Question Number	Scheme	Marks
6(a)	Attempts $\pm\left((5\mathbf{i}+2\mathbf{j}+\mathbf{k})-(\mathbf{i}+2\mathbf{j}-3\mathbf{k})\right)=\pm(4\mathbf{i}+4\mathbf{k})$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{4^2+4^2}$ or $\lvert\overrightarrow{AC}\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{32}$ and $\lvert\overrightarrow{AC}\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Question Number	Scheme	Marks
8(a)(i)	$\pm\left((3\mathbf{i}+4\mathbf{j}-6\mathbf{k})-(2\mathbf{i}-\mathbf{j}+5\mathbf{k})\right)$	M1
	$=\mathbf{i}+5\mathbf{j}-11\mathbf{k}$	A1
(ii)	e.g. $\mathbf{r}=2\mathbf{i}-\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ or $\mathbf{r}=3\mathbf{i}+4\mathbf{j}-6\mathbf{k}+\lambda(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ o.e.	A1ft
		(3)
(a)		

(b)	$\overrightarrow{CA}=(2-p)\mathbf{i}-5\mathbf{j}+6\mathbf{k}$ $\quad\overrightarrow{CB}=(3-p)\mathbf{i}-5\mathbf{k}$	M1
	$(2-p)(3-p)-30=0$	**d**M1
	$\Rightarrow 6-5p+p^2-30=0\Rightarrow p^2-5p-24=0\Rightarrow p=...$	**dd**M1
	$p=-3,\ \ p=8$	A1
		(4)
(c)	$\lvert\overrightarrow{CA}\rvert=\sqrt{(2-8)^2+(-5)^2+6^2}$ or $\lvert\overrightarrow{CB}\rvert=\sqrt{(3-8)^2+(-5)^2}$	M1
	$\lvert\overrightarrow{CA}\rvert=\sqrt{97}$ or $\lvert\overrightarrow{CB}\rvert=\sqrt{50}$	A1
	Area of triangle $=\dfrac{1}{2}\times\lvert\overrightarrow{CA}\rvert\times\lvert\overrightarrow{CB}\rvert=\dfrac{1}{2}\times\sqrt{50}\times\sqrt{97}=$ awrt 34.8 units^2 or e.g. $\dfrac{1}{2}\times\lvert\overrightarrow{CA}\rvert\times\lvert\overrightarrow{AB}\rvert\sin BAC=\dfrac{1}{2}\times\sqrt{50}\times\sqrt{147}\sin\left(\cos^{-1}\dfrac{\sqrt{50}}{\sqrt{147}}\right)=$ awrt 34.8 units^2	dM1A1
		(4)
		(11 marks)

Question	Scheme	Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}5-1\\3--4\\-2-3\end{pmatrix}=\begin{pmatrix}4\\7\\-5\end{pmatrix}=4\mathbf{i}+7\mathbf{j}-5\mathbf{k}$	M1
	e.g. $\mathbf{r}=\mathbf{i}-4\mathbf{j}+3\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(4\mathbf{i}+7\mathbf{j}-5\mathbf{k})$	M1A1
		(3)
(b)	$\overrightarrow{AC}=\begin{pmatrix}3-1\\p--4\\-1-3\end{pmatrix}=\begin{pmatrix}2\\p+4\\-4\end{pmatrix}=2\mathbf{i}+(p+4)\mathbf{j}-4\mathbf{k}$	M1
	$\begin{pmatrix}2\\p+4\\-4\end{pmatrix}\cdot\begin{pmatrix}4\\7\\-5\end{pmatrix}=8+7p+28+20=0\Rightarrow p=-8$	M1A1
		(3)
(c)	$\|AB\|=\sqrt{4^2+7^2+(-5)^2}=\sqrt{90}$ or $\|AC\|=\sqrt{2^2+(-4)^2+(-4)^2}=6$	M1
	Area $\frac{1}{2}\times"\sqrt{90}"\times"6"=9\sqrt{10}$	dM1A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
6(a)	$(2,3,-7)$	B1
		(1)
(b)	Attempts $\begin{pmatrix}1\\2\\2\end{pmatrix}\bullet\begin{pmatrix}4\\-1\\8\end{pmatrix}=1\times4+2\times-1+2\times8=(18)$	M1
	Attempts $\mathbf{a.b}=\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$: $18=\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+(-1)^2+8^2}\cos\theta$	**d**M1
	$\cos\theta=\frac{2}{3}$	A1
		(3)

(c)	Uses $\lambda = 6$ to find length PQ E.g. $\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k} \Rightarrow PQ = \sqrt{6^2 + 12^2 + 12^2} = (18)$ Or $PQ = 6 \times \sqrt{1^2 + 2^2 + 2^2} = (18)$	M1
	Area $QPR = \frac{1}{2}ab\sin C = \frac{1}{2} \times 18^2 \times \sqrt{1 - \left(\frac{2}{3}\right)^2} = 54\sqrt{5}$	M1 A1
		(3)
(d)	Attempts a correct method of finding at least one value for μ e.g. $(4\mu)^2 + \mu^2 + (8\mu)^2 = 6^2 + 12^2 + 12^2$ $\Rightarrow \mu = \frac{\text{"18"}}{\sqrt{4^2 + (-1)^2 + 8^2}} = (\pm)2$	M1
	Attempts one correct position $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} \pm 2 \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$	**d**M1
	Possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$	A1
		(3)
		(10 marks)

Question Number	Scheme	Marks
7 (a)	Finds $\left\| \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \right\| = \sqrt{4^2 + 4^2 + 2^2} = 6$ and attempts $\frac{1}{\text{"6"}} \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}$	M1
	$\overrightarrow{OA} = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$ oe	A1
		(2)
(b)	Co-ordinates or position vector of point $X = \begin{pmatrix} 1 + 4\lambda \\ -10 + 4\lambda \\ -9 + 2\lambda \end{pmatrix}$	M1
	$\overrightarrow{OX} . \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0 \Rightarrow 4(1 + 4\lambda) + 4(-10 + 4\lambda) + 2(-9 + 2\lambda) = 0$	dM1
	$36\lambda = 54 \Rightarrow \lambda = 1.5$	ddM1 A1
	$X = (7, -4, -6)$	A1
		(5)
(c)	Finds $OX = \sqrt{7^2 + (-4)^2 + (-6)^2} = \sqrt{101}$ and $OA = 1$	M1
	Area $OXA = \frac{1}{2} \times 1 \times \sqrt{101} = \frac{\sqrt{101}}{2}$	dM1 A1
		(3)
		(10 marks)

Question	Scheme	Marks
6(a)	$\left\lvert\overrightarrow{OA}\right\rvert^2=(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2$	**M1**
	$\left\lvert\overrightarrow{OA}\right\rvert=5\sqrt{10}\Rightarrow(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2=250$	**M1**
	$\Rightarrow 64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ $\Rightarrow 81\lambda^2+52\lambda-220=0*$	**A1***
		(3)
(b)	$81\lambda^2+52\lambda-220=0\Rightarrow(81\lambda-110)(\lambda+2)=0\Rightarrow\lambda=...\left(-2,\dfrac{110}{81}\right)$ $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1+8\times\text{"their}\lambda\text{"}\\2-\text{"their}\lambda\text{"}\\5+4\times\text{"their}\lambda\text{"}\end{pmatrix}$	**M1**
	E.g $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1\\2\\5\end{pmatrix}-2\begin{pmatrix}8\\-1\\4\end{pmatrix}=\begin{pmatrix}-15\\4\\-3\end{pmatrix}*$; or $\overrightarrow{OA}=\begin{pmatrix}1+8\times\frac{110}{81}\\2-\frac{110}{81}\\5+4\times\frac{110}{81}\end{pmatrix}=\begin{pmatrix}\frac{961}{81}\\\frac{52}{81}\\\frac{845}{81}\end{pmatrix}$	**A1*;** **A1**
		(3)
(c)	$\cos\theta=\pm\dfrac{\begin{pmatrix}-15\\4\\-3\end{pmatrix}.\begin{pmatrix}8\\-1\\4\end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}}=...$	**M1**
	$=\pm\dfrac{-136}{45\sqrt{10}}\ (=\pm0.9557...)$ or e.g. $\theta=\text{awrt }0.299(rad)/17.1°$	**A1**
	Area $OAB=\dfrac{1}{2}5\sqrt{10}\times4\sqrt{10}\sin\theta=...$	**M1**
	$=\text{awrt }29.4$	**A1**
		(4)
		(10 marks)

Question Number	Scheme	Marks
8	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\b\end{pmatrix}\Rightarrow \begin{matrix}-1+2\lambda=2+4\mu\ (1)\\5-\lambda=-2-3\mu\ (2)\\4+5\lambda=-5+\mu b\ (3)\end{matrix}$	
	Uses equations (1) and (2) to find either λ **or** μ e.g. $(1)+2(2)\Rightarrow\mu=...$ or $3(1)+4(2)\Rightarrow\lambda=...$	M1
	Uses equations (1) and (2) to find both λ and μ	**d**M1
	$\mu=-\dfrac{11}{2}$ and $\lambda=-\dfrac{19}{2}$	A1
	$4+5\lambda=-5+\mu b\Rightarrow 4+5\times-\dfrac{19}{2}=-5-\dfrac{11}{2}b$ or $4+5\lambda=-5+7\mu\Rightarrow 4+5\times-\dfrac{19}{2}=-5-\dfrac{11}{2}\times 7$	**dd**M1
	$\Rightarrow 11b=77\Rightarrow b=7$ or obtains $-\dfrac{87}{2}=-\dfrac{87}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6)

Question	Scheme	Marks
6(a)	$\left\lvert\overrightarrow{OA}\right\rvert^2=(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2$	**M1**
	$\left\lvert\overrightarrow{OA}\right\rvert=5\sqrt{10}\Rightarrow(1+8\lambda)^2+(2-\lambda)^2+(5+4\lambda)^2=250$	**M1**
	$\Rightarrow 64\lambda^2+16\lambda+1+\lambda^2-4\lambda+4+16\lambda^2+40\lambda+25=250$ $\Rightarrow 81\lambda^2+52\lambda-220=0$*	**A1***
		(3)
(b)	$81\lambda^2+52\lambda-220=0\Rightarrow(81\lambda-110)(\lambda+2)=0\Rightarrow\lambda=...\left(-2,\dfrac{110}{81}\right)$ $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1+8\times\text{"their}\lambda\text{"}\\2-\text{"their}\lambda\text{"}\\5+4\times\text{"their}\lambda\text{"}\end{pmatrix}$	**M1**
	E.g $\Rightarrow\overrightarrow{OA}=\begin{pmatrix}1\\2\\5\end{pmatrix}-2\begin{pmatrix}8\\-1\\4\end{pmatrix}=\begin{pmatrix}-15\\4\\-3\end{pmatrix}$ * ; or $\overrightarrow{OA}=\begin{pmatrix}1+8\times\frac{110}{81}\\2-\frac{110}{81}\\5+4\times\frac{110}{81}\end{pmatrix}=\begin{pmatrix}\frac{961}{81}\\\frac{52}{81}\\\frac{845}{81}\end{pmatrix}$	**A1*;** **A1**
		(3)

(c)	$\cos\theta = \pm\dfrac{\begin{pmatrix}-15\\4\\-3\end{pmatrix}.\begin{pmatrix}8\\-1\\4\end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}} = \ldots$	M1
	$= \pm\dfrac{-136}{45\sqrt{10}}\ \left(=\pm 0.9557\ldots\right)$ or e.g. $\theta =$ awrt $0.299(rad)/17.1°$	A1
	Area $OAB = \dfrac{1}{2}5\sqrt{10}\times 4\sqrt{10}\sin\theta = \ldots$	M1
	$=$ awrt 29.4	A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
9	$\begin{pmatrix}2-\lambda\\8+2\lambda\\10+3\lambda\end{pmatrix} = \begin{pmatrix}-4+5\mu\\-1+4\mu\\2+8\mu\end{pmatrix}$ Attempts to solve any two of the three equations Either (1) and (2) $\left.\begin{matrix}2-\lambda=-4+5\mu\\8+2\lambda=-1+4\mu\end{matrix}\right\}\Rightarrow \lambda=-\dfrac{3}{2}, \mu=\dfrac{3}{2}$ (1) and (3) $\left.\begin{matrix}2-\lambda=-4+5\mu\\10+3\lambda=2+8\mu\end{matrix}\right\}\Rightarrow \lambda=\dfrac{8}{23}, \mu=\dfrac{26}{23}$ (2) and (3) $\left.\begin{matrix}8+2\lambda=-1+4\mu\\10+3\lambda=2+8\mu\end{matrix}\right\}\Rightarrow \lambda=-10, \mu=-\dfrac{11}{4}$	M1, A1
	Substitutes their values of λ and μ into both sides of the "third" equation E.g. $\lambda=-\dfrac{3}{2}$ into $10+3\lambda=\dfrac{11}{2}$ and $\mu=\dfrac{3}{2}$ into $2+8\mu=14$	dM1
	Concludes that lines don't intersect with correct calculations and minimal reason	A1
	Additionally states that $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ is not parallel to $\begin{pmatrix}5\\4\\8\end{pmatrix}$ with a minimal reason So lines are skew CSO *	A1*
		(5) (5 marks)

Question	Scheme	Marks
6(a)	$(3\mathbf{i}+p\mathbf{j}+7\mathbf{k})+\lambda(2\mathbf{i}-5\mathbf{j}+4\mathbf{k})=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$ $3+2\lambda=8+4\mu \quad (1)$ $\Rightarrow p-5\lambda=-2+\mu \quad (2)$ $7+4\lambda=5+2\mu \quad (3)$	M1
	e.g. $(3)-2\times(1): 1=-11-6\mu \Rightarrow \mu=-2 \quad \left(\text{or } \lambda=-\frac{3}{2}\right)$	M1
	$\mu=-2,\ \lambda=-\frac{3}{2} \Rightarrow p=-2-2-\frac{15}{2}=\ldots$	M1
	$p=-\frac{23}{2}$	A1
		(4)
(b)	Intersect at $(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+-2(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=..$	M1
	$=-4\mathbf{j}+\mathbf{k}$	A1
		(2)
(c)	$(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=2\times4-5\times1+4\times2=\ldots$	M1
	$\cos\theta=\frac{\text{"11"}}{\sqrt{2^2+5^2+4^2}\sqrt{4^2+1^2+2^2}}=\ldots\left(=\frac{11}{3\sqrt{105}}=0.3578\ldots\right)$	M1
	$\theta=\cos^{-1}\frac{11}{3\sqrt{105}}=69.0°$	A1
		(3)
(d)	$\lambda=2\Rightarrow\overrightarrow{OA}=\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)$	B1ft
	$\overrightarrow{AB}=\pm\left((8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})-\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)\right)$ $=\pm\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right)$	M1
	$\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=0\Rightarrow\mu=..\left(=-\frac{1}{6}\right)$	M1
	$\overrightarrow{OB}=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\text{"}-\frac{1}{6}\text{"}(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=\ldots$	dM1
	$=\frac{22}{3}\mathbf{i}-\frac{13}{6}\mathbf{j}+\frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3},-\frac{13}{6},\frac{14}{3}\right)$	A1
		(5)

Question Number	Scheme	Marks
6(a)	$(2, 3, -7)$	B1
		(1)
(b)	Attempts $\begin{pmatrix}1\\2\\2\end{pmatrix} \bullet \begin{pmatrix}4\\-1\\8\end{pmatrix} = 1\times4+2\times-1+2\times8=(18)$	M1
	Attempts $\mathbf{a.b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$: $18=\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+(-1)^2+8^2}\cos\theta$	**d**M1
	$\cos\theta = \dfrac{2}{3}$	A1
		(3)
(c)	Uses $\lambda = 6$ to find length PQ E.g. $\overrightarrow{PQ} = 6\mathbf{i}+12\mathbf{j}+12\mathbf{k} \Rightarrow PQ = \sqrt{6^2+12^2+12^2}=(18)$ Or $PQ = 6\times\sqrt{1^2+2^2+2^2}=(18)$	M1
	Area $QPR = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}\times18^2\times\sqrt{1-\left(\dfrac{2}{3}\right)^2} = 54\sqrt{5}$	M1 A1
		(3)
(d)	Attempts a correct method of finding at least one value for μ e.g. $(4\mu)^2+\mu^2+(8\mu)^2 = 6^2+12^2+12^2$ $\Rightarrow \mu = \dfrac{\text{"18"}}{\sqrt{4^2+(-1)^2+8^2}} = (\pm)2$	M1
	Attempts one correct position $\mathbf{r} = \begin{pmatrix}2\\3\\-7\end{pmatrix} \pm 2\begin{pmatrix}4\\-1\\8\end{pmatrix}$	**d**M1
	Possible coordinates $(10, 1, 9)$ and $(-6, 5, -23)$	A1
		(3)
		(10 marks)

Question	Scheme	Marks
2	$\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\quad\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\quad\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$	
(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=...$	**M1**
	$=\begin{pmatrix}5\\6\\-2\end{pmatrix}$, $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$	**A1**
		(2)
(b)	$\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=...$	**M1**
	$\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$	**dM1**
	$a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=...$	**ddM1**
	$a=-1,\ 6$	**A1**
		(4)
		(6 marks)

Question Number	Scheme	Marks
9(a)	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ Either $A: \lambda = 1$ or $B: \lambda = 3$ leading to $a = -3,\ b = 5$	 M1 A1, A1 **(3)**
(b)	$\overrightarrow{AB} = = \pm((\text{'}5\mathbf{i}\text{'} - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ \text{'}-3\mathbf{j}\text{'} + 5\mathbf{k}));= 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1, A1 **(2)**
(c)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix}4\\-3\\2\end{pmatrix} - \begin{pmatrix}1\\-3\\5\end{pmatrix} = \begin{pmatrix}3\\0\\-3\end{pmatrix}$ or $\overrightarrow{CA} = \begin{pmatrix}-3\\0\\3\end{pmatrix}$	M1
	$\cos C\hat{A}B = \dfrac{\begin{pmatrix}4\\2\\-2\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-3\end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2}\,.\sqrt{(3)^2 + (0)^2 + (-3)^2}}$	dM1
	$\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24}\,.\sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$	A1 **(3)**
(d)	Area $CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ;= 3\sqrt{3}$	M1, A1 **(2)**
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} + 5\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}9\\1\\1\end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} - 3\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-7\\-7\\9\end{pmatrix}$	M1, A1
	For both of $\overrightarrow{OD_1} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} + 5\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}9\\1\\1\end{pmatrix}$ and $\overrightarrow{OD_2} = \begin{pmatrix}-1\\-4\\6\end{pmatrix} - 3\begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-7\\-7\\9\end{pmatrix}$	M1, A1 **(4)** **(14 marks)**

Question Number	Scheme	Marks
6(a)	Attempts $\pm\left(\left(5\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)-\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)\right)=\pm\left(4\mathbf{i}+4\mathbf{k}\right)$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{4^2+4^2}$ or $\lvert\overrightarrow{AC}\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{32}$ and $\lvert\overrightarrow{AC}\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Question Number	Scheme	Marks
5(a)	e.g. $\dfrac{r}{h}=\dfrac{12}{30}\Rightarrow r=\dfrac{12}{30}h\Rightarrow V=\dfrac{1}{3}\pi r^2h=\dfrac{1}{3}\pi\left(\dfrac{2}{5}h\right)^2h$	M1
	$V=\dfrac{4\pi h^3}{75}$ *	A1*
		(2)
(b)	$V=\dfrac{4\pi h^3}{75}\Rightarrow\dfrac{\mathrm{d}V}{\mathrm{d}h}=\dfrac{12\pi h^2}{75}\left(=\dfrac{4\pi h^2}{25}\right)$	B1
	$V=1.5\times60\times2\pi$ $180\pi=\dfrac{4\pi h^3}{75}\Rightarrow h^3=3375\Rightarrow h=\ldots(15)$	M1
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\mathrm{d}h}{\mathrm{d}V}\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{75}{12\pi\times15^2}\times2\pi=\ldots$	M1
	$\dfrac{1}{18}$ (cm s^{-1})	A1
		(4)
		Total 6

Question	Scheme	Marks
2	$\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\quad\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\quad\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$	
(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=\ldots$	**M1**
	$=\begin{pmatrix}5\\6\\-2\end{pmatrix}$, $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$	**A1**
		(2)
(b)	$\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=\ldots$	**M1**
	$\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$	**dM1**
	$a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=\ldots$	**ddM1**
	$a=-1,\ 6$	**A1**
		(4)
		(6 marks)

Question Number	Scheme	Marks
6 (a)(i)	$\overrightarrow{AB} = (\pm)\left[(8\mathbf{i}+3\mathbf{j}-7\mathbf{k})-(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})\right] = ...$	M1
	$\overrightarrow{AB} = 6\mathbf{i}+6\mathbf{j}-12\mathbf{k}$	A1
(ii)	$\mathbf{r} = \begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r} = \begin{pmatrix}8\\3\\-7\end{pmatrix}+\lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$	B1ft
		(3)
(b)	Attempts $\pm\overrightarrow{CP} = \pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$	M1
	$\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0 \Rightarrow \begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0 \Rightarrow 1(\lambda-1)+1(\lambda-8)-2(-2\lambda+3)=0$ Alt: $(\lambda-1)^2+(\lambda-8)^2+(-2\lambda+3)^2 = 6\lambda^2-30\lambda+74 = 6\left(\lambda-\frac{5}{2}\right)^2+\frac{73}{2}$	dM1
	$\Rightarrow \lambda = \frac{5}{2}$ $\left[\text{use of } \overrightarrow{AB} \text{ in } \overrightarrow{CP} \text{ gives } \lambda = \frac{5}{12}, \text{use of } \overrightarrow{OB}\ \lambda = \frac{-7}{2} \text{ or } \frac{-7}{12}\right]$	A1
	$\overrightarrow{OP} = \begin{pmatrix}2\\-3\\5\end{pmatrix}+\frac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix} = \frac{9}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}$	ddM1, A1 (5)
		(8 marks)

Question Number	Scheme	Marks
6 (a)(i)	$\overrightarrow{AB}=(\pm)\left[(8\mathbf{i}+3\mathbf{j}-7\mathbf{k})-(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})\right]=...$	**M1**
	$\overrightarrow{AB}=6\mathbf{i}+6\mathbf{j}-12\mathbf{k}$	**A1**
(ii)	$\mathbf{r}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r}=\begin{pmatrix}8\\3\\-7\end{pmatrix}+\lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$	**B1ft**
		(3)
(b)	Attempts $\pm\overrightarrow{CP}=\pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$	**M1**
	$\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow\begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow 1(\lambda-1)+1(\lambda-8)-2(-2\lambda+3)=0$ Alt: $(\lambda-1)^2+(\lambda-8)^2+(-2\lambda+3)^2=6\lambda^2-30\lambda+74=6\left(\lambda-\frac{5}{2}\right)^2+\frac{73}{2}$	**dM1**
	$\Rightarrow\lambda=\frac{5}{2}$ $\left[\text{use of }\overrightarrow{AB}\text{ in }\overrightarrow{CP}\text{ gives }\lambda=\frac{5}{12},\text{use of }\overrightarrow{OB}\ \lambda=\frac{-7}{2}\text{ or }\frac{-7}{12}\right]$	**A1**
	$\overrightarrow{OP}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\frac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix}=\frac{9}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}$	**ddM1, A1** **(5)**
		(8 marks)

Question Number	Scheme	Marks
4(a)	Attempts direction vector by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r}=4\mathbf{i}+8\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+6\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1
		(2)
(b)	E.g $\overrightarrow{PC}=\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}-\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix}$	M1
	Uses $\overrightarrow{PC}.(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ E.g. $\Rightarrow 1(2+\lambda)-2(10-2\lambda)+2\times2\lambda=0\Rightarrow\lambda=...$	dM1
	E.g I $\lambda=2\Rightarrow\mathbf{c}=(4+2)\mathbf{i}+(8-4)\mathbf{j}+(1+4)\mathbf{k}$ E.g II $\mu=1\Rightarrow\mathbf{c}=(5+1)\mathbf{i}+(6-2)\mathbf{j}+(3+2)\mathbf{k}$	ddM1
	$(6,\ 4,\ 5)$	A1
		(4)
(c)	$\overrightarrow{OP'}=\mathbf{p}+2\overrightarrow{PC}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}+2(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$	M1
	$(10,\ 10,\ 9)$	A1
		(2)
(d)	$\lvert\overrightarrow{PP'}\rvert=2\lvert\overrightarrow{PC}\rvert=2\sqrt{4^2+6^2+4^2}=...$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

Question Number	Scheme	Marks
2(a)	$\overrightarrow{BA}.\overrightarrow{BC}=-6\times2+2\times5-3\times8=(-26)$	M1
	Uses $\overrightarrow{BA}.\overrightarrow{BC}=\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\cos\theta\Rightarrow-26=\sqrt{49}\times\sqrt{93}\cos\theta\Rightarrow\theta=...$	**d**M1
	$\theta=112.65°$	A1
		(3)
(b)	Attempts to use $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert\sin\theta$ with their θ	M1
	Area = awrt 62.3	A1
		(2)
		(5 marks)

Question	Scheme	Marks
6(a)	$(3\mathbf{i}+p\mathbf{j}+7\mathbf{k})+\lambda(2\mathbf{i}-5\mathbf{j}+4\mathbf{k})=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$ $3+2\lambda=8+4\mu$ (1) $\Rightarrow p-5\lambda=-2+\mu$ (2) $7+4\lambda=5+2\mu$ (3)	M1
	e.g. $(3)-2\times(1): 1=-11-6\mu \Rightarrow \mu=-2$ $\left(\text{or } \lambda=-\frac{3}{2}\right)$	M1
	$\mu=-2$, $\lambda=-\frac{3}{2} \Rightarrow p=-2-2-\frac{15}{2}=...$	M1
	$p=-\frac{23}{2}$	A1
		(4)
(b)	Intersect at $(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+-2(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=..$	M1
	$=-4\mathbf{j}+\mathbf{k}$	A1
		(2)
(c)	$(2\mathbf{i}-5\mathbf{j}+4\mathbf{k}).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=2\times4-5\times1+4\times2=...$	M1
	$\cos\theta=\frac{\text{"11"}}{\sqrt{2^2+5^2+4^2}\sqrt{4^2+1^2+2^2}}=...\left(=\frac{11}{3\sqrt{105}}=0.3578...\right)$	M1
	$\theta=\cos^{-1}\frac{11}{3\sqrt{105}}=69.0°$	A1
		(3)
(d)	$\lambda=2\Rightarrow\overrightarrow{OA}=\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)$	B1ft
	$\overrightarrow{AB}=\pm\left((8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\mu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})-\left(7\mathbf{i}-\frac{43}{2}\mathbf{j}+15\mathbf{k}\right)\right)$ $=\pm\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right)$	M1
	$\left((1+4\mu)\mathbf{i}+\left(\frac{39}{2}+\mu\right)\mathbf{j}+(-10+2\mu)\mathbf{k}\right).(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=0\Rightarrow\mu=..\left(=-\frac{1}{6}\right)$	M1
	$\overrightarrow{OB}=(8\mathbf{i}-2\mathbf{j}+5\mathbf{k})+\text{"}-\frac{1}{6}\text{"}(4\mathbf{i}+\mathbf{j}+2\mathbf{k})=...$	dM1
	$=\frac{22}{3}\mathbf{i}-\frac{13}{6}\mathbf{j}+\frac{14}{3}\mathbf{k}$ or $\left(\frac{22}{3},-\frac{13}{6},\frac{14}{3}\right)$	A1
		(5)

Question Number	Scheme	Marks
7. (a)	Attempts $\overrightarrow{BA}\cdot\overrightarrow{BC}$ $\begin{pmatrix}-6\\2\\-3\end{pmatrix}\bullet\begin{pmatrix}-4\\3\\-5\end{pmatrix}=24+6+15=45$	M1
	Attempts to apply $\mathbf{a}\bullet\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta \Rightarrow 45=7\times\sqrt{50}\cos\theta$	dM1
	$\cos\theta=\dfrac{9\sqrt{2}}{14}$ oe	A1
		(3)
(b)	Attempts $\sin\theta=\sqrt{1-\left(\dfrac{9\sqrt{2}}{14}\right)^2}$	M1
	Attempts to apply $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta=7\times\sqrt{50}\times\dfrac{\sqrt{34}}{14}$	M1
	Area *ABCD* = $5\sqrt{17}$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
4(a)	Attempts direction vector by subtracting $(5\mathbf{i}+6\mathbf{j}+3\mathbf{k})$ and $(4\mathbf{i}+8\mathbf{j}+\mathbf{k})$ either way around.	M1
	E.g. $\mathbf{r}=4\mathbf{i}+8\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$, $\mathbf{r}=5\mathbf{i}+6\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1
		(2)
(b)	E.g $\overrightarrow{PC}=\begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}-\begin{pmatrix}2\\-2\\1\end{pmatrix}=\begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix}$	M1
	Uses $\overrightarrow{PC}.(\mathbf{i}-2\mathbf{j}+2\mathbf{k})=0$ E.g. $\Rightarrow 1(2+\lambda)-2(10-2\lambda)+2\times2\lambda=0\Rightarrow\lambda=...$	dM1
	E.g I $\lambda=2\Rightarrow\mathbf{c}=(4+2)\mathbf{i}+(8-4)\mathbf{j}+(1+4)\mathbf{k}$ E.g II $\mu=1\Rightarrow\mathbf{c}=(5+1)\mathbf{i}+(6-2)\mathbf{j}+(3+2)\mathbf{k}$	ddM1
	$(6,\ 4,\ 5)$	A1
		(4)
(c)	$\overrightarrow{OP'}=\mathbf{p}+2\overrightarrow{PC}=2\mathbf{i}-2\mathbf{j}+\mathbf{k}+2(4\mathbf{i}+6\mathbf{j}+4\mathbf{k})$	M1
	$(10,\ 10,\ 9)$	A1
		(2)
(d)	$\lvert\overrightarrow{PP'}\rvert=2\lvert\overrightarrow{PC}\rvert=2\sqrt{4^2+6^2+4^2}=...$	M1
	$4\sqrt{17}$	A1
		(2)
		Total 10

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}))$	M1
	e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$ e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$	A1
		(2)
(b)	$3+3\mu=-6\Rightarrow\mu=-3$	M1
	$p+12=4\Rightarrow p=\ldots$ or $q-3=-1\Rightarrow q=\ldots$	dM1
	$p=-8,\quad q=2$	A1
		(3)
(c)	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=12+4+3$	M1
	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=\ldots$	M1
	$(\cos\theta=)\ \dfrac{19}{26}$	A1
		(3)
(d)	$AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\dfrac{19}{26}\right)^2}$	M1
	$\dfrac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}))$	M1
	e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$ e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$	A1
		(2)
(b)	$3+3\mu=-6\Rightarrow\mu=-3$	M1
	$p+12=4\Rightarrow p=\ldots$ or $q-3=-1\Rightarrow q=\ldots$	dM1
	$p=-8,\quad q=2$	A1
		(3)
(c)	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=12+4+3$	M1
	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=\ldots$	M1
	$(\cos\theta=)\ \dfrac{19}{26}$	A1
		(3)
(d)	$AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\dfrac{19}{26}\right)^2}$	M1
	$\dfrac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \ldots$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = \ldots$ $\left(NB \quad \sqrt{172} = 2\sqrt{43}, \ \sqrt{189} = 3\sqrt{21} \right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2,\ BC^2 = 6^2 + 4^2 + 3^2,\ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC \cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172}\cos\theta \Rightarrow \cos\theta = \ldots$	ddM1
	$\theta =$ awrt $33.7°$	A1
(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

Question	Scheme	Marks
8(a)	Attempts $\pm\left(\begin{pmatrix}6\\0\\7\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}\right)=\pm\begin{pmatrix}0\\-6\\5\end{pmatrix}$	M1
	$\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ oe	A1
		(2)
(b)	Meet if $\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}=\begin{pmatrix}3\\1\\4\end{pmatrix}+\mu\begin{pmatrix}1\\5\\9\end{pmatrix}$ so $\begin{cases}6=3+\mu\\6-6\lambda=1+5\mu\\2+5\lambda=4+9\mu\end{cases}$	M1
	From first equation $\mu=3$ Alt: solves equations 2 and 3 to give either $\mu=\frac{13}{79}$ or $\lambda=\frac{55}{79}$	A1
	Then need $\lambda=\frac{5-5\times3}{6}=-\frac{5}{3}$ from 2nd equation, $\lambda=\frac{2+9\times3}{5}=\frac{29}{5}$ from 3rd equation Alt: checks their μ in the first equation	M1
	Values of λ do not agree and hence lines do not meet.	A1
		(4)
(c)	$\overrightarrow{OC}=\begin{pmatrix}3\\1\\4\end{pmatrix}-\begin{pmatrix}1\\5\\9\end{pmatrix}\left(=\begin{pmatrix}2\\-4\\-5\end{pmatrix}\right)$	M1
	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}2\\-4\\-5\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}=\begin{pmatrix}-4\\-10\\-7\end{pmatrix}$ Accept $\pm$. **Depends on first mark.**	dM1
	$\frac{\pm\overrightarrow{AC}\cdot(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\lvert\overrightarrow{AC}\rvert\lvert\mathbf{i}+5\mathbf{j}+9\mathbf{k}\rvert}=\pm\frac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}$ **Depends on both previous marks.**	ddM1A1
	$\Rightarrow\theta=\arccos\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$ or e.g. $\Rightarrow\theta=90°-\arcsin\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$	A1
		(5)
		(11 marks)

Question Number	Scheme	Marks
3 (a)	Attempts $(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})-(8\mathbf{i}-5\mathbf{j}+3\mathbf{k})$	M1
	$\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$	A1
		(2)
(b)	Attempts $\overrightarrow{PQ}\cdot\overrightarrow{RQ}=2\times-6+-3\times2+4\times1$	M1
	Full attempt to find $\cos PQR$ E.g. $2\times-6+-3\times2+4\times1=\sqrt{29}\sqrt{41}\cos PQR$	dM1
	Angle PQR = 114°	A1
		(3)
		(5 marks)

Question Number	Scheme	Marks
8(a)	$\pm(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}))$	M1
	e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$ e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda(4\mathbf{i}-\mathbf{j}+3\mathbf{k})$	A1
		(2)
(b)	$3+3\mu=-6\Rightarrow\mu=-3$	M1
	$p+12=4\Rightarrow p=\ldots$ or $q-3=-1\Rightarrow q=\ldots$	dM1
	$p=-8,\quad q=2$	A1
		(3)
(c)	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=12+4+3$	M1
	$(4\mathbf{i}-\mathbf{j}+3\mathbf{k})\cdot(3\mathbf{i}-4\mathbf{j}+\mathbf{k})=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=\ldots$	M1
	$(\cos\theta=)\ \dfrac{19}{26}$	A1
		(3)
(d)	$AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\dfrac{19}{26}\right)^2}$	M1
	$\dfrac{3\sqrt{910}}{26}$ o.e.	A1
		(2)
		Total 10

Question Number	Scheme	Notes	Marks
5(a)	$4+2\lambda=13+5\mu$ $4-3\lambda=-1+\mu$ $-5+6\lambda=4-3\mu$	For writing down any 2 of these equations.	M1
	E.g. $4+2\lambda=13+5\mu$ $4-3\lambda=-1+\mu$ $\Rightarrow \lambda=\ldots$ or $\mu=\ldots$	Full method for finding λ or μ	M1
	$\lambda=2,\ \mu=-1$	Both correct values	A1
	$-5+6\lambda=-5+12=7$ $4-3\mu=4+3=7$ So lines intersect	Shows that the parameters satisfy the third equation and makes a conclusion.	B1
	$\lambda=2\rightarrow(4+4)\mathbf{i}+(4-6)\mathbf{j}+(-5+12)\mathbf{k}$ **or** $\mu=-1\rightarrow(13-5)\mathbf{i}+(-1-1)\mathbf{j}+(4+3)\mathbf{k}$	Uses their λ or μ to find A.	M1
	$8\mathbf{i}-2\mathbf{j}+7\mathbf{k}$	Correct vector or coordinates	A1
			(6)
(b)	$\begin{pmatrix}2\\-3\\6\end{pmatrix}\bullet\begin{pmatrix}5\\1\\-3\end{pmatrix}=10-3-18=\sqrt{2^2+3^2+6^2}\sqrt{5^2+1^2+3^2}\cos\theta$ Full attempt at the scalar product between the direction vectors		M1
	$\cos\theta=\pm\dfrac{11}{7\sqrt{35}}$	Correct magnitude for cos θ (may be implied by e.g. $\theta=105.4\ldots$ or $74.6\ldots$	A1
	$\theta=74.6°$	Awrt 74.6	A1
			(3)
(c)	$\lvert 2\mathbf{i}-3\mathbf{j}+6\mathbf{k}\rvert=\sqrt{2^2+3^2+6^2}=7$	Finds the magnitude of the direction of l_1	M1
	$35\div7=5\Rightarrow\lambda=5$ $8\mathbf{i}-2\mathbf{j}+7\mathbf{k}\pm5(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})$	Correct strategy for one of the points	M1
	$(18,-17,37)$ or $(-2,13,-23)$	One correct point (ignore labels)	A1
	$P(18,-17,37)$ and $Q(-2,13,-23)$	Correct points with correct labels	A1
			(4)
			Total 13

Question Number	Scheme	Marks
9(a)	$A(1,a,5)$, $B(b,-1,3)$, $l: \mathbf{r} = -\mathbf{i}-4\mathbf{j}+6\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}-\mathbf{k})$	
	Either $A:\lambda=1$ or $B:\lambda=3$	M1
	leading to $a=-3,\ b=5$	A1, A1
		(3)
(b)	$\overrightarrow{AB} = = \pm(('5\mathbf{i}'-\mathbf{j}+3\mathbf{k})-(\mathbf{i}\ '-3\mathbf{j}'+5\mathbf{k}));= 4\mathbf{i}+2\mathbf{j}-2\mathbf{k}$	M1, A1
		(2)
(c)	$\overrightarrow{AC} = \overrightarrow{OC}-\overrightarrow{OA} = \begin{pmatrix}4\\-3\\2\end{pmatrix}-\begin{pmatrix}1\\-3\\5\end{pmatrix}=\begin{pmatrix}3\\0\\-3\end{pmatrix}$ or $\overrightarrow{CA}=\begin{pmatrix}-3\\0\\3\end{pmatrix}$	M1
	$\cos C\hat{A}B = \dfrac{\begin{pmatrix}4\\2\\-2\end{pmatrix}\bullet\begin{pmatrix}3\\0\\-3\end{pmatrix}}{\sqrt{(4)^2+(2)^2+(-2)^2}\,.\sqrt{(3)^2+(0)^2+(-3)^2}}$	dM1
	$\cos C\hat{A}B = \dfrac{12+0+6}{\sqrt{24}.\sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$	A1
		(3)
(d)	Area $CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ;=3\sqrt{3}$	M1, A1
		(2)
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix}-1\\-4\\6\end{pmatrix}+5\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}9\\1\\1\end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix}-1\\-4\\6\end{pmatrix}-3\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}-7\\-7\\9\end{pmatrix}$	M1, A1
	For both of $\overrightarrow{OD_1} = \begin{pmatrix}-1\\-4\\6\end{pmatrix}+5\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}9\\1\\1\end{pmatrix}$ and $\overrightarrow{OD_2} = \begin{pmatrix}-1\\-4\\6\end{pmatrix}-3\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}-7\\-7\\9\end{pmatrix}$	M1, A1
		(4)
		(14 marks)

Question Number	Scheme	Marks
8(a)(i)	$\pm((3\mathbf{i}+4\mathbf{j}-6\mathbf{k})-(2\mathbf{i}-\mathbf{j}+5\mathbf{k}))$	M1
	$=\mathbf{i}+5\mathbf{j}-11\mathbf{k}$	A1
(ii)	e.g. $\mathbf{r}=2\mathbf{i}-\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ or $\mathbf{r}=3\mathbf{i}+4\mathbf{j}-6\mathbf{k}+\lambda(\mathbf{i}+5\mathbf{j}-11\mathbf{k})$ o.e.	A1ft
		(3)
(a)		

(b)	$\overrightarrow{CA} = (2-p)\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} \qquad \overrightarrow{CB} = (3-p)\mathbf{i} - 5\mathbf{k}$	M1
	$(2-p)(3-p) - 30 = 0$	**d**M1
	$\Rightarrow 6 - 5p + p^2 - 30 = 0 \Rightarrow p^2 - 5p - 24 = 0 \Rightarrow p = \ldots$	**dd**M1
	$p = -3, \quad p = 8$	A1
		(4)
(c)	$\lvert\overrightarrow{CA}\rvert = \sqrt{(2-8)^2 + (-5)^2 + 6^2}$ or $\lvert\overrightarrow{CB}\rvert = \sqrt{(3-8)^2 + (-5)^2}$	M1
	$\lvert\overrightarrow{CA}\rvert = \sqrt{97}$ or $\lvert\overrightarrow{CB}\rvert = \sqrt{50}$	A1
	Area of triangle $= \frac{1}{2} \times \lvert\overrightarrow{CA}\rvert \times \lvert\overrightarrow{CB}\rvert = \frac{1}{2} \times \sqrt{50} \times \sqrt{97} =$ awrt 34.8 units^2 or e.g. $\frac{1}{2} \times \lvert\overrightarrow{CA}\rvert \times \lvert\overrightarrow{AB}\rvert \sin BAC = \frac{1}{2} \times \sqrt{50} \times \sqrt{147} \sin\left(\cos^{-1}\frac{\sqrt{50}}{\sqrt{147}}\right) =$ awrt 34.8 units^2	**d**M1A1
		(4)
		(11 marks)

Question Number	Scheme	Marks
8 (a)	$\begin{pmatrix}4\\-3\\2\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\-1\end{pmatrix}=\begin{pmatrix}2\\0\\-9\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-3\end{pmatrix}\Rightarrow \begin{matrix}4+3\lambda=2+2\mu\\-3-2\lambda=0-1\mu\\2-1\lambda=-9-3\mu\end{matrix}$ **any two** of these	M1
	Full method to find either λ **or** μ eg (1) +3(3) $\Rightarrow \mu=...$	M1
	Either $\lambda=-4$ or $\mu=-5$	A1
	Position vector of intersection is $\begin{pmatrix}4\\-3\\2\end{pmatrix}-4\begin{pmatrix}3\\-2\\-1\end{pmatrix}=$ OR $\begin{pmatrix}2\\0\\-9\end{pmatrix}-5\begin{pmatrix}2\\-1\\-3\end{pmatrix}=$	dM1
	$=\begin{pmatrix}-8\\5\\6\end{pmatrix}$	A1
		(5)
(b)	Co-ordinates or position vector of point $Q=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}$	
	$\overrightarrow{PQ}=\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}-\begin{pmatrix}10\\-7\\0\end{pmatrix}=\begin{pmatrix}2\mu-8\\7-1\mu\\-9-3\mu\end{pmatrix}$	M1
	Uses $\overrightarrow{PQ}.\begin{pmatrix}2\\-1\\-3\end{pmatrix}=0\Rightarrow 4\mu-16-7+\mu+27+9\mu=0\Rightarrow \mu=-\frac{2}{7}$	dM1 A1
	Substitutes their μ into $\begin{pmatrix}2+2\mu\\0-1\mu\\-9-3\mu\end{pmatrix}\Rightarrow Q=\left(\frac{10}{7},\frac{2}{7},-\frac{57}{7}\right)$	ddM1 A1
		(5)
		(10 marks)

Question Number	Scheme	Marks
7 (a)	Co-ordinates or position vector of a point on l = $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix}$	
	$\overrightarrow{AX} = \begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-4\lambda \\ 5-3\lambda \\ -5+5\lambda \end{pmatrix}$	M1
	Uses $\overrightarrow{AX}.\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow\ 20+16\lambda-15+9\lambda-25+25\lambda=0 \Rightarrow \lambda=\frac{2}{5}$	dM1 A1
	(i) Substitutes their λ into $\begin{pmatrix} 4-4\lambda \\ 2-3\lambda \\ -3+5\lambda \end{pmatrix} \Rightarrow X = \left(\frac{12}{5}, \frac{4}{5}, -1\right)$	dM1 A1
	(ii) $\overrightarrow{AX} = -\frac{33}{5}\mathbf{i} + \frac{19}{5}\mathbf{j} - 3\mathbf{k}$	
	Shortest distance $= \sqrt{\left(-\frac{33}{5}\right)^2 + \left(\frac{19}{5}\right)^2 + (-3)^2} = \sqrt{67}$	M1 A1
		(7)
(b)	Uses $\overrightarrow{AX} = \overrightarrow{XB}$ or similar correct method	M1
	point B has position vector $-\frac{21}{5}\mathbf{i} + \frac{23}{5}\mathbf{j} - 4\mathbf{k}$	A1
		(2)
		(9 marks)

8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = ...$ either way around	M1
	$\overline{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overline{AC}.\overline{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overline{AC}.\overline{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = ...$ $\left(NB \quad \sqrt{172} = 2\sqrt{43}, \ \sqrt{189} = 3\sqrt{21} \right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overline{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2, \ BC^2 = 6^2 + 4^2 + 3^2, \ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC \cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172} \cos\theta \Rightarrow \cos\theta = ...$	ddM1
	$\theta =$ awrt $33.7°$	A1
(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)