Question Number	Scheme	Marks
9(a)	$(V)=\pi\int_0^4\left(\cos x+\frac{1}{5}e^x\right)^2\,dx$	B1
	$\left(\cos x+\frac{1}{5}e^x\right)^2=\cos^2 x+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}$	M1
	$=\frac{\cos 2x+1}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}$	dM1
	$=\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$	A1
		(4)
(b)	**Way I** $\int e^x\cos x\,dx=\pm e^x\sin x\pm\int e^x\sin x\,dx$ $=e^x\sin x-\int e^x\sin x\,dx$ **Way Two** $\int e^x\cos x\,dx=e^x\cos x\pm\int e^x\sin x\,dx$ $=e^x\cos x+\int e^x\sin x\,dx$	M1 A1
	$\int e^x\cos x\,dx=e^x\cos x\pm e^x\sin x-\int e^x\cos x\,dx$ $2\int e^x\cos x\,dx=e^x\cos x\pm e^x\sin x$	dM1
	$\int e^x\cos x\,dx=\frac{1}{2}e^x(\cos x+\sin x)+(c)$	A1
		(4)
(c)	$\pi\int_0^4\left(\frac{1}{2}+\frac{\cos 2x}{2}+\frac{2}{5}e^x\cos x+\frac{1}{25}e^{2x}\right)dx$ $=(\pi)\left[\frac{1}{2}x+\frac{\sin 2x}{4}+\frac{2}{5}\left(\frac{1}{2}e^x(\sin x+\cos x)\right)+\frac{1}{50}e^{2x}\right]_0^4$ $2\pi\left(2+\frac{\sin 8}{4}+\frac{2}{5}\left(\frac{1}{2}e^4(\sin(4)+\cos(4))\right)+\frac{1}{50}e^8-\frac{11}{50}\right)$ $=\text{awrt } 290\,(\text{cm}^3)$	M1A1ft dM1 A1
		(4)
		(12 marks)

(a)

Question	Scheme	Marks
1	$\int x\cos 3x\,dx=\frac{1}{3}x\sin 3x-\frac{1}{3}\int\sin 3x\,dx$	**M1**
	$=\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x$	**dM1 A1**
	$\int_0^{\frac{\pi}{6}}x\cos 3x\,dx=\left[\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}=\frac{1}{3}\frac{\pi}{6}\sin\frac{\pi}{2}+\frac{1}{9}\cos\frac{\pi}{2}-\left(0+\frac{1}{9}\right)$	**M1**
	$=\frac{\pi}{18}-\frac{1}{9}$	**A1**
		(5)
		(5 marks)

Question Number	Scheme	Marks
10(a)	$\frac{dx}{dt} = 6t$ o.e.	B1
	$\text{Area} = \left(\int \sin t \sin 2t \times 6t \, dt = \right) \int \sin t \times 2\sin t \cos t \times 6t \, dt$	M1
	$= 12\int_0^{\frac{\pi}{2}} t\sin^2 t \cos t \, dt$	A1
		(3)
(b)	$= (12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}} - (12)\int_0^{\frac{\pi}{2}} \frac{1}{3}\sin^3 t \, dt$	M1
	$= (12)\left[\frac{t}{3}\sin^3 t\right]_0^{\frac{\pi}{2}} - \left(\frac{12}{3}\right)\int_0^{\frac{\pi}{2}} \sin t\left(1-\cos^2 t\right) dt$	M1
	$= \left[4t\sin^3 t + 4\cos t - \frac{4}{3}\cos^3 t\right]_0^{\frac{\pi}{2}}$	A1
	$= 2\pi - \left(4 - \frac{4}{3}\right)$	M1
	$= 2\pi - \frac{8}{3}$	A1
		(5)
		Total 8

Question Number	Scheme	Marks
4 (a)	$a = 3, b = 5$	**B1**
	E.g $u = \sqrt{2x+1} \Rightarrow \frac{dx}{du} = u$ or $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ o.e.	**B1**
	$\int \sqrt{8x+4}\, e^{\sqrt{2x+1}} \, dx = \int 2u\, e^u \, u \, du$	**M1**
	$= \int_3^5 2u^2 e^u \, du$	**A1**
		(4)
(b)	$\int \sqrt{8x+4}\, e^{2x+1} \, dx = \int 2u^2 e^u \, du$	
	$= 2u^2 e^u - \int 4u\, e^u \, du$	**M1**
	$= = 2u^2 e^u - \left(4ue^u - 4e^u\right) = 2u^2 e^u - 4ue^u + 4e^u$	**dM1 A1ft**
	$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}} \, dx = \left[2u^2 e^u - 4ue^u + 4e^u\right]_3^5 = \left(50e^5 - 20e^5 + 4e^5\right) - \left(18e^3 - 12e^3 + 4e^3\right)$	**ddM1**
	$= 34e^5 - 10e^3$	**A1**
		(5)
		(9 marks)

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2}\mathrm{d}x = \int x^{-2}\ln x\,\mathrm{d}x = \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,\mathrm{d}x$	M1A1
	$= \frac{x^{-1}}{-1}\ln x + \int x^{-2}\,\mathrm{d}x$	
	$= \frac{x^{-1}}{-1}\ln x + \frac{x^{-1}}{-1}(+c)$	dM1A1
	$\int_1^{\mathrm{e}} \frac{\ln x}{x^2}\mathrm{d}x = \left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^{\mathrm{e}} = \left(\frac{-1}{\mathrm{e}}\ln \mathrm{e} - \frac{1}{\mathrm{e}}\right) - \left(\frac{-1}{1}\ln 1 - \frac{1}{1}\right)$	ddM1
	$= 1 - \frac{2}{\mathrm{e}}$	A1
		(6)

Question Number	Scheme	Marks
7(a)	$\frac{1}{(4-x)(2-x)} = \frac{A}{(4-x)} + \frac{B}{(2-x)} \Rightarrow 1 \equiv A(2-x) + B(4-x) \Rightarrow A = \ldots$ or $B = \ldots$	M1
	$A = -\frac{1}{2}, B = \frac{1}{2}$ giving $\frac{-\frac{1}{2}}{(4-x)} + \frac{\frac{1}{2}}{(2-x)}$	A1
		[2]
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(4-x)(2-x),\ t \geqslant 0 \Rightarrow \int \frac{1}{(4-x)(2-x)}\,\mathrm{d}x = \int k\,\mathrm{d}t$	
	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt\ (+c)$	M1, A1
	$\{t=0, x=0 \Rightarrow\}\ \frac{1}{2}\ln 4 - \frac{1}{2}\ln 2 = 0 + c\ \left\{\Rightarrow c = \frac{1}{2}\ln 2\right\}$	M1
	$\frac{1}{2}\ln(4-x) - \frac{1}{2}\ln(2-x) = kt + \frac{1}{2}\ln 2 \Rightarrow \ln\left(\frac{4-x}{4-2x}\right) = 2kt$	
	$\frac{4-x}{4-2x} = \mathrm{e}^{2kt}$	M1
	$4 - x = 4\mathrm{e}^{2kt} - 2x\mathrm{e}^{2kt} \Rightarrow 4 - 4\mathrm{e}^{2kt} = x - 2x\mathrm{e}^{2kt}$	
	$\Rightarrow 4 - 4\mathrm{e}^{2kt} = x(1 - 2\mathrm{e}^{2kt}) \Rightarrow x = \frac{4 - 4\mathrm{e}^{2kt}}{1 - 2\mathrm{e}^{2kt}}$ (*)	dM1, A1*
		[6]
(c)	$1 = \frac{4 - 4\mathrm{e}^{0.2t}}{1 - 2\mathrm{e}^{0.2t}} \Rightarrow 2\mathrm{e}^{0.2t} = 3$	M1
	$\Rightarrow 0.2t = \ln\frac{3}{2} \Rightarrow t = $ awrt 2.03	A1
		[2]
		10 marks

Question	Scheme	Marks
7(a)	$V=(\pi)\int_{(0)}^{(4)}\left(\frac{1}{4}(4-x)e^{x}\right)^{2}dx$	M1
	$\int(f(x))^{2}dx=\frac{1}{16}\int(4-x)^{2}e^{2x}dx=\ldots$	M1
	$V=\frac{\pi}{16}\int_{0}^{4}\left(x^{2}-8x+16\right)e^{2x}dx$	A1
		(3)
(b)	$=\left("\frac{\pi}{16}"\right)\left((x^{2}-8x+16)\frac{e^{2x}}{2}-\int(2x-8)\times\frac{e^{2x}}{2}dx\right)$	M1 A1
	$=\left("\frac{\pi}{16}"\right)\left((x^{2}-8x+16)\frac{e^{2x}}{2}-\left[(2x-8)\frac{e^{2x}}{4}-\int 2\times\frac{e^{2x}}{4}dx\right]\right)$ $=\left("\frac{\pi}{16}"\right)\left((x^{2}-8x+16)\frac{e^{2x}}{2}-(2x-8)\frac{e^{2x}}{4}+\frac{e^{2x}}{4}\right)$ **Depends on the first method mark.**	dM1
	$V="\frac{\pi}{16}"\left[(x^{2}-8x+16)\frac{e^{2x}}{2}-(2x-8)\frac{e^{2x}}{4}+\frac{e^{2x}}{4}\right]_{0}^{4}="\frac{\pi}{16}"\left(0+0+\frac{e^{8}}{4}-\frac{16}{2}-\frac{8}{4}-\frac{1}{4}\right)$ **Depends on both previous method marks.**	ddM1
	$=\frac{\pi}{64}\left(e^{8}-41\right)\ (\text{cm}^{3})$	A1
		(5)

Question Number	Scheme	Marks
7(a)	$u=\tan x\rightarrow\frac{du}{dx}=\sec^{2}x$	B1
	$u^{2}=\tan^{2}x=\sec^{2}x-1\rightarrow\int\frac{u+u^{3}}{(5+u^{2})^{3}}\times\frac{1}{1+u^{2}}\,du$	M1dM1
	$\int\frac{u}{(5+u^{2})^{3}}\,du$ *	A1*
		(4)
(b)	$\int\frac{u}{(5+u^{2})^{3}}\,du=-\frac{1}{4}(5+u^{2})^{-2}+c=-\frac{1}{4}(5+\tan^{2}x)^{-2}(+c)$	M1A1
		(2)
		(6 marks)

Question Number	Scheme	Marks
5(i)	$\int x^2 e^{4x}\,dx = \frac{1}{4}x^2 e^{4x} - \int \frac{1}{2}x e^{4x}\,dx$	M1 A1
	$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \int \frac{1}{8}e^{4x}\,dx$	dM1
	$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + c$	A1
		(4)

(ii)	$\frac{2x+11}{(2x+1)(2-x)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2-x)} \Rightarrow A = ..., B = ...$	M1
	$= \frac{4}{(2x+1)} + \frac{3}{(2-x)}$	A1
	$\int \frac{2x+11}{(2x+1)(2-x)}\,dx = 2\ln\lvert 2x+1\rvert - 3\ln\lvert 2-x\rvert$	dM1 A1ft
	$\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,dx = (2\ln 15 - 3\ln 5) - (2\ln 9 - 3\ln 2)$	ddM1
	$= \ln\frac{8}{45}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
7	$x = 4\sin\theta \Rightarrow \frac{dx}{d\theta} = 4\cos\theta$	B1
	$\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx = \int \frac{4\cos\theta}{(16-16\sin^2\theta)^{\frac{3}{2}}}\,d\theta = \int \frac{4\cos\theta}{64\cos^3\theta}\,d\theta$	M1 A1
	$= \frac{1}{16}\tan\theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60°) - f(30°)$	M1
	$= \frac{1}{16}\tan\frac{\pi}{3} - \frac{1}{16}\tan\frac{\pi}{6} = \frac{\sqrt{3}}{24}$	A1
		(6 marks)

Question	Scheme	Marks
7(a)	$\frac{dx}{dt} = k - 3x \Rightarrow \int \frac{1}{k-3x}\, dx = \int dt$ oe	M1
	$\Rightarrow -\frac{1}{3}\ln(k-3x) = t(+c)$	M1 A1
	$t = 0, x = 0 \Rightarrow -\frac{1}{3}\ln k = c$	M1
	$-\ln(k-3x) = 3t - \ln k \Rightarrow \ln\frac{k-3x}{k} = -3t \Rightarrow 1 - \frac{3}{k}x = e^{-3t} \Rightarrow x = ...$	M1
	$x = \frac{k}{3}(1 - e^{-3t})$	A1
		(6)
(b)	$t \to \infty \Rightarrow e^{-3t} \Rightarrow 0 \Rightarrow x \to \frac{k}{3} \Rightarrow \frac{k}{3} = 7 \Rightarrow k = ...$	M1
	$k = 21$	A1
		(2)
(c)	$x = 5 \Rightarrow 7(1 - e^{-3t}) = 5 \Rightarrow e^{-3t} = ...$	M1
	$\Rightarrow t = -\frac{1}{3}\ln\frac{2}{7}$	dM1
	Awrt 0.42 seconds	A1
		(3)
		(11 marks)