| Question Number | Scheme | Marks |
|---|---|---|
| **9 (a)** | $\frac{dy}{dt}=\frac{-1}{(t+1)^2}$, $\frac{dx}{dt}=\frac{2}{2t+5}$ $\Rightarrow\left(\frac{dy}{dx}=\right)-\frac{2t+5}{2(t+1)^2}$ | M1A1 |
| | Sets $-\frac{2t+5}{2(t+1)^2}=-4\Rightarrow 8t^2+14t+3=0$ | M1A1 |
| | $\Rightarrow(4t+1)(2t+3)=0\Rightarrow t=-\frac{1}{4}\Rightarrow(y=)\frac{4}{3}$ | dM1A1 |
| | | **(6)** |
| **(b) (i)** | $a=2, b=5$ | B1 |
| | Area $=\int y\frac{dx}{dt}\,dt=\int_2^5\frac{1}{t+1}\times\frac{2}{2t+5}\,dt=\int_2^5\frac{2}{(t+1)(2t+5)}\,dt$ | M1A1 |
| **(ii)** | $\frac{2}{(t+1)(2t+5)}\equiv\frac{A}{(t+1)}+\frac{B}{(2t+5)}\Rightarrow A=\ldots, B=\ldots$ | M1 |
| | $=\frac{2/3}{(t+1)}-\frac{4/3}{(2t+5)}$ | A1 |
| | $\int\frac{2/3}{(t+1)}-\frac{4/3}{(2t+5)}\,dt=\frac{2}{3}\ln\lvert t+1\rvert-\frac{2}{3}\ln\lvert 2t+5\rvert$ | M1A1ft |
| | $\int_2^5\frac{2}{(t+1)(2t+5)}\,dt=\left(\frac{2}{3}\ln 6-\frac{2}{3}\ln 15\right)-\left(\frac{2}{3}\ln 3-\frac{2}{3}\ln 9\right)$ | dM1A1 |
| | $=\frac{2}{3}\ln\left(\frac{6}{5}\right)=\frac{1}{3}\ln\left(\frac{36}{25}\right)$ | |
| | | **(9)** **(15 marks)** |

| Question | Scheme | Marks |
|---|---|---|
| 3 | $8x^3-3y^2+2xy=9$ | |
| | **Either** $3y^2\to ky\dfrac{dy}{dx}$ **or** $2xy\to \alpha y+\beta x\dfrac{dy}{dx}$ | **M1** |
| | **Both** $3y^2\to ky\dfrac{dy}{dx}$ **and** $2xy\to \alpha y+\beta x\dfrac{dy}{dx}$ | **dM1** |
| | $8x^3-3y^2+2xy=9\to 24x^2-6y\dfrac{dy}{dx}+2y+2x\dfrac{dy}{dx}=0$ | **A1** |
| | $(2,5):96-30\dfrac{dy}{dx}+10+4\dfrac{dy}{dx}=0\Rightarrow \dfrac{dy}{dx}=...\left(=\dfrac{53}{13}\right)$ | **M1** |
| | $\dfrac{dy}{dx}=\dfrac{53}{13}$ or $\dfrac{dx}{dy}=\dfrac{13}{53}$ o.e. | **A1** |
| | Normal: $y-5="-\dfrac{13}{53}"\left(x-2\right)$ | **M1** |
| | $13x+53y-291=0$ | **A1** |
| | | **(7)** |
| | | **(7 marks)** |

| Question | Scheme | Marks |
|---|---|---|
| 1 | $\int x\cos 3x\,dx=\dfrac{1}{3}x\sin 3x-\dfrac{1}{3}\int \sin 3x\,dx$ | **M1** |
| | $=\dfrac{1}{3}x\sin 3x+\dfrac{1}{9}\cos 3x$ | **dM1 A1** |
| | $\displaystyle\int_0^{\frac{\pi}{6}} x\cos 3x\,dx=\left[\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}=\frac{1}{3}\frac{\pi}{6}\sin\frac{\pi}{2}+\frac{1}{9}\cos\frac{\pi}{2}-\left(0+\frac{1}{9}\right)$ | **M1** |
| | $=\dfrac{\pi}{18}-\dfrac{1}{9}$ | **A1** |
| | | **(5)** |
| | | **(5 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| 2(a) | States or uses $S=6x^2$ **or** $\dfrac{dS}{dt}=4$ | B1 |
| | States or uses $S=6x^2$ **and** $\dfrac{dS}{dt}=4$ | B1 |
| | Attempts to use $\dfrac{dS}{dt}=\dfrac{dS}{dx}\times\dfrac{dx}{dt}\Rightarrow 4=12x\times\dfrac{dx}{dt}\Rightarrow\dfrac{dx}{dt}=...$ | M1 |
| | $\Rightarrow\dfrac{dx}{dt}=\dfrac{1}{3x}$ oe | A1 |
| | | **(4)** |

| | | |
|---|---|---|
| **(b)** | States or uses $\frac{dV}{dx} = 3x^2$ | B1 |
| | Attempts to use $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} \Rightarrow \frac{dV}{dt} = 3x^2 \times \frac{1}{3x} = x$ | M1 |
| | $\Rightarrow \frac{dV}{dt} = V^{\frac{1}{3}}$ | A1 |
| | | **(3)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)** | $\frac{dV}{dr} = 4\pi r^2$ | **B1** **(1)** |
| **(b) (i)** | $\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c$ (oe) | **M1 A1** |
| | $t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = ...$ | **M1** |
| | $V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3}$ * | **A1 *** |
| **(ii)** | $150\ \text{cm}^3$ | **B1** |
| | | **(5)** |
| **(c)** | $t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$ | **M1** |
| | $100 = \frac{4}{3}\pi r^3 \Rightarrow r = 2.88\ \text{cm}$ | **dM1 A1cao** **(3)** |
| **(d)** | $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$ | **M1** |
| | $t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = ...$ | **dM1** |
| | $\Rightarrow \frac{dr}{dt} = \text{awrt } 0.11\ \text{cm s}^{-1}$ | **A1** **(3)** |
| | | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **4(a)** | e.g. $5 + 17x - 10x^2 = A(2x+1)(1-x) + Bx(2x+1) + Cx(1-x)$<br>$\Rightarrow A = ...,\ B = ....,\ C = ....$ | M1 |
| | $A = 5,\ B = 4,\ C = 8 \Rightarrow f(x) = \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}$ | A1A1 |
| | | **(3)** |
| **(b)** | $\int \frac{5}{x} + \frac{4}{(1-x)} + \frac{8}{(2x+1)}\ dx = 5\ln\lvert x\rvert - 4\ln\lvert 1-x\rvert + 4\ln\lvert 2x+1\rvert\ (+c)$ | M1A1ftA1 |
| | $\left[5\ln\lvert x\rvert - 4\ln\lvert 1-x\rvert + 4\ln\lvert 2x+1\rvert\right]_2^4$<br>$= (5\ln 4 - 4\ln 3 + 4\ln 9) - (5\ln 2 - 4\ln 1 + 4\ln 5) = 5\ln 2 + 4\ln\left(\frac{3}{5}\right)$ | dM1A1 |
| | | **(5)** |
| | | **(8 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **9(a)(i)** | $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{3\sin^2 t\cos t}{-4\sin 2t}$ | M1A1 |
| | $\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{3\sin^2 t\cos t}{-8\sin t\cos t}=-\dfrac{3}{8}\sin t$<br>or e.g.<br>$\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{\frac{3}{2}\sin 2t\sin t}{-4\sin 2t}=-\dfrac{3}{8}\sin t$ | A1 |
| **(ii)** | $t=\dfrac{\pi}{6}\Rightarrow x=1,\quad y=\dfrac{1}{8}$ | B1 |
| | $m=-\dfrac{3}{8}\sin\left(\dfrac{\pi}{6}\right)=-\dfrac{3}{16}\Rightarrow y-\dfrac{1}{8}=-\dfrac{3}{16}(x-1)$ | M1 |
| | $3x+16y-5=0$ * | A1* |
| | | **(6)** |
| **(b)** | $3(2\cos 2t)+16(\sin^3 t)-5=0$ | M1 |
| | $6(1-2\sin^2 t)+16\sin^3 t-5=0$ | dM1 |
| | $16\sin^3 t-12\sin^2 t+1=0$ | A1 |
| | $(2\sin t-1)^2(4\sin t+1)=0\Rightarrow\sin t=-\dfrac{1}{4}$ | ddM1 |
| | $\sin t=-\dfrac{1}{4}\Rightarrow x=2\cos 2t=..$ **and** $y=\sin^3 t=...$ | dddM1 |
| | $Q\left(\dfrac{7}{4},-\dfrac{1}{64}\right)$ | A1 |
| | | **(6)** |
| | | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **3(a)** | $\dfrac{8x-5}{(2x-1)(4x-3)} \equiv \dfrac{A}{2x-1}+\dfrac{B}{4x-3} \Rightarrow 8x-5=A(4x-3)+B(2x-1) \Rightarrow A=\ldots\ B=\ldots$ | M1 |
| | $A=1$ or $B=2$ | A1 |
| | $\dfrac{8x-5}{(2x-1)(4x-3)} \equiv \dfrac{1}{2x-1}+\dfrac{2}{4x-3}$ | A1 |
| | | **(3)** |
| **(b)** | $\displaystyle\int\left(\frac{1}{2x-1}+\frac{2}{4x-3}\right)\mathrm{d}x=\frac{1}{2}\ln\lvert 2x-1\rvert+\frac{1}{2}\ln\lvert 4x-3\rvert(+c)$<br>Follow through their $A$ and $B$ | M1, A1ft, A1ft |
| | | **(3)** |
| **(c)** | $\left[\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(4x-3)\right]_k^{3k}=\frac{1}{2}\ln(6k-1)(12k-3)-\frac{1}{2}\ln(2k-1)(4k-3)$<br>$=\frac{1}{2}\ln\dfrac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$ | M1 |
| | $\frac{1}{2}\ln\dfrac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=\frac{1}{2}\ln 20 \Rightarrow \dfrac{(6k-1)(12k-3)}{(2k-1)(4k-3)}=20$<br>$\Rightarrow (6k-1)(12k-3)=20(2k-1)(4k-3) \Rightarrow 88k^2-170k+57=0$ | dM1A1 |
| | $88k^2-170k+57=0 \Rightarrow k=\ldots$ | ddM1 |
| | $k=\dfrac{3}{2}$ | A1 |
| | | **(5)** |
| | | **Total 11** |