| Question Number | Scheme | Marks |
|---|---|---|
| **8(a)** | $\pm\left(-6\mathbf{i}+4\mathbf{j}-\mathbf{k}-\left(-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\right)\right)$ | M1 |
| | e.g. $\mathbf{r}=-10\mathbf{i}+5\mathbf{j}-4\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$<br>e.g. $\mathbf{r}=-6\mathbf{i}+4\mathbf{j}-\mathbf{k}\pm\lambda\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)$ | A1 |
| | | **(2)** |
| **(b)** | $3+3\mu=-6\Rightarrow\mu=-3$ | M1 |
| | $p+12=4\Rightarrow p=\ldots$ or $q-3=-1\Rightarrow q=\ldots$ | dM1 |
| | $p=-8,\quad q=2$ | A1 |
| | | **(3)** |
| **(c)** | $\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)\square\left(3\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)=12+4+3$ | M1 |
| | $\left(4\mathbf{i}-\mathbf{j}+3\mathbf{k}\right)\square\left(3\mathbf{i}-4\mathbf{j}+\mathbf{k}\right)=\sqrt{26}\sqrt{26}\cos\theta=19\Rightarrow\cos\theta=\ldots$ | M1 |
| | $(\cos\theta=)\ \dfrac{19}{26}$ | A1 |
| | | **(3)** |
| **(d)** | $AC=AB\sin\theta=\sqrt{26}\sqrt{1-\left(\dfrac{19}{26}\right)^2}$ | M1 |
| | $\dfrac{3\sqrt{910}}{26}$ o.e. | A1 |
| | | **(2)** |
| | | **Total 10** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6 (a)(i)** | $\overrightarrow{AB}=(\pm)\left[\left(8\mathbf{i}+3\mathbf{j}-7\mathbf{k}\right)-\left(2\mathbf{i}-3\mathbf{j}+5\mathbf{k}\right)\right]=\ldots$ | **M1** |
| | $\overrightarrow{AB}=6\mathbf{i}+6\mathbf{j}-12\mathbf{k}$ | **A1** |
| **(ii)** | $\mathbf{r}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r}=\begin{pmatrix}8\\3\\-7\end{pmatrix}+\lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$ | **B1ft** |
| | | **(3)** |
| **(b)** | Attempts $\pm\overrightarrow{CP}=\pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$ | **M1** |
| | $\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow\begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix}=0\Rightarrow1\left(\lambda-1\right)+1\left(\lambda-8\right)-2\left(-2\lambda+3\right)=0$<br>Alt: $\left(\lambda-1\right)^2+\left(\lambda-8\right)^2+\left(-2\lambda+3\right)^2=6\lambda^2-30\lambda+74=6\left(\lambda-\dfrac{5}{2}\right)^2+\dfrac{73}{2}$ | **dM1** |
| | $\Rightarrow\lambda=\dfrac{5}{2}\quad\left[\text{use of }\overrightarrow{AB}\text{ in }\overrightarrow{CP}\text{ gives }\lambda=\dfrac{5}{12},\text{use of }\overrightarrow{OB}\ \lambda=\dfrac{-7}{2}\text{ or }\dfrac{-7}{12}\right]$ | **A1** |
| | $\overrightarrow{OP}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+\dfrac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix}=\dfrac{9}{2}\mathbf{i}-\dfrac{1}{2}\mathbf{j}$ | **ddM1, A1**<br>**(5)** |
| | | **(8 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6(a)** | $\left(2,3,-7\right)$ | B1 |
| | | **(1)** |
| **(b)** | Attempts $\begin{pmatrix}1\\2\\2\end{pmatrix}\bullet\begin{pmatrix}4\\-1\\8\end{pmatrix}=1\times4+2\times-1+2\times8=\left(18\right)$ | M1 |
| | Attempts $\mathbf{a}.\mathbf{b}=\left\|\mathbf{a}\right\|\left\|\mathbf{b}\right\|\cos\theta$: $18=\sqrt{1^2+2^2+2^2}\times\sqrt{4^2+\left(-1\right)^2+8^2}\cos\theta$ | **d**M1 |
| | $\cos\theta=\dfrac{2}{3}$ | A1 |
| | | **(3)** |
| **(c)** | Uses $\lambda=6$ to find length $PQ$ E.g.<br>$\overrightarrow{PQ}=6\mathbf{i}+12\mathbf{j}+12\mathbf{k}\Rightarrow PQ=\sqrt{6^2+12^2+12^2}=\left(18\right)$<br>Or $PQ=6\times\sqrt{1^2+2^2+2^2}=\left(18\right)$ | M1 |
| | Area $QPR=\dfrac{1}{2}ab\sin C=\dfrac{1}{2}\times18^2\times\sqrt{1-\left(\dfrac{2}{3}\right)^2}=54\sqrt{5}$ | M1 A1 |
| | | **(3)** |
| **(d)** | Attempts a correct method of finding at least one value for $\mu$ e.g.<br>$\left(4\mu\right)^2+\mu^2+\left(8\mu\right)^2=6^2+12^2+12^2$<br>$\Rightarrow\mu=\dfrac{\text{"18"}}{\sqrt{4^2+\left(-1\right)^2+8^2}}=\left(\pm\right)2$ | M1 |
| | Attempts one correct position $\mathbf{r}=\begin{pmatrix}2\\3\\-7\end{pmatrix}\pm2\begin{pmatrix}4\\-1\\8\end{pmatrix}$ | **d**M1 |
| | Possible coordinates $\left(10,1,9\right)$ and $\left(-6,5,-23\right)$ | A1 |
| | | **(3)** |
| | | **(10 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6(a)** | Attempts $\pm\left(\left(5\mathbf{i}+2\mathbf{j}+\mathbf{k}\right)-\left(\mathbf{i}+2\mathbf{j}-3\mathbf{k}\right)\right)=\pm\left(4\mathbf{i}+4\mathbf{k}\right)$ | M1 |
| | e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-3\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ or $\mathbf{r}=5\mathbf{i}+2\mathbf{j}+\mathbf{k}+\lambda(4\mathbf{i}+4\mathbf{k})$ oe | A1 |
| | | **(2)** |
| **(b)** | $\overrightarrow{AC}=2\mathbf{i}+(\alpha-2)\mathbf{j}+8\mathbf{k}$ | B1 |
| | $\left\|\overrightarrow{AB}\right\|=\sqrt{4^2+4^2}$ or $\left\|\overrightarrow{AC}\right\|=\sqrt{2^2+(\alpha-2)^2+8^2}$ | M1 |
| | $\left\|\overrightarrow{AB}\right\|=\sqrt{32}$ and $\left\|\overrightarrow{AC}\right\|=\sqrt{68+(\alpha-2)^2}$ | A1 |
| | $\cos 45°=\dfrac{8+32}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}$ | dM1 |
| | $\cos 45°=\dfrac{40}{\sqrt{32}\times\sqrt{68+(\alpha-2)^2}}\Rightarrow\alpha^2-4\alpha-28=0$ | ddM1 |
| | $\alpha=2\pm4\sqrt{2}$ | A1 |
| | | **(6)** |
| | | **(8 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)** | Finds $\left\|\begin{pmatrix}4\\4\\2\end{pmatrix}\right\|=\sqrt{4^2+4^2+2^2}=6$ and attempts $\dfrac{1}{"6"}\begin{pmatrix}4\\4\\2\end{pmatrix}$ | M1 |
| | $\overrightarrow{OA}=\begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ oe | A1 |
| | | **(2)** |
| **(b)** | Co-ordinates or position vector of point $X=\begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$ | M1 |
| | $\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix}=0\Rightarrow4(1+4\lambda)+4(-10+4\lambda)+2(-9+2\lambda)=0$ | dM1 |
| | $36\lambda=54\Rightarrow\lambda=1.5$ | ddM1 A1 |
| | $X=(7,-4,-6)$ | A1 |
| | | **(5)** |
| **(c)** | Finds $OX=\sqrt{7^2+(-4)^2+(-6)^2}=\sqrt{101}$ and $OA=1$ | M1 |
| | Area $OXA=\dfrac{1}{2}\times1\times\sqrt{101}=\dfrac{\sqrt{101}}{2}$ | dM1 A1 |
| | | **(3)** |
| | | **(10 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **9(a)** | $A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ | |
| | Either $A: \lambda = 1$ or $B: \lambda = 3$ | M1 |
| | leading to $a = -3$, $b = 5$ | A1, A1 |
| | | **(3)** |
| **(b)** | $\overrightarrow{AB} = = \pm\left(('5\mathbf{i}' - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ '- 3\mathbf{j}' + 5\mathbf{k})\right); = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ | M1, A1 |
| | | **(2)** |
| **(c)** | $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$ or $\overrightarrow{CA} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$ | M1 |
| | $\cos C\hat{A}B = \dfrac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2}.\sqrt{(3)^2 + (0)^2 + (-3)^2}}$ | dM1 |
| | $\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24}.\sqrt{18}} = \dfrac{\sqrt{3}}{2} \Rightarrow C\hat{A}B = 30^\circ$ | A1 |
| | | **(3)** |
| **(d)** | Area $CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ; = 3\sqrt{3}$ | M1, A1 |
| | | **(2)** |
| **(e)** | For one of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$ | M1, A1 |
| | For both of $\overrightarrow{OD_1} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ and | |
| | $\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$ | M1, A1 |
| | | **(4)** |
| | | **(14 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6 (a)(i)** | $\overrightarrow{AB} = (\pm)\left[\left(8\mathbf{i}+3\mathbf{j}-7\mathbf{k}\right)-\left(2\mathbf{i}-3\mathbf{j}+5\mathbf{k}\right)\right] = \ldots$ | **M1** |
| | $\overrightarrow{AB} = 6\mathbf{i}+6\mathbf{j}-12\mathbf{k}$ | **A1** |
| **(ii)** | $\mathbf{r} = \begin{pmatrix}2\\-3\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r} = \begin{pmatrix}8\\3\\-7\end{pmatrix}+\lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$ | **B1ft** |
| | | **(3)** |
| **(b)** | Attempts $\pm\overrightarrow{CP} = \pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$ | **M1** |
| | $\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \Rightarrow \begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \Rightarrow 1\left(\lambda-1\right)+1\left(\lambda-8\right)-2\left(-2\lambda+3\right)=0$<br>Alt: $\left(\lambda-1\right)^2+\left(\lambda-8\right)^2+\left(-2\lambda+3\right)^2 = 6\lambda^2-30\lambda+74 = 6\left(\lambda-\frac{5}{2}\right)^2+\frac{73}{2}$ | **dM1** |
| | $\Rightarrow \lambda = \frac{5}{2}$ $\left[\text{use of } \overrightarrow{AB} \text{ in } \overrightarrow{CP} \text{ gives } \lambda = \frac{5}{12}, \text{use of } \overrightarrow{OB}\ \lambda = \frac{-7}{2} \text{ or } \frac{-7}{12}\right]$ | **A1** |
| | $\overrightarrow{OP} = \begin{pmatrix}2\\-3\\5\end{pmatrix}+\frac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix} = \frac{9}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}$ | **ddM1, A1**<br>**(5)** |
| | | **(8 marks)** |

| Question | Scheme | Marks |
|---|---|---|
| 2 | $\overrightarrow{OA}=\begin{pmatrix}7\\2\\-5\end{pmatrix}$ $\overrightarrow{AB}=\begin{pmatrix}-2\\4\\3\end{pmatrix}$ $\overrightarrow{OC}=\begin{pmatrix}a\\5\\-1\end{pmatrix}$ | |
| (a) | $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\begin{pmatrix}7\\2\\-5\end{pmatrix}+\begin{pmatrix}-2\\4\\3\end{pmatrix}=...$ | M1 |
| | $=\begin{pmatrix}5\\6\\-2\end{pmatrix}$ , $5\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ or $(5,6,-2)$ | A1 |
| | | (2) |
| (b) | $\overrightarrow{BC}=\overrightarrow{OC}-"\overrightarrow{OB}"=\begin{pmatrix}a\\5\\-1\end{pmatrix}-"\begin{pmatrix}5\\6\\-2\end{pmatrix}"=...$ | M1 |
| | $\overrightarrow{OC}."\overrightarrow{BC}"=0\Rightarrow\begin{pmatrix}a\\5\\-1\end{pmatrix}."\begin{pmatrix}a-5\\-1\\1\end{pmatrix}"=0\Rightarrow a(a-5)-5-1=0$ | dM1 |
| | $a^2-5a-6=0\Rightarrow(a-6)(a+1)=0\Rightarrow a=...$ | ddM1 |
| | $a=-1,\ 6$ | A1 |
| | | (4) |
| | | (6 marks) |