Question Number	Scheme	Marks
7 (a)	$\frac{dV}{dr} = 4\pi r^2$	**B1** **(1)**
(b) (i)	$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c$ (oe)	**M1 A1**
	$t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = ...$	**M1**
	$V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3}$ *	**A1 ***
(ii)	$150\ \text{cm}^3$	**B1**
		(5)
(c)	$t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$	**M1**
	$100 = \frac{4}{3}\pi r^3 \Rightarrow r = 2.88\ \text{cm}$	**dM1 A1cao** **(3)**
(d)	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$	**M1**
	$t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = ...$	**dM1**
	$\Rightarrow \frac{dr}{dt} = \text{awrt } 0.11\ \text{cm s}^{-1}$	**A1** **(3)**
		(12 marks)

Question	Scheme	Marks
3	$8x^3 - 3y^2 + 2xy = 9$	
	Either $3y^2 \to ky\frac{dy}{dx}$ **or** $2xy \to \alpha y + \beta x\frac{dy}{dx}$	**M1**
	Both $3y^2 \to ky\frac{dy}{dx}$ **and** $2xy \to \alpha y + \beta x\frac{dy}{dx}$	**dM1**
	$8x^3 - 3y^2 + 2xy = 9 \to 24x^2 - 6y\frac{dy}{dx} + 2y + 2x\frac{dy}{dx} = 0$	**A1**
	$(2, 5): 96 - 30\frac{dy}{dx} + 10 + 4\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = ...\left(= \frac{53}{13}\right)$	**M1**
	$\frac{dy}{dx} = \frac{53}{13}$ or $\frac{dx}{dy} = \frac{13}{53}$ o.e.	**A1**
	Normal: $y - 5 = "-\frac{13}{53}"(x - 2)$	**M1**
	$13x + 53y - 291 = 0$	**A1**
		(7)
		(7 marks)

Question Number	Scheme	Marks
4 (a)	Attempt at chain rule $y^2 \to \ldots y\frac{dy}{dx}$	M1
	Attempt at product rule $2xy \to \alpha x\frac{dy}{dx} + \beta y$	M1
	Correct differentiation $8x + 2y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 24$ oe	A1
	Collects terms in $(2y - 2x)\frac{dy}{dx} = 24 - 8x + 2y$	M1
	$\left(\frac{dy}{dx} =\right) \frac{12 - 4x + y}{y - x}$ oe	A1
		(5)
(b)	Sets $\frac{12 - 4x + y}{y - x} = 2 \Rightarrow y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$	M1
	Substitute $y = 12 - 2x$ or $x = 6 - \frac{1}{2}y$ into $4x^2 + y^2 - 2xy = 24x$	dM1
	$x^2 - 8x + 12 = 0$ oe or $y^2 - 8y = 0$	A1
	Solves $x = 6, y = 0$ or $x = 2, y = 8$	ddM1
	Chooses $a(x) = 2, b(y) = 8$ or states $(2, 8)$ only	A1
		(5) (10 marks)

Question	Scheme	Marks
1(a)	$A = \frac{1}{9}$	B1
		(1)
(b)	$"3^{-2}"\left(1 + \underline{(-2)\left(\frac{kx}{3}\right)} + \underline{\frac{(-2)(-3)}{2}\left(\frac{kx}{3}\right)^2} + \ldots\right)$ or x: $3^{-2}(-2)\left(\frac{k}{3}\right)\left(= -\frac{2k}{27}\right)$ and x^2: $3^{-2}\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2 = \frac{k^2}{27}$	B1
	$\frac{(-2)(-3)}{2}\left(\frac{k}{3}\right)^2 = 3 \times (-2)\left(\frac{k}{3}\right) \Rightarrow \ldots k^2 = \ldots k$	M1
	$k^2 + 6k = 0$ *	A1*
		(3)
(c)(i)	$k = -6$	B1
(ii)	$3^{-2}\frac{(-2)(-3)(-4)}{3!}\left(\frac{"-6"}{3}\right)^3 = \frac{32}{9}$	M1A1
		(3)
		(7 marks)

Question	Scheme	Marks
4(a)	$A=\frac{1}{2}5^2\left(\theta-\sin\theta\right)\Rightarrow\frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{25}{2}\left(1\pm\cos\theta\right)$	**M1**
	$\frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{25}{2}\left(1-\cos\theta\right)$	**A1**
		(2)
(b)	$\frac{\mathrm{d}\theta}{\mathrm{d}t}=0.1$	**B1**
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{d}t}$ (oe)	**B1**
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{25}{2}\left(1-\cos\frac{\pi}{3}\right)\times0.1$	**M1**
	$=\frac{5}{8}$ (cm^2 per second)	**A1**
		(4)
		(6 marks)

Question Number	Scheme	Marks
2(a)	$\frac{\mathrm{d}V}{\mathrm{d}x}=3x^2$	M1 (B1 on EPEN)
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{5}{3x^2}$	**d**M1A1
		(3)
	Alternative 1	
	$V=x^3\Rightarrow x=V^{\frac{1}{3}}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}V}=\frac{1}{3}V^{-\frac{2}{3}}$	M1 (B1 on EPEN)
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{5}{3V^{\frac{2}{3}}}=\frac{5}{3x^2}$	**d**M1A1
	Alternative 2	
	$\frac{\mathrm{d}V}{\mathrm{d}t}=5\Rightarrow V=5t+c$	M1 (B1 on EPEN)
	$V=x^3\Rightarrow x^3=5t+c\Rightarrow3x^2\frac{\mathrm{d}x}{\mathrm{d}t}=5$ or $V=x^3\Rightarrow x^3=5t+c\Rightarrow3x^2=5\frac{\mathrm{d}t}{\mathrm{d}x}$	**d**M1
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5}{3x^2}$	A1
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{5}{3\times4^2}=\frac{5}{48}$	M1A1
		(2)
		(5 marks)
Note this is now being marked as M1dM1A1 not B1M1A1		

Question Number	Scheme	Marks
9(a)	$(V)=\pi\int_0^4\left(\cos x+\frac{1}{5}e^x\right)^2\,dx$	B1
	$\left(\cos x+\frac{1}{5}e^x\right)^2=\cos^2 x+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}$	M1
	$=\frac{\cos 2x+1}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}$	dM1
	$=\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$	A1
		(4)
(b)	**Way I** $\int e^x\cos x\,dx=\pm e^x\sin x\pm\int e^x\sin x\,dx$ $=e^x\sin x-\int e^x\sin x\,dx$ **Way Two** $\int e^x\cos x\,dx=e^x\cos x\pm\int e^x\sin x\,dx$ $=e^x\cos x+\int e^x\sin x\,dx$	M1 A1
	$\int e^x\cos x\,dx=e^x\cos x\pm e^x\sin x-\int e^x\cos x\,dx$ $2\int e^x\cos x\,dx=e^x\cos x\pm e^x\sin x$	dM1
	$\int e^x\cos x\,dx=\frac{1}{2}e^x(\cos x+\sin x)+(c)$	A1
		(4)
(c)	$\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$ $=(\pi)\left[\frac{1}{2}x+\frac{\sin 2x}{4}+\frac{2}{5}\left(\frac{1}{2}e^x(\sin x+\cos x)\right)+\frac{1}{50}e^{2x}\right]_0^4$ $2\pi\left(2+\frac{\sin 8}{4}+\frac{2}{5}\left(\frac{1}{2}e^4(\sin(4)+\cos(4))\right)+\frac{1}{50}e^8-\frac{11}{50}\right)$ $=\text{awrt } 290\,(\text{cm}^3)$	M1A1ft dM1 A1
		(4)
		(12 marks)

(a)

Question Number	Scheme	Marks
3	Differentiates wrt x $\quad 3^x\ln 3+6\underline{\underline{\frac{dy}{dx}}}=\underline{\frac{3}{2}y^2}+\underline{3xy\frac{dy}{dx}}$	B1 B1, M1, A1
	Substitutes $(2, 3)$ **AND** rearranges to get $\frac{dy}{dx}$ or vice versa $\Rightarrow 9\ln 3+6\frac{dy}{dx}=\frac{27}{2}+18\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{9\ln 3-\frac{27}{2}}{12}=\frac{6\ln 3-9}{8}=\frac{-9+\ln 729}{8}$	M1 A1, A1 **(7)**
		(7 marks)

Question Number	Scheme	Marks
4(a)	$\sqrt{1-4x^2} = 1-\frac{1}{2}\times 4x^2$	B1
	$\frac{\frac{1}{2}\times-\frac{1}{2}\times\left(-4x^2\right)^2}{2}$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}\times\left(-4x^2\right)^3}{3!}$	M1
	$=1-2x^2-2x^4-4x^6+\ldots$	A1, A1 **(4)**
(b)	Substitutes $x=\frac{1}{4}$ into both sides of (a) e.g. $\sqrt{\frac{3}{4}} \approx 1-2\times\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)^4-4\left(\frac{1}{4}\right)^6$	M1
	$\sqrt{3} \approx 1.7324$ cao	A1 **(2)** **(6 marks)**

Question Number	Scheme	Marks
1	Attempt at chain rule $y^2 \to \ldots y\frac{dy}{dx}$	M1
	Attempt at product rule $3x^2y \to \ldots x^2\frac{dy}{dx}+\ldots xy$	M1
	Correct differentiation $2-8y\frac{dy}{dx}+6xy+3x^2\frac{dy}{dx}=8x$	A1
	Substitutes $x=3, y=2 \Rightarrow \frac{dy}{dx}=-\frac{14}{11}$	M1, A1
	Correct method for normal at $(3,2) \Rightarrow y-2=\frac{11}{14}(x-3)$	dM1
	$11x-14y-5=0$	A1 **(7)** **(7 marks)**

Question Number	Scheme	Marks
6	$u=\sqrt{x^3+1} \Rightarrow u^2=x^3+1 \Rightarrow 2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=3x^2$ or $u=\sqrt{x^3+1} \Rightarrow u^2=x^3+1 \Rightarrow 2u=3x^2\dfrac{\mathrm{d}x}{\mathrm{d}u}$ or $u=\sqrt{x^3+1} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2}\left(x^3+1\right)^{-\frac{1}{2}}\times 3x^2$ or $x=(u^2-1)^{\frac{1}{3}} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=\dfrac{2}{3}u(u^2-1)^{-\frac{2}{3}}$	B1
	e.g. $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int \frac{9x^5}{u}\frac{2u}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$ or e.g. $\displaystyle\int \frac{9x^5}{\sqrt{x^3+1}}\mathrm{d}x=\int \frac{9x^5}{\sqrt{x^3+1}}\frac{2\sqrt{x^3+1}}{3x^2}\mathrm{d}u=\int 6x^3\,\mathrm{d}u=6\int\left(u^2-1\right)\mathrm{d}u$	M1A1
	$6\displaystyle\int\left(u^2-1\right)\mathrm{d}u=2u^3-6u\ (+c)=2\left(x^3+1\right)^{\frac{3}{2}}-6\left(x^3+1\right)^{\frac{1}{2}}\ (+c)$	M1
	$=2\left(x^3+1\right)^{\frac{1}{2}}\left[\left(x^3+1\right)-3\right]+c=2\left(x^3+1\right)^{\frac{1}{2}}\left(x^3-2\right)+c$	A1
		(5)
		Total 5