Question	Scheme	Marks
9(a)	Volume, $V = \pi\int_{\frac{1}{3}}^{3} y^2\,\mathrm{d}x = \pi\int_{\frac{1}{3}}^{3} \dfrac{x^{-\frac{1}{2}}}{(1+x)\left(\arctan\left(\sqrt{x}\right)\right)^2}\,\mathrm{d}x$	B1
	$\tan u = \sqrt{x} \Rightarrow \sec^2 u\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{2}x^{-\frac{1}{2}}$ oe	M1 A1
	$\Rightarrow V = (\pi)\displaystyle\int \dfrac{2\times\frac{1}{2}x^{-\frac{1}{2}}}{\left(\arctan\left(\sqrt{x}\right)\right)^2(1+x)}\,\mathrm{d}x = (\pi)\int \dfrac{2}{u^2\left(1+\tan^2 u\right)}\sec^2 u\,\mathrm{d}u$	dM1
	$= \pi\displaystyle\int \dfrac{2}{u^2\cancel{\sec^2 u}}\cancel{\sec^2 u}\,\mathrm{d}u = 2\pi\int\dfrac{1}{u^2}\,\mathrm{d}u$	A1
	$\left.\begin{aligned} x=3 &\Rightarrow u = \arctan\sqrt{3} = \dfrac{\pi}{3}\\ x=\dfrac{1}{3} &\Rightarrow u = \arctan\dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}\end{aligned}\right\} \Rightarrow V = 2\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\dfrac{1}{u^2}\,\mathrm{d}u$	B1
		(6)
(b)	$V = (2\pi)\left[-\dfrac{1}{u}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1
	$= (2\pi)\left[-\dfrac{1}{\pi/3} - \left(-\dfrac{1}{\pi/6}\right)\right]$ Or $= (2\pi)\left[-\dfrac{1}{\arctan\sqrt{x}}\right]_{\frac{1}{3}}^{3} = (2\pi)\left(-\dfrac{1}{\arctan\sqrt{3}} - \left(-\dfrac{1}{\arctan\sqrt{1/3}}\right)\right)$	dM1
	$= 2\pi\left(-\dfrac{3}{\pi}+\dfrac{6}{\pi}\right) = 6$	A1
		(3)
		(9 marks)

Question	Scheme	Marks
4.	$\int_1^4 \frac{10}{5x+2x\sqrt{x}}\,dx$	
	$u=\sqrt{x} \Rightarrow x=u^2 \Rightarrow \frac{dx}{du}=2u$	B1
	$\int \frac{10}{5x+2x\sqrt{x}}\,dx = \int \frac{10}{5u^2+2u^3}\,2u\,du$	M1 A1
	$= \int \frac{20}{5u+2u^2}\,du = \int \frac{4}{u} - \frac{8}{5+2u}\,du$	dM1 A1
	$= 4\ln u - 4\ln(5+2u)$	ddM1
	$\text{Limits} = \left[4\ln u - 4\ln(5+2u)\right]_1^2 = 4\ln 2 - 4\ln 9 + 4\ln 7 = \ldots$	M1
	$= 4\ln\left(\frac{14}{9}\right)$	A1 **(8 marks)**

Question Number	Scheme	Notes	Marks
6 Way 1	$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx\,(+c)$ M1: For applying parts to obtain $\alpha e^{2x}\sin 3x \pm \beta\int e^{2x}\sin 3x\,dx\,(+c)$ A1: Correct expression		M1A1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left\{-\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx\right\}(+c)$ Applies parts again to $\int e^{2x}\sin 3x\,dx$ and obtains $\alpha e^{2x}\cos 3x \pm \beta\int e^{2x}\cos 3x\,dx$		M1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx\,(+c)$ Fully correct application of parts twice		A1
	$\int e^{2x}\cos 3x\,dx + \frac{4}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\,(+c)$ $\Rightarrow \frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\,(+c) \Rightarrow \int e^{2x}\cos 3x\,dx = \ldots$ Fully correct strategy for finding $\int e^{2x}\cos 3x\,dx$		M1
	$= \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + k$	Cao	A1
			(6)
Way 2	$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\int e^{2x}\sin 3x\,dx\,(+c)$ M1: For applying parts to obtain $\alpha e^{2x}\cos 3x \pm \beta\int e^{2x}\sin 3x\,dx\,(+c)$ A1: Correct expression		M1A1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{2}\left\{\frac{1}{2}e^{2x}\sin 3x - \frac{3}{2}\int e^{2x}\cos 3x\,dx\right\}(+c)$ Applies parts again to $\int e^{2x}\sin 3x\,dx$ and obtains $\alpha e^{2x}\sin 3x \pm \beta\int e^{2x}\cos 3x\,dx$		M1
	$\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3x\,dx\,(+c)$ Fully correct application of parts twice		A1
	$\int e^{2x}\cos 3x\,dx + \frac{9}{4}\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x\,(+c)$ $\Rightarrow \frac{13}{4}\int e^{2x}\cos 3x\,dx = \frac{1}{2}e^{2x}\cos 3x + \frac{3}{4}e^{2x}\sin 3x\,(+c) \Rightarrow \int e^{2x}\cos 3x\,dx = \ldots$ Fully correct strategy for finding $\int e^{2x}\cos 3x\,dx$		M1
	$= \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + k$	Cao	A1
			Total 6

Question Number	Scheme	Marks
2(a)	$2^x \to 2^x \ln 2$	B1
	$y^2 \to \ldots y\frac{dy}{dx}$	M1
	$4x^2y \to \ldots x^2\frac{dy}{dx} + \ldots xy$	M1
	$3 + 10y\frac{dy}{dx} + 4x^2\frac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$\left(10y + 4x^2\right)\frac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \frac{dy}{dx} = \ldots$	M1
	$\frac{dy}{dx} = \frac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

2(b)	At $x = 0 \quad 5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10 \times 2^0 \ln 2 - 3 - 8 \times 0 \times 3}{10 \times 3 + 4 \times 0^2} = \frac{10 \ln 2 - 3}{30}$	M1 A1
		(2)

Question Number	Scheme	Marks
4(i)	$\frac{dV}{dt} = 70\pi$	B1
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 70\pi = 4\pi \times 5^2 \times \frac{dr}{dt}$ or e.g. $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \Rightarrow \frac{dr}{dt} = 70\pi \times \frac{1}{4\pi \times 5^2}$	M1
	$\left(\frac{dr}{dt}\right) = 0.7 \text{ (cm s}^{-1}\text{)}$ oe e.g. $\frac{7}{10}$	A1
		(4)

(ii)	$\frac{dh}{dt} = \frac{k}{h^3} \Rightarrow \int h^3 \, dh = \int k \, dt \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0, \ h = 4 \Rightarrow \frac{1}{4} \times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5, \ h = 6 \Rightarrow \frac{1}{4} \times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = \ldots$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
6	$u=\sqrt{x^3+1} \Rightarrow u^2=x^3+1 \Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=3x^2$ or $u=\sqrt{x^3+1} \Rightarrow u^2=x^3+1 \Rightarrow 2u=3x^2\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or $u=\sqrt{x^3+1} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}\times 3x^2$ or $x=(u^2-1)^{\frac{1}{3}} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int \frac{9x^5}{u}\frac{2u}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$ or e.g. $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int \frac{9x^5}{\sqrt{x^3+1}}\frac{2\sqrt{x^3+1}}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$	M1A1
	$6\displaystyle\int\left(u^2-1\right)\mathrm{d}u=2u^3-6u\ (+c)=2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}\ (+c)$	M1
	$=2\left(x^3+1\right)^{\frac{1}{2}}\left[\left(x^3+1\right)-3\right]+c=2\left(x^3+1\right)^{\frac{1}{2}}\left(x^3-2\right)+c$	A1
		(5)
		Total 5

Question Number	Scheme	Marks
9 (a)	$\dfrac{\mathrm{d}A}{\mathrm{d}t}=\dfrac{A^{\frac{3}{2}}}{5t^2}\Rightarrow\displaystyle\int\frac{\mathrm{d}A}{A^{\frac{3}{2}}}=\int\frac{\mathrm{d}t}{5t^2}$ oe	M1
	$-2A^{-\frac{1}{2}}=-\dfrac{1}{5}t^{-1}(+c)$	M1 M1 A1
	Substitutes $t=3, A=2.25 \Rightarrow c=\ldots\left(-\dfrac{19}{15}\right)$	M1
	Uses their $\dfrac{2}{\sqrt{A}}=\dfrac{1}{5t}+\dfrac{19}{15}\Rightarrow A=\left(\dfrac{30t}{19t+3}\right)^2$	M1, A1
		(7)
(b)	As $t\to\infty, A\to\left(\dfrac{30}{19}\right)^2=\dfrac{900}{361}$ or awrt 2.49 cm^2	M1 A1
		(2)
		(9 marks)

Question Number	Scheme	Marks
7(a)	Substitutes $x = 3$ and $t = 0$ into $x = \dfrac{k(9t+5)}{4t+3}$	M1
	$\Rightarrow 3 = \dfrac{5k}{3} \Rightarrow k = 1.8$ *	A1*
		(2)
(b)	4050	B1
		(1)
(c)	$\dfrac{3}{x(9-2x)} \equiv \dfrac{A}{x} + \dfrac{B}{9-2x}$	M1
	Either $A = \dfrac{1}{3}$ or $B = \dfrac{2}{3}$	A1
	$\dfrac{3}{9x-2x^2} \equiv \dfrac{1}{3x} + \dfrac{2}{3(9-2x)}$	A1
		(3)
(d)	$3\dfrac{dx}{dt} = x(9-2x) \Rightarrow \displaystyle\int \frac{3}{x(9-2x)}\,dx = \int dt$ **Or** $3\dfrac{dx}{dt} = x(9-2x) \Rightarrow \displaystyle\int \frac{1}{x(9-2x)}\,dx = \int \frac{1}{3}\,dt$ $\Rightarrow \dfrac{1}{3}\ln x - \dfrac{1}{3}\ln(9-2x) = t(+c)$ **or** $\dfrac{1}{9}\ln x - \dfrac{1}{9}\ln(9-2x) = \dfrac{1}{3}t(+c)$	M1 A1ft
	Substitutes $t = 0, x = 3 \Rightarrow c = 0$	M1
	$\dfrac{1}{3}\ln x - \dfrac{1}{3}\ln(9-2x) = t \Rightarrow \left(\dfrac{x}{9-2x}\right) = e^{3t}$	**dd**M1
	$\left(\dfrac{9-2x}{x}\right) = e^{-3t} \Rightarrow \dfrac{9}{x} - 2 = e^{-3t} \Rightarrow x = \dfrac{9}{2+e^{-3t}}$ *	A1*
		(5)
(e)	4500	B1
		(1)
		(12 marks)

Question Number	Scheme	Marks
6.(a)	$x=\frac{3}{2}\Rightarrow a=\frac{\pi}{6}$	B1
	$\int y^2\,\mathrm{d}x=\int y^2\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t=\int(2\sin 2t)^2\,3\cos t\,(\mathrm{d}t)$	M1
	Uses $\sin 2t=2\sin t\cos t\ \Rightarrow\int(4\sin t\cos t)^2\,3\cos t\,\mathrm{d}t$	dM1
	Volume $=\int\pi y^2\mathrm{d}x=48\pi\int_0^{\left(\frac{\pi}{6}\right)}\sin^2 t\cos^3 t\,\mathrm{d}t$	A1
		(4)
(b)	$u=\sin t\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}t}=\cos t$	B1
	$k\int\sin^2 t\cos^3 t\,\mathrm{d}t=k\int u^2\cos^2 t\,\mathrm{d}u$	
	$=k\int u^2(1-\sin^2 t)\,\mathrm{d}u=k\int u^2(1-u^2)\,\mathrm{d}u$	M1
	$=k\left[\frac{u^3}{3}-\frac{u^5}{5}\right]$	dM1
	Volume $=48\pi\left[\frac{u^3}{3}-\frac{u^5}{5}\right]_0^{\frac{1}{2}}=\frac{17\pi}{10}$	ddM1A1
		(5)
		(9 marks)

Question Number	Scheme	Marks
7. (a)	Attempts $\overrightarrow{BA}\square\overrightarrow{BC}$ $\begin{pmatrix}-6\\2\\-3\end{pmatrix}\bullet\begin{pmatrix}-4\\3\\-5\end{pmatrix}=24+6+15=45$	M1
	Attempts to apply $\mathbf{a}\bullet\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta\Rightarrow 45=7\times\sqrt{50}\cos\theta$	dM1
	$\cos\theta=\dfrac{9\sqrt{2}}{14}$ oe	A1
		(3)
(b)	Attempts $\sin\theta=\sqrt{1-\left(\dfrac{9\sqrt{2}}{14}\right)^2}$	M1
	Attempts to apply $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta=7\times\sqrt{50}\times\dfrac{\sqrt{34}}{14}$	M1
	Area $ABCD=5\sqrt{17}$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
3	$8y\dfrac{dy}{dx}=xe^{-y^2}\rightarrow\int 8ye^{y^2}dy=\int x\ dx$	B1
	$4e^{y^2}=\dfrac{x^2}{2}\ (+c)$	M1A1
	$4=\dfrac{5^2}{2}+c\Rightarrow c=...$	**d**M1
	$y^2=\ln\left(\dfrac{x^2-17}{8}\right)$	A1
		(5 marks)

B1: Separates the variables correctly with some indication of integration