme	Marks
9(a)	$A(1, a, 5), B(b, -1, 3), l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ Either $A:\lambda=1$ or $B:\lambda=3$ leading to $a = -3, b = 5$	M1 A1, A1 (3)
(b)	$\overrightarrow{AB} = \pm((5\mathbf{i} - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})) = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1, A1 (2)
(c)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} \text{ or } \overrightarrow{CA} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$ $\cos \hat{CAB} = \frac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2} \cdot \sqrt{(3)^2 + (0)^2 + (-3)^2}}$ $\cos \hat{CAB} = \frac{12 + 0 + 6}{\sqrt{24} \cdot \sqrt{18}} = \frac{\sqrt{3}}{2} \Rightarrow \hat{CAB} = 30^\circ$	M1 dM1 A1 (3)
(d)	$\text{Area } CAB = \frac{1}{2} \sqrt{24} \sqrt{18} \sin 30^\circ = 3\sqrt{3}$	M1, A1 (2)
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$ For both of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1 M1, A1 (4) (14 marks)

Question	Scheme	Marks
6(a)	$ \overrightarrow{OA} ^2 = (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2$	M1
	$ \overrightarrow{OA} = 5\sqrt{10} \Rightarrow (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2 = 250$	M1
	$\Rightarrow 64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ $\Rightarrow 81\lambda^2 + 52\lambda - 220 = 0^*$	A1*
		(3)
(b)	$81\lambda^2 + 52\lambda - 220 = 0 \Rightarrow (81\lambda - 110)(\lambda + 2) = 0 \Rightarrow \lambda = \dots \left(-2, \frac{110}{81}\right)$ $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1+8 \times \text{"their } \lambda \text{"} \\ 2 - \text{"their } \lambda \text{"} \\ 5+4 \times \text{"their } \lambda \text{"} \end{pmatrix}$	M1
	E.g $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 2 \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}^*$; or $\overrightarrow{OA} = \begin{pmatrix} 1+8 \times \frac{110}{81} \\ 2 - \frac{110}{81} \\ 5+4 \times \frac{110}{81} \end{pmatrix} = \begin{pmatrix} \frac{961}{81} \\ \frac{52}{81} \\ \frac{845}{81} \end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos \theta = \pm \frac{\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{15^2 + 4^2 + 3^2} \sqrt{8^2 + 1^2 + 4^2}} = \dots$	M1
	$= \pm \frac{-136}{45\sqrt{10}} (= \pm 0.9557\dots)$ or e.g. $\theta = \text{awrt } 0.299(\text{rad}) / 17.1^\circ$	A1
	$\text{Area } OAB = \frac{1}{2} 5\sqrt{10} \times 4\sqrt{10} \sin \theta = \dots$	M1
	$= \text{awrt } 29.4$	A1
		(4)
(10 marks)		

Question Number	Scheme	Marks
8(a)	A and B are where $y = 0$ so $t^3 - 9t = 0 \Rightarrow t(t^2 - 9) = 0 \Rightarrow t = 3$ (0 and -3) Substitutes $t = 3$ in $x = 3^2 + 2 \times 3 = 15$ B(15, 0) $A = (3, 0)$	M1 A1* B1 (3)
8(a) ALT	Uses answer $x = 15$: $t^2 + 2t = 15 \Rightarrow t = 3$ (-5) Substitutes $t = 3$ in $y = t^3 - 9t = 27 - 27 = 0$ ✓ B(15, 0) $A = (3, 0)$	M1 A1* B1 (3)
(b)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 9}{2t + 2}$ Substitutes $t = 3$ into $\frac{dy}{dx} = \frac{3t^2 - 9}{2t + 2} \Rightarrow \frac{dy}{dx} = \frac{9}{4}$ Uses their $\frac{9}{4}$ with (15, 0) to produce tangent equation $9x - 4y - 135 = 0$	M1 A1 dM1 A1 (4)
(c)	Substitutes $x = t^2 + 2t$, $y = t^3 - 9t$, into their $9x - 4y - 135 = 0$ $\Rightarrow 9(t^2 + 2t) - 4(t^3 - 9t) - 135 = 0$ $\Rightarrow 4t^3 - 9t^2 - 54t + 135 = 0$ $\Rightarrow (t^2 - 6t + 9)(4t + 15) = 0$ $t = -\frac{15}{4}$ Coordinates of X are $\left(\frac{105}{16}, -\frac{1215}{64}\right)$	M1 dM1 A1 dM1 A1 (5) (12 marks)

Question Number	Scheme	Marks
1(a)	$(2-5x)^{-2} = \frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$ or e.g. $\frac{8}{4\left(1-\frac{5x}{2}\right)^2}$	B1
	$= 8 \times \frac{1}{4} \left(1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3 \dots \right)$	M1A1
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
		(4)
Alt (a) by direct expansion	$= 8 \times \left(2^{-2} + (-2)2^{-3}(-5x)^1 + \frac{-2 \times -3}{2!} 2^{-4}(-5x)^2 + \frac{-2 \times -3 \times -4}{3!} 2^{-5}(-5x)^3 \right)$	B1, <u>M1A1</u>
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
(b)	$ x < \frac{2}{5}$ o.e.	B1
		(1)
		(5 marks)

Question	Scheme	Marks
4(a)	$16x^3 - 9kx^2y + 8y^3 = 875$	
	$(8)y^3 \rightarrow (8 \times) 3y^2 \frac{dy}{dx}$	B1
	$-9kx^2y \rightarrow \dots kxy \pm \dots - 9kx^2 \frac{dy}{dx}$	M1
	$48x^2 - 18kxy - 9kx^2 \frac{dy}{dx} + 24y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}(24y^2 - 9kx^2) = 18kxy - 48x^2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2} \quad *$	A1*
		(4)
(b)	$\frac{dy}{dx} = 0, x = \frac{5}{2} \Rightarrow \frac{6k\left(\frac{5}{2}\right)y - 16\left(\frac{5}{2}\right)^2}{8y^2 - 3k\left(\frac{5}{2}\right)^2} = 0 \quad \text{or}$	M1
	$x = \frac{5}{2} \Rightarrow 16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2 y + 8y^3 = 875$	
	$15ky - 100 = 0 \quad \text{or} \quad 250 - \frac{225}{4}ky + 8y^3 = 875$	A1
	E.g. $16\left(\frac{5}{2}\right)^3 - 9k\left(\frac{5}{2}\right)^2\left(\frac{20}{3k}\right) + 8\left(\frac{20}{3k}\right)^3 = 875 \Rightarrow k^3 = \dots \left(= \frac{64}{27} \right) \Rightarrow k = \dots$	M1
	$k = \frac{4}{3}$	A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
3 (a)	Attempts a correct identity $3x^3 + 8x^2 - 3x - 6 \equiv (Ax + B)x(x + 3) + C(x + 3) + Dx$ o.e. Correct method for two constants e.g. $x = 0, x = -3 \Rightarrow C = \dots, D = \dots$ Two correct constants e.g. $C = -2$ and $D = 2$ Correct method to find all four constants (e.g. by comparing coefficients) $A = 3, B = -1, C = -2, D = 2$	M1 dM1 A1 ddM1 A1 (5)
(b)	$g(x) = 3x - 1 - \frac{2}{x} + \frac{2}{x+3}$ $\Rightarrow g'(x) = 3 + \frac{2}{x^2} - \frac{2}{(x+3)^2}$	M1 A1ft (2)
(c)	Explains that (if $x > 0$) $\frac{2}{x^2} > \frac{2}{(x+3)^2}$ so $\frac{2}{x^2} - \frac{2}{(x+3)^2} > 0$ and $\Rightarrow g'(x) > 3$	B1 (1)
3 (a) ALT	Via division Attempts to divide $3x^3 + 8x^2 - 3x - 6$ by $x^2 + 3x$ forming a linear quotient Correct method for two constants implied by quotient of $3x + \dots$ Correct quotient $3x - 1$ Correct method to find all four constants (e.g. uses PF on $\frac{rem}{x(x+3)}$) $A = 3, B = -1, C = -2, D = 2$	M1 dM1 A1 ddM1 A1 (5)

Question Number	Scheme	Marks
1(a)	$(2-5x)^{-2} = \frac{1}{4} \left(1 - \frac{5x}{2}\right)^{-2} \text{ or e.g. } \frac{8}{4 \left(1 - \frac{5x}{2}\right)^2}$	B1
	$= 8 \times \frac{1}{4} \left(1 + (-2) \times \left(-\frac{5x}{2}\right) + \frac{(-2) \times (-3)}{2!} \times \left(-\frac{5x}{2}\right)^2 + \frac{(-2) \times (-3) \times (-4)}{3!} \times \left(-\frac{5x}{2}\right)^3 \dots \right)$	M1A1
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
		(4)
Alt (a) by direct expansion	$8 \times (2-5x)^{-2}$ $= 8 \times \left(2^{-2} + (-2)2^{-3}(-5x)^1 + \frac{-2 \times -3}{2!} 2^{-4}(-5x)^2 + \frac{-2 \times -3 \times -4}{3!} 2^{-5}(-5x)^3 \right)$	B1, <u>M1A1</u>
	$\frac{8}{(2-5x)^2} = 2 + 10x + \frac{75}{2}x^2 + 125x^3 \dots$	A1
(b)	$ x < \frac{2}{5} \text{ o.e.}$	B1
		(1)
		(5 marks)

Question	Scheme	Marks
4(a)	$A = \frac{1}{2}5^2(\theta - \sin \theta) \Rightarrow \frac{dA}{d\theta} = \frac{25}{2}(1 \pm \cos \theta)$	M1
	$\frac{dA}{d\theta} = \frac{25}{2}(1 - \cos \theta)$	A1
		(2)
(b)	$\frac{d\theta}{dt} = 0.1$	B1
	$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt} \text{ (oe)}$	B1
	$\frac{dA}{dt} = \frac{25}{2} \left(1 - \cos \frac{\pi}{3}\right) \times 0.1$	M1
	$= \frac{5}{8} \text{ (cm}^2 \text{ per second)}$	A1
		(4)
(6 marks)		

Question	Scheme	Marks
5(a)	$1 = 2 \sin u \Rightarrow p = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$	B1
	$x = 2 \sin u \Rightarrow \frac{dx}{du} = 2 \cos u \text{ oe}$	M1
	$\int \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx = \int \frac{6 \sin u + 2}{(4-4 \sin^2 u)^{\frac{3}{2}}} 2 \cos u du = \int \frac{6 \sin u + 2}{(4 \cos^2 u)^{\frac{3}{2}}} 2 \cos u du$	M1
	$= \int \frac{12 \sin u}{8 \cos^2 u} + \frac{2}{4 \cos^2 u} du = \int_0^{\frac{\pi}{6}} \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du *$	A1*
		(4)
(b)	$\int \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du = \frac{3}{2} \sec u + \frac{1}{2} \tan u$	M1A1
	$\left[\frac{3}{2} \sec u + \frac{1}{2} \tan u \right]_0^{\frac{\pi}{6}} = \left(\frac{3}{2} \sec \left(\frac{\pi}{6} \right) + \frac{1}{2} \tan \left(\frac{\pi}{6} \right) \right) - \left(\frac{3}{2} \sec 0 + \frac{1}{2} \tan 0 \right) = \dots$	M1
	$= \sqrt{3} + \frac{\sqrt{3}}{6} - \frac{3}{2} = \frac{7\sqrt{3}}{6} - \frac{3}{2} \left(= \frac{7\sqrt{3}-9}{6} \right)$	A1
		(4)
(8 marks)		
Notes		

Question Number	Scheme	Marks
9(a)(i)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3 \sin^2 t \cos t}{-4 \sin 2t}$	M1A1
	$\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{3 \sin^2 t \cos t}{-8 \sin t \cos t} = -\frac{3}{8} \sin t$ or e.g. $\frac{3 \sin^2 t \cos t}{-4 \sin 2t} = \frac{\frac{3}{2} \sin 2t \sin t}{-4 \sin 2t} = -\frac{3}{8} \sin t$	A1
(ii)	$t = \frac{\pi}{6} \Rightarrow x = 1, y = \frac{1}{8}$	B1
	$m = -\frac{3}{8} \sin\left(\frac{\pi}{6}\right) = -\frac{3}{16} \Rightarrow y - \frac{1}{8} = -\frac{3}{16}(x - 1)$	M1
	$3x + 16y - 5 = 0$ *	A1*
		(6)
(b)	$3(2 \cos 2t) + 16(\sin^3 t) - 5 = 0$	M1
	$6(1 - 2 \sin^2 t) + 16 \sin^3 t - 5 = 0$	dM1
	$16 \sin^3 t - 12 \sin^2 t + 1 = 0$	A1
	$(2 \sin t - 1)^2 (4 \sin t + 1) = 0 \Rightarrow \sin t = -\frac{1}{4}$	ddM1
	$\sin t = -\frac{1}{4} \Rightarrow x = 2 \cos 2t = \dots$ and $y = \sin^3 t = \dots$	dddM1
	$\mathcal{Q}\left(\frac{7}{4}, -\frac{1}{64}\right)$	A1
		(6)
		(12 marks)

Question Number	Scheme	Marks
2 (a)	E.g. $x = \frac{t-1}{2t+1} \Rightarrow t = \frac{x+1}{1-2x}$ or $y = \frac{6}{2t+1} \Rightarrow t = \frac{6-y}{2y}$	M1
	E.g. $y = \frac{6}{2t+1} \Rightarrow y = \frac{6}{2 \times \left(\frac{x+1}{1-2x}\right) + 1}$ or $t = \frac{6-y}{2y} \Rightarrow x = \frac{\frac{6-y}{2y} - 1}{2 \times \frac{6-y}{2y} + 1}$	A1
	E.g. $y = \frac{6}{2 \times \left(\frac{x+1}{1-2x}\right) + 1} \Rightarrow y = \frac{6(1-2x)}{2 \times (x+1) + 1(1-2x)} = ax + b$	dM1
	E.g. $y = \frac{6(1-2x)}{3}, y = 2(1-2x)$ oe so linear *	A1*
		(4)
(b)	$y = 2(1-2x)$ and $y = x + 12 \Rightarrow 2(1-2x) = x + 12 \Rightarrow x = \dots$	M1
	$x = -2$	A1cao
		(2)

Question	Scheme	Marks
1	$\int x \cos 3x \, dx = \frac{1}{3} x \sin 3x - \frac{1}{3} \int \sin 3x \, dx$	M1
	$= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x$	dM1 A1
	$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx = \left[\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x \right]_0^{\frac{\pi}{6}} = \frac{1}{3} \frac{\pi}{6} \sin \frac{\pi}{2} + \frac{1}{9} \cos \frac{\pi}{2} - \left(0 + \frac{1}{9} \right)$	M1
	$= \frac{\pi}{18} - \frac{1}{9}$	A1
		(5)
(5 marks)		

Question Number	Scheme	Notes	Marks
9(a)	$V = \pi \int y^2 dx = \pi \int y^2 \frac{dx}{d\theta} d\theta$ $= \pi \int (3 \sin \theta - \sin 2\theta)^2 (-5 \sin \theta) d\theta$	Applies $V = \pi \int y^2 \frac{dx}{d\theta} d\theta$ with or without the π	M1
	$= \pi \int (3 \sin \theta - 2 \sin \theta \cos \theta)^2 (-5 \sin \theta) d\theta$	Applies $\sin 2\theta = 2 \sin \theta \cos \theta$	M1
		Fully correct integral in terms of $\sin \theta$ and $\cos \theta$ only (π not needed)	A1
	$= \pi \int \sin^2 \theta (3 - 2 \cos \theta)^2 (-5 \sin \theta) d\theta$ $V = -5\pi \int \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = -5\pi \int_{\pi}^0 \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$ $V = 5\pi \int_0^{\pi} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta^*$	Completes correctly with correct limits and no incorrect statements previously. The factor of π must be present throughout.	A1*
			(4)
(b)	$u = \cos \theta \Rightarrow V = 5\pi \int \sin^3 \theta (3 - 2u)^2 \frac{du}{-\sin \theta}$	Applies the substitution correctly	M1
	$\theta = 0 \Rightarrow u = 1, \theta = \pi \Rightarrow u = -1$	Attempts to change θ limits to u limits	M1
	$V = -5\pi \int \sin^2 \theta (3 - 2u)^2 du = -5\pi \int (1 - u^2)(3 - 2u)^2 du$ Correct integral in terms of u only		A1
	$(1 - u^2)(3 - 2u)^2 = (1 - u^2)(9 - 12u + 4u^2)$ $= 9 - 12u - 5u^2 + 12u^3 - 4u^4$	Attempt to expand	M1
		Correct expansion	A1
	$V = 5\pi \int_{-1}^1 (9 - 12u - 5u^2 + 12u^3 - 4u^4) du$ $= 5\pi \left[9u - 6u^2 - \frac{5u^3}{3} + 3u^4 - \frac{4u^5}{5} \right]_{-1}^1 = \dots$ Integrates and applies their u limits		M1
	$= \frac{196}{3} \pi$	Cao	A1
			(7)
			Total 11

Question Number	Scheme	Marks
3(a)	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{A}{2x-1} + \frac{B}{4x-3} \Rightarrow 8x-5 = A(4x-3) + B(2x-1) \Rightarrow A = \dots B = \dots$	M1
	$A = 1 \text{ or } B = 2$	A1
	$\frac{8x-5}{(2x-1)(4x-3)} \equiv \frac{1}{2x-1} + \frac{2}{4x-3}$	A1
		(3)
(b)	$\int \left(\frac{1}{2x-1} + \frac{2}{4x-3} \right) dx = \frac{1}{2} \ln 2x-1 + \frac{1}{2} \ln 4x-3 (+c)$ Follow through their A and B	M1, A1ft, A1ft
		(3)
(c)	$\left[\frac{1}{2} \ln(2x-1) + \frac{1}{2} \ln(4x-3) \right]_k^{3k} = \frac{1}{2} \ln(6k-1)(12k-3) - \frac{1}{2} \ln(2k-1)(4k-3)$ $= \frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$	M1
	$\frac{1}{2} \ln \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = \frac{1}{2} \ln 20 \Rightarrow \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)} = 20$ $\Rightarrow (6k-1)(12k-3) = 20(2k-1)(4k-3) \Rightarrow 88k^2 - 170k + 57 = 0$	dM1A1
	$88k^2 - 170k + 57 = 0 \Rightarrow k = \dots$	ddM1
	$k = \frac{3}{2}$	A1
		(5)
		Total 11

Question Number	Scheme	Marks
1 (a)	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times (kx)^2 + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times (kx)^3 \dots$	
(i)	$\frac{1}{2}k = \frac{1}{8} \Rightarrow k = \frac{1}{4}$	M1A1
(ii)	$A = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{2!} \times "k"^{n^2} = -\frac{1}{128} \quad B = \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{3!} \times "k"^{n^3} = \frac{1}{1024}$	M1 A1 A1
		(5)
(b)	Substitutes $x = 0.6 \Rightarrow \sqrt{1.15} = 1 + \frac{1}{8} \times 0.6 - \frac{1}{128} \times 0.6^2 + \frac{1}{1024} \times 0.6^3 = 1.072398$	M1 A1
		(2)
		(7 marks)
1(b) alt	$(1+kx)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right) \times (kx) - \left(\frac{1}{8}\right) \times (kx)^2 + \left(\frac{1}{16}\right) \times (kx)^3$ Substitutes " kx " = 0.15 $\Rightarrow \sqrt{1.15} = 1 + \frac{1}{2} \times 0.15 - \frac{1}{8} \times 0.15^2 + \frac{1}{16} \times 0.15^3 = 1.072398$	M1A1
		(2)

Question Number	Scheme	Marks
2(a)	$x = 2 \Rightarrow 4 - 8y + y^2 = 13 \Rightarrow (y^2 - 8y - 9 = 0 \Rightarrow) y = \dots$	M1
	$y = 9$	A1
		(2)
(b)	$2^x \rightarrow 2^x \ln 2$	B1
	$-4xy \rightarrow -4x \frac{dy}{dx} - 4y \quad \text{OR} \quad y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$2^x \ln 2 - 4x \frac{dy}{dx} - 4y + 2y \frac{dy}{dx} = 0$	A1
	$\frac{dy}{dx} (2y - 4x) = 4y - 2^x \ln 2 \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{4y - 2^x \ln 2}{2y - 4x} \quad \text{or} \quad \frac{dy}{dx} = \frac{2^x \ln 2 - 4y}{4x - 2y} \quad \text{or} \quad \frac{dy}{dx} = \frac{\ln 2 e^{x \ln 2} - 4y}{4x - 2y}$	A1
		(5)
(c)	$(2, 9) \rightarrow \frac{dy}{dx} = \frac{4(9) - 2^2 \ln 2}{2(9) - 4(2)}$ $\Rightarrow y - "9" = \frac{36 - 4 \ln 2}{10} (x - 2)$	M1
	$y = 0 \Rightarrow 0 - "9" = \frac{36 - 4 \ln 2}{10} (x - 2) \Rightarrow x = \dots$	dM1
	$x = \frac{4 \ln 2 + 9}{2 \ln 2 - 18} \quad \text{o.e. e.g. } x = \frac{-8 \ln 2 - 18}{-4 \ln 2 + 36}$	A1
		(3)
		Total 10

Question	Scheme	Marks
3(a)	$y^2x + 3y = 4x^2 + k$	
	$y^2x \rightarrow \dots \times y \frac{dy}{dx} (+\dots)$ or $3y \rightarrow 3 \frac{dy}{dx}$	M1
	$y^2x \rightarrow 2xy \frac{dy}{dx} + y^2$	A1
	$\dots + 3y = 4x^2 + k \rightarrow \dots + 3 \frac{dy}{dx} = 8x$	A1
	$2xy \frac{dy}{dx} + y^2 + 3 \frac{dy}{dx} = 8x \Rightarrow \frac{dy}{dx} = \dots$	dM1
	$\frac{dy}{dx} = \frac{8x - y^2}{2xy + 3}$ oe	A1
		(5)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 8x - 4 = 0 \Rightarrow x = \dots$	M1
	$p = \frac{1}{2}$	A1
	$k = 2^2 \times \frac{1}{2} + 3 \times 2 - 4 \times \frac{1}{4} = \dots$	M1
	$k = 7$	A1
		(4)

Question	Scheme	Marks
6	Assume the sequence is geometric	B1
	So $(r =) \frac{1+2k}{k} = \frac{3+3k}{1+2k}$	M1
	$\Rightarrow (1+2k)^2 = k(3+3k) \Rightarrow k^2 + k + 1 = 0$	A1
	But $k^2 + k + 1 = \left(k + \frac{1}{2}\right)^2 - \frac{1}{4} + 1 \geq \frac{3}{4} > 0$ (since $\left(k + \frac{1}{2}\right)^2 \geq 0$ for all (real) k)	dM1
	This is a contradiction and hence the original assumption is not true. The sequence is not geometric.	A1
		(5)
	(5 marks)	

Question Number	Scheme	Marks
2(a)	$2^x \rightarrow 2^x \ln 2$	B1
	$y^2 \rightarrow \dots y \frac{dy}{dx}$	M1
	$4x^2 y \rightarrow \dots x^2 \frac{dy}{dx} + \dots xy$	M1
	$3 + 10y \frac{dy}{dx} + 4x^2 \frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$(10y + 4x^2) \frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \dots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

2(b)	At $x = 0$ $5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10 \times 2^0 \ln 2 - 3 - 8 \times 0 \times 3}{10 \times 3 + 4 \times 0^2} = \frac{10 \ln 2 - 3}{30}$	M1 A1
		(2)

Question Number	Scheme	Marks
7(a)	$\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)} \Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \dots \text{ or } B = \dots$	M1
	$A = -\frac{1}{2}, B = \frac{1}{2} \text{ giving } \frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$	A1
		[2]
(b)	$\frac{dx}{dt} = k(4-x)(2-x), t \geq 0 \Rightarrow \int \frac{1}{(4-x)(2-x)} dx = \int k dt$	
	$\frac{1}{2} \ln(4-x) - \frac{1}{2} \ln(2-x) = kt \quad (+c)$	M1, A1
	$\{t=0, x=0 \Rightarrow\} \frac{1}{2} \ln 4 - \frac{1}{2} \ln 2 = 0 + c \Rightarrow c = \frac{1}{2} \ln 2$	M1
	$\frac{1}{2} \ln(4-x) - \frac{1}{2} \ln(2-x) = kt + \frac{1}{2} \ln 2 \Rightarrow \ln\left(\frac{4-x}{4-2x}\right) = 2kt$	
	$\frac{4-x}{4-2x} = e^{2kt}$	M1
	$4-x = 4e^{2kt} - 2xe^{2kt} \Rightarrow 4-4e^{2kt} = x-2xe^{2kt}$	
	$\Rightarrow 4-4e^{2kt} = x(1-2e^{2kt}) \Rightarrow x = \frac{4-4e^{2kt}}{1-2e^{2kt}} (*)$	dM1, A1*
		[6]
(c)	$1 = \frac{4-4e^{0.2t}}{1-2e^{0.2t}} \Rightarrow 2e^{0.2t} = 3$	M1
	$\Rightarrow 0.2t = \ln \frac{3}{2} \Rightarrow t = \text{awrt } 2.03$	A1
		[2]
10 marks		

Question Number	Scheme	Marks
3.	States or implies Volume = $\int_{\sqrt{5}}^5 \pi \left(\frac{\sqrt{3x}}{\sqrt{3x^2+5}} \right)^2 dx$	B1
	$\int \left(\frac{\sqrt{3x}}{\sqrt{3x^2+5}} \right)^2 dx = \int \frac{3x}{(3x^2+5)} dx = \frac{1}{2} \ln(3x^2+5)$	M1A1
	Volume = $\left\{ \pi \right\} \left(\frac{1}{2} \ln(3 \times 25 + 5) - \frac{1}{2} \ln(3 \times 5 + 5) \right)$	M1
	$= \pi \ln 2$	A1
(5 marks)		

Question Number	Scheme	Marks
2(a)	$\frac{dV}{dx} = 3x^2$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3x^2}$	dM1A1
		(3)
Alternative 1		
	$V = x^3 \Rightarrow x = V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV} = \frac{1}{3}V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{5}{3V^{\frac{2}{3}}} = \frac{5}{3x^2}$	dM1A1
Alternative 2		
	$\frac{dV}{dt} = 5 \Rightarrow V = 5t + c$	M1 (B1 on EPEN)
	$V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 \frac{dx}{dt} = 5$ or $V = x^3 \Rightarrow x^3 = 5t + c \Rightarrow 3x^2 = 5 \frac{dt}{dx}$	dM1
	$\frac{dx}{dt} = \frac{5}{3x^2}$	A1
(b)	$\frac{dx}{dt} = \frac{5}{3 \times 4^2} = \frac{5}{48}$	M1A1
		(2)
		(5 marks)

Note this is now being marked as M1dM1A1 not R1M1A1

Question	Scheme	Marks
5(a)	$\frac{dx}{dt} = \frac{1}{2}(9-4t)^{-\frac{1}{2}} \times -4 = \left(\frac{-2}{\sqrt{9-4t}} \right)$	M1
	$A = \int_{(x=0)}^{(x=3)} y \frac{dx}{dt} (dt) = \int_{(t=\frac{9}{4})}^{(t=0)} \frac{t^3}{\sqrt{9+4t}} \times \left(\frac{-2}{\sqrt{9-4t}} \right) (dt)$	M1
	$= -2 \int_{(\frac{9}{4})}^{(0)} \frac{t^3}{\sqrt{81-16t^2}} (dt) \text{ or e.g. } -2 \int_{(\frac{9}{4})}^{(0)} \frac{t^3}{\sqrt{(9-4t)(9+4t)}} (dt)$ Depends on both previous M marks.	ddM1
	$= 2 \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81-16t^2}} dt$	A1
		(4)
(b)	$\frac{du}{dt} = -32t \text{ or } du = -32t dt \text{ oe}$ or $u = 81 - 16t^2 \Rightarrow t^2 = \frac{1}{16}(81-u) \Rightarrow t = \frac{1}{4}(81-u)^{\frac{1}{2}} \Rightarrow \frac{dt}{du} = -\frac{1}{8}(81-u)^{-\frac{1}{2}}$	B1
	$A = 2 \int_0^{\frac{9}{4}} \frac{t^2}{\sqrt{81-16t^2}} t dt = 2 \int_{(81)}^{(0)} \frac{(81-u)}{16\sqrt{u}} \frac{du}{-32}$ or $A = 2 \int_0^{\frac{9}{4}} \frac{t^2}{\sqrt{81-16t^2}} t dt = 2 \int_{(81)}^{(0)} \frac{\left(\frac{1}{4}(81-u)^{\frac{1}{2}} \right)^3}{\sqrt{u}} \times -\frac{1}{8}(81-u)^{-\frac{1}{2}} du$	M1
	$= k \int_{(81)}^{(0)} 81u^{\frac{1}{2}} - u^{\frac{1}{2}} du$	A1ft
	$= K \left[\frac{81u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_{(81)}^{(0)}$	M1
	$= -\frac{1}{256} \left(0 - \left(162\sqrt{81} - \frac{2}{3}\sqrt{81^3} \right) \right) = \dots$ Depends on all previous M marks.	ddM1
	$= \frac{243}{64}$	A1
		(6)
(10 marks)		

Question Number	Scheme	Marks
10 Main	Assume that $p^2 - 4q + 2 = 0$	B1
	$p^2 = 4q - 2$	M1
	$p^2 = 4q - 2 (= 2(2q - 1))$ So p^2 is even so p is even oe e.g. so $p = 2k$	A1
	If $p = 2k$ then e.g. $4k^2 = 4q - 2$ or e.g. $4k^2 = 4q - 2 \Rightarrow q = k^2 + \frac{1}{2}$ or e.g. $q = \frac{2k^2 + 1}{2}$ or e.g. $4k^2 = 4q - 2 \Rightarrow k^2 = q - \frac{1}{2}$ or e.g. $k^2 = \frac{2q - 1}{2}$ or e.g. Let $4k^2 = 4q - 2 \Rightarrow 2q - 2k^2 = 1$	dM1
	$4k^2$ is a multiple of 4 and $4q - 2$ isn't a multiple of 4 or e.g. $q = k^2 + \frac{1}{2}$ which is not an integer or e.g. $k^2 = q - \frac{1}{2}$ which is not an integer or e.g. $2q - 2k^2 = 1$ which is not possible So $p^2 - 4q + 2 \neq 0$	A1
		(5 marks)

Wes 1

Question Number	Scheme	Marks
1	$\int 2x \ln(3x) dx = x^2 \ln(3x) - \int \frac{1}{x} \times x^2 dx$	M1
	$= \dots + \dots x^2$	dM1
	$x^2 \ln(3x) - \frac{x^2}{2}$	A1
	$9 \ln 9 - \frac{9}{2} - \left(\ln 3 - \frac{1}{2} \right) = 17 \ln 3 - 4$	ddM1A1
		(5 marks)

Question Number	Scheme	Notes	Marks
7(a)	$\frac{dV}{dt} = 300 - kV \Rightarrow \int \frac{dV}{300 - kV} = \int dt$	Correct separation of variables	B1
	$\int \frac{dV}{300 - kV} = -\frac{1}{k} \ln(300 - kV)$	$\int \frac{dV}{300 - kV} = \alpha \ln(300 - kV)$	M1
	$-\frac{1}{k} \ln(300 - kV) = t + c$	Correct equation including a constant of integration	A1
	$-\frac{1}{k} \ln(300 - kV) = t + c \Rightarrow \ln(300 - kV) = -kt + d$ $\Rightarrow 300 - kV = e^{-kt+d}$ Correct processing to remove the "ln"		M1
	$kV = 300 - e^{-kt+d} \Rightarrow V = \frac{300}{k} - Be^{-kt}$ $V = \frac{300}{k} + Ae^{-kt} *$	Correct proof	A1*
			(5)
(b)	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$V = \frac{300}{k} - \frac{300}{k} e^{-kt} \Rightarrow \frac{dV}{dt} = 300e^{-kt}$	$\frac{dV}{dt} = \alpha e^{-kt}$	M1
	$300e^{-10k} = 200 \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ and correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
			(4)
(b) Way 2	$V = 0, t = 0 \Rightarrow 0 = \frac{300}{k} + A \Rightarrow A = -\frac{300}{k}$	Uses $V = 0$ when $t = 0$ to find A in terms of k	M1
	$\frac{dV}{dt} = 200, t = 10 \Rightarrow 200 = 300 - kV$ $\Rightarrow kV = 100$	Uses $\frac{dV}{dt} = 200$ when $t = 10$ to find a value for kV	M1
	$V = \frac{300}{k} + Ae^{-kt} \Rightarrow kV = 300 - 300e^{-10k}$ $\Rightarrow 100 = 300 - 300e^{-10k} \Rightarrow e^{-10k} = \frac{2}{3} \Rightarrow k = \dots$	Substitutes for kV, kA and $t = 10$ and uses correct processing to find k	M1
	$k = -\frac{1}{10} \ln \frac{2}{3}$	Oe e.g. $\frac{1}{10} \ln \frac{3}{2}$	A1
(c)	$6000 = \frac{3000}{\ln 1.5} - \frac{3000}{\ln 1.5} e^{-\frac{t}{10} \ln 1.5}$ $\Rightarrow e^{-\frac{t}{10} \ln 1.5} = 1 - 2 \ln 1.5$ $\Rightarrow -\frac{t}{10} \ln 1.5 = \ln(1 - 2 \ln 1.5)$	Correct strategy using $V = 6000$ to reach $at = \dots$	M1
	$t = 41$	Correct value	A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx (+c)$		M1A1
	M1: For applying parts to obtain $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left\{ -\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{4}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c)$ $\Rightarrow \frac{13}{9} \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \int e^{2x} \sin 3x \, dx (+c)$		M1A1
	M1: For applying parts to obtain $\alpha e^{2x} \cos 3x \pm \beta \int e^{2x} \sin 3x \, dx (+c)$ A1: Correct expression		
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{2} \left\{ \frac{1}{2} e^{2x} \sin 3x - \frac{3}{2} \int e^{2x} \cos 3x \, dx \right\} (+c)$ Applies parts again to $\int e^{2x} \sin 3x \, dx$ and obtains $\alpha e^{2x} \sin 3x \pm \beta \int e^{2x} \cos 3x \, dx$		M1
	$\int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x - \frac{9}{4} \int e^{2x} \cos 3x \, dx (+c)$ Fully correct application of parts twice		A1
	$\int e^{2x} \cos 3x \, dx + \frac{9}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c)$ $\Rightarrow \frac{13}{4} \int e^{2x} \cos 3x \, dx = \frac{1}{2} e^{2x} \cos 3x + \frac{3}{4} e^{2x} \sin 3x (+c) \Rightarrow \int e^{2x} \cos 3x \, dx = \dots$ Fully correct strategy for finding $\int e^{2x} \cos 3x \, dx$		M1
	$= \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + k$	Cao	A1
			Total 6

Question	Scheme	Marks
6(a)	$ \overrightarrow{OA} ^2 = (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2$	M1
	$ \overrightarrow{OA} = 5\sqrt{10} \Rightarrow (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2 = 250$	M1
	$\Rightarrow 64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ $\Rightarrow 81\lambda^2 + 52\lambda - 220 = 0^*$	A1*
		(3)
(b)	$81\lambda^2 + 52\lambda - 220 = 0 \Rightarrow (81\lambda - 110)(\lambda + 2) = 0 \Rightarrow \lambda = \dots \left(-2, \frac{110}{81}\right)$ $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1+8 \times \text{"their } \lambda \text{"} \\ 2 - \text{"their } \lambda \text{"} \\ 5+4 \times \text{"their } \lambda \text{"} \end{pmatrix}$	M1
	E.g $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 2 \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}^*$; or $\overrightarrow{OA} = \begin{pmatrix} 1+8 \times \frac{110}{81} \\ 2 - \frac{110}{81} \\ 5+4 \times \frac{110}{81} \end{pmatrix} = \begin{pmatrix} \frac{961}{81} \\ \frac{52}{81} \\ \frac{845}{81} \end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos \theta = \pm \frac{\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{15^2 + 4^2 + 3^2} \sqrt{8^2 + 1^2 + 4^2}} = \dots$	M1
	$= \pm \frac{-136}{45\sqrt{10}} (= \pm 0.9557\dots)$ or e.g. $\theta = \text{awrt } 0.299(\text{rad}) / 17.1^\circ$	A1
	$\text{Area } OAB = \frac{1}{2} 5\sqrt{10} \times 4\sqrt{10} \sin \theta = \dots$	M1
	$= \text{awrt } 29.4$	A1
		(4)
(10 marks)		

Question	Scheme	Marks
2	$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix} \quad \overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} \quad \overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$	
(a)	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \dots$	M1
	$= \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix}, 5\mathbf{i} + 6\mathbf{j} - 2\mathbf{k} \text{ or } (5, 6, -2)$	A1
		(2)
(b)	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} = \dots$	M1
	$\overrightarrow{OC} \cdot \overrightarrow{BC} = 0 \Rightarrow \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a-5 \\ -1 \\ 1 \end{pmatrix} = 0 \Rightarrow a(a-5) - 5 - 1 = 0$	dM1
	$a^2 - 5a - 6 = 0 \Rightarrow (a-6)(a+1) = 0 \Rightarrow a = \dots$	ddM1
	$a = -1, 6$	A1
		(4)
(6 marks)		

Question Number	Scheme	Marks
9 (a)	Let $p = 3k + 2$ then $(3k + 2)^3 = 27k^3 + 54k^2 + 36k + 8$	M1
	$= 3 \times (9k^3 + 18k^2 + 12k + 3) - 1$ not a multiple of 3	A1
	So p cannot be of form $3k + 1$ or $3k + 2$, since p^3 is a multiple of 3. Hence p must be a multiple of 3, a contradiction of our assumption, hence for all integers p , when p^3 is a multiple of 3, then p is a multiple of 3	A1
		(3)
(b)	Assumption: there exist (integers) p and q such that $\sqrt[3]{3} = \frac{p}{q}$ (where p and q have no (non-trivial) common factors.)	B1
	Then $\sqrt[3]{3} = \frac{p}{q} \Rightarrow p^3 = 3q^3$	M1
	So p^3 is a multiple of 3 and (so) p is a multiple of 3	A1
	But $p = 3k \Rightarrow 27k^3 = 3q^3 \Rightarrow q^3 = 9k^3$	dM1
	Hence q^3 is a multiple of 3 so q is a multiple of 3, but as p and q have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.*	A1*
		(5)
(8 marks)		

Question	Scheme	Marks
2	$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix} \quad \overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} \quad \overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$	
(a)	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} = \dots$	M1
	$= \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix}, 5\mathbf{i} + 6\mathbf{j} - 2\mathbf{k} \text{ or } (5, 6, -2)$	A1
		(2)
(b)	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} = \dots$	M1
	$\overrightarrow{OC} \cdot \overrightarrow{BC} = 0 \Rightarrow \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a-5 \\ -1 \\ 1 \end{pmatrix} = 0 \Rightarrow a(a-5) - 5 - 1 = 0$	dM1
	$a^2 - 5a - 6 = 0 \Rightarrow (a-6)(a+1) = 0 \Rightarrow a = \dots$	ddM1
	$a = -1, 6$	A1
		(4)
(6 marks)		