Question Number	Scheme	Marks
8	$\begin{pmatrix}-1\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\5\end{pmatrix}=\begin{pmatrix}2\\-2\\-5\end{pmatrix}+\mu\begin{pmatrix}4\\-3\\b\end{pmatrix}\Rightarrow\begin{array}{l}-1+2\lambda=2+4\mu\ (1)\\5-\lambda=-2-3\mu\ (2)\\4+5\lambda=-5+\mu b\ (3)\end{array}$	
	Uses equations (1) and (2) to find either λ **or** μ e.g. $(1)+2(2)\Rightarrow\mu=\ldots$ or $3(1)+4(2)\Rightarrow\lambda=\ldots$	M1
	Uses equations (1) and (2) to find both λ and μ	**d**M1
	$\mu=-\frac{11}{2}$ and $\lambda=-\frac{19}{2}$	A1
	$4+5\lambda=-5+\mu b\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}b$ or $4+5\lambda=-5+7\mu\Rightarrow 4+5\times-\frac{19}{2}=-5-\frac{11}{2}\times 7$	**dd**M1
	$\Rightarrow 11b=77\Rightarrow b=7$ or obtains $-\frac{87}{2}=-\frac{87}{2}$	A1
	States that when $b=7$, lines intersect or when $b\neq 7$, lines do not intersect Lines are not parallel so when $b\neq 7$ lines are skew. *	A1 **Cso**
		(6)

8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \ldots$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = \ldots$ $\left(NB \quad \sqrt{172} = 2\sqrt{43}, \ \sqrt{189} = 3\sqrt{21}\right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2, \ BC^2 = 6^2 + 4^2 + 3^2, \ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC \cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172} \cos\theta \Rightarrow \cos\theta = \ldots$	ddM1
	$\theta =$ awrt $33.7°$	A1

(d)	Attempts a correct method of finding one position for D. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for D. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

Question Number	Scheme	Marks
6(a)	Attempts $\pm\left(\left(5\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)-\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)\right)=\pm\left(4\mathbf{i}+4\mathbf{k}\right)$	M1
	e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe	A1
		(2)
(b)	$\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$	B1
	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{4^2+4^2}$ or $\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{2^2+(\alpha-2)^2+8^2}$	M1
	$\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{32}$ and $\left\lvert\overrightarrow{AC}\right\rvert=\sqrt{68+(\alpha-2)^2}$	A1
	$\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$	dM1
	$\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$	ddM1
	$\alpha=2\pm4\sqrt{2}$	A1
		(6)
		(8 marks)

Question Number	Scheme	Marks
9(a)	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$	
	Either $A: \lambda = 1$ or $B: \lambda = 3$	M1
	leading to $a = -3$, $b = 5$	A1, A1
		(3)
(b)	$\overrightarrow{AB} = = \pm\left(('5\mathbf{i}' - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ '- 3\mathbf{j}' + 5\mathbf{k})\right); = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$	M1, A1
		(2)
(c)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$ or $\overrightarrow{CA} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$	M1
	$\cos C\hat{A}B = \dfrac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2}.\sqrt{(3)^2 + (0)^2 + (-3)^2}}$	dM1
	$\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24}.\sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$	A1
		(3)
(d)	Area $CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ; = 3\sqrt{3}$	M1, A1
		(2)
(e)	For one of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1
	For both of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ and	
	$\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	M1, A1
		(4)
		(14 marks)

Question	Scheme	Marks
6(a)	$\left\|\overrightarrow{OA}\right\|^2 = (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2$	M1
	$\left\|\overrightarrow{OA}\right\| = 5\sqrt{10} \Rightarrow (1+8\lambda)^2 + (2-\lambda)^2 + (5+4\lambda)^2 = 250$	M1
	$\Rightarrow 64\lambda^2 + 16\lambda + 1 + \lambda^2 - 4\lambda + 4 + 16\lambda^2 + 40\lambda + 25 = 250$ $\Rightarrow 81\lambda^2 + 52\lambda - 220 = 0^*$	A1*
		(3)
(b)	$81\lambda^2 + 52\lambda - 220 = 0 \Rightarrow (81\lambda - 110)(\lambda + 2) = 0 \Rightarrow \lambda = \ldots \left(-2, \frac{110}{81}\right)$ $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1+8\times"\text{their}\lambda" \\ 2-"\text{their}\lambda" \\ 5+4\times"\text{their}\lambda" \end{pmatrix}$	M1
	E.g $\Rightarrow \overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} - 2\begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}^*$; or $\overrightarrow{OA} = \begin{pmatrix} 1+8\times\frac{110}{81} \\ 2-\frac{110}{81} \\ 5+4\times\frac{110}{81} \end{pmatrix} = \begin{pmatrix} \frac{961}{81} \\ \frac{52}{81} \\ \frac{845}{81} \end{pmatrix}$	A1*; A1
		(3)
(c)	$\cos\theta = \pm \frac{\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix} . \begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{15^2+4^2+3^2}\sqrt{8^2+1^2+4^2}} = \ldots$	M1
	$= \pm\frac{-136}{45\sqrt{10}}$ $\left(= \pm 0.9557\ldots\right)$ or e.g. $\theta =$ awrt $0.299\,(rad)$ / $17.1°$	A1
	Area $OAB = \frac{1}{2} 5\sqrt{10} \times 4\sqrt{10} \sin\theta = \ldots$	M1
	= awrt 29.4	A1
		(4)
		(10 marks)

Question Number	Scheme	Marks
9	$\begin{pmatrix} 2-\lambda \\ 8+2\lambda \\ 10+3\lambda \end{pmatrix} = \begin{pmatrix} -4+5\mu \\ -1+4\mu \\ 2+8\mu \end{pmatrix}$	
	Attempts to solve any two of the three equations	
	Either (1) and (2) $\left.\begin{matrix} 2-\lambda=-4+5\mu \\ 8+2\lambda=-1+4\mu \end{matrix}\right\} \Rightarrow \lambda=-\frac{3}{2}, \mu=\frac{3}{2}$	
	(1) and (3) $\left.\begin{matrix} 2-\lambda=-4+5\mu \\ 10+3\lambda=2+8\mu \end{matrix}\right\} \Rightarrow \lambda=\frac{8}{23}, \mu=\frac{26}{23}$	M1, A1
	(2) and (3) $\left.\begin{matrix} 8+2\lambda=-1+4\mu \\ 10+3\lambda=2+8\mu \end{matrix}\right\} \Rightarrow \lambda=-10, \mu=-\frac{11}{4}$	
	Substitutes their values of λ and μ into both sides of the "third" equation E.g. $\lambda=-\frac{3}{2}$ into $10+3\lambda=\frac{11}{2}$ and $\mu=\frac{3}{2}$ into $2+8\mu=14$	dM1
	Concludes that lines don't intersect with correct calculations and minimal reason	A1
	Additionally states that $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ is not parallel to $\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$ with a minimal reason So lines are skew CSO *	A1*
		(5) **(5 marks)**