8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{matrix} 3x - y = 1 \\ x + y = 25 \end{matrix}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5 \quad \text{Not integers}$$

Show the calculations and statements that are needed to complete the proof.

(4)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$(8 - 3x)^{-\frac{1}{3}} \qquad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

(b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

8.

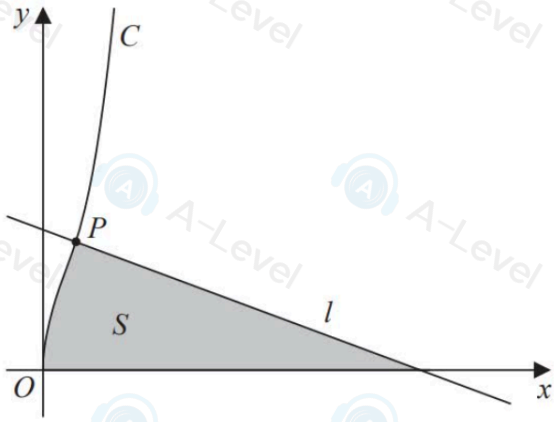

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

(6)

7. Water is flowing into a large container and is leaking from a hole at the base of the container.

At time t seconds after the water starts to flow, the volume, $V\,\text{cm}^3$, of water in the container is modelled by the differential equation

$$\frac{dV}{dt} = 300 - kV$$

where k is a constant.

(a) Solve the differential equation to show that, according to the model,

$$V = \frac{300}{k} + Ae^{-kt}$$

where A is a constant.

(5)

Given that the container is initially empty and that when $t = 10$, the volume of water is increasing at a rate of $200\,\text{cm}^3\text{s}^{-1}$

(b) find the exact value of k.

(4)

(c) Hence find, according to the model, the time taken for the volume of water in the container to reach 6 litres. Give your answer to the nearest second.

(2)

4.

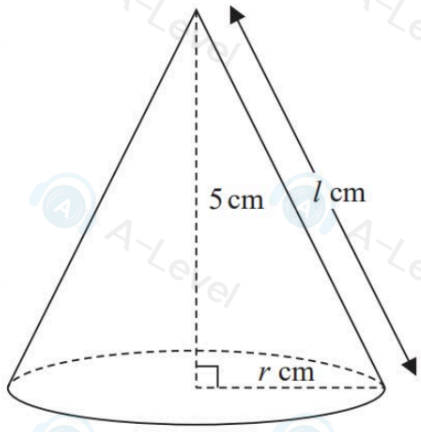

Figure 2

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r

(1)

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi rl$]

(4)

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

4. (a) Prove by contradiction that for all positive numbers k

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

(1)

4. Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $|\overrightarrow{PP'}|$, giving your answer as a simplified surd.

(2)

3.

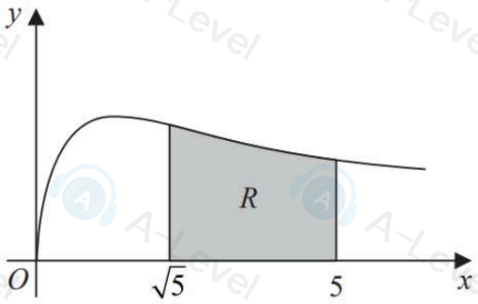

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

blank

4.

$$f(x) = \frac{4 - 4x}{x(x - 2)^2} \qquad x > 2$$

(a) Express $f(x)$ in partial fractions.

(4)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Find

$$\int_3^5 f(x)\,dx$$

giving your answer in the form $a + \ln b$, where a and b are rational numbers to be found.

(2)

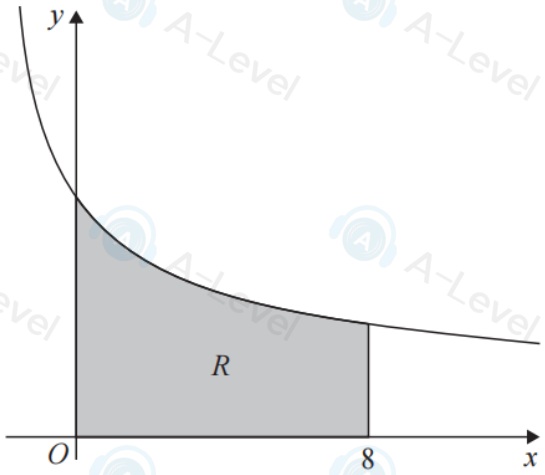

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve shown in Figure 1 has equation

$$y = \frac{4}{x+2} \qquad x > -2$$

The region R, bounded by the curve, the y-axis, the x-axis and the line with equation $x = 8$ is shown shaded in Figure 1

Region R is rotated through 360 degrees about the x-axis.

Use calculus to find the exact value of the volume of the solid generated, writing your answer in simplest form.

(5)