**2.** **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

Use integration by parts to find the exact value of $\int_1^e \frac{\ln x}{x^2}\,dx$

Write your answer in the form $a + \frac{b}{e}$, where $a$ and $b$ are integers.

**(6)**

**(Total for Question 2 is 6 marks)**

---

**3:** Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y\frac{dy}{dx} = xe^{-y^2} \qquad y \geqslant 0 \quad x \geqslant 5$$

giving your answer in the form $y^2 = f(x)$

**(5)**

---

**8.** (a) Find, in ascending powers of $x$, the first three non-zero terms of the binomial series expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient as a simplified fraction.

**(4)**

Using the expansion from part (a),

(b) state the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{4-x}}$

**(1)**

(c) Hence, or otherwise, show that

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} \approx a + bx^2$$

where $a$ and $b$ are fully simplified fractions to be found.

**(2)**

(d) Use

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} = a + bx^2$$

with $x = 1$ and the values of $a$ and $b$ to find a fully simplified rational approximation for $\sqrt{135}$

Show your working and make your method clear.

**(3)**

**5.**

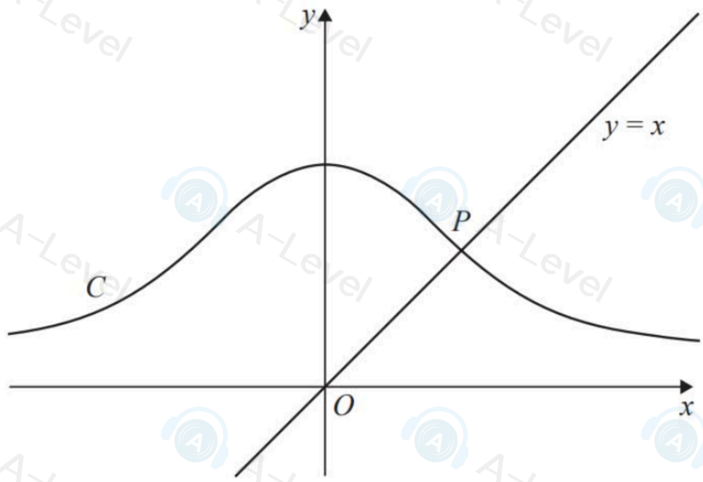


**Figure 2**

Figure 2 shows a sketch of the curve $C$ with equation

$$y^3 - x^2 + 4x^2y = k$$

where $k$ is a positive constant greater than 1

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$ and $y$.

**(5)**

The point $P$ lies on $C$.

Given that the normal to $C$ at $P$ has equation $y = x$, as shown in Figure 2,

(b) find the value of $k$.

**(5)**




**2.** A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

**(4)**

(b) Hence or otherwise, find the $x$ coordinate of the point of intersection of this line and the line with equation $y = x + 12$

**(2)**

Leave blank

**4.**

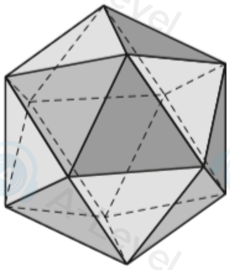

**Figure 1**

A regular icosahedron of side length $x$ cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length $x$ cm.

(a) Show that the surface area, $A$ cm$^2$, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

**(2)**

Given that the volume, $V$ cm$^3$, of the icosahedron is given by

$$V = \frac{5}{12}\left(3 + \sqrt{5}\right)x^3$$

(b) show that $\dfrac{\mathrm{d}V}{\mathrm{d}A} = \dfrac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$

**(3)**

The surface area of the icosahedron is increasing at a constant rate of $0.025\,\text{cm}^2\,\text{s}^{-1}$

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

**(3)**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**3.** Given that $y = 4$ at $x = \frac{\pi}{6}$, solve the differential equation

$$y\cos^2(2x)\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sin(2x) \qquad y > 0 \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

giving your answer in the form $y^2 = \mathrm{g}(x)$

**(6)**

DO NOT WRITE IN T

**8.** Relative to a fixed origin $O$, the line $l$ has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

where $\lambda$ is a scalar parameter.

The point $A$ and the point $B$ lie on line $l$

Given that

- $A$ has coordinates $(-2, a, 4)$
- $B$ has coordinates $(b, 3, 1)$

(a) find the value of the constant $a$ and the value of the constant $b$.

**(2)**

(b) Hence find vector $\overrightarrow{AB}$

**(2)**

The point $C$ has coordinates $(4, 7, -2)$.

(c) Find the size of angle $CAB$, giving your answer in degrees to one decimal place.

**(4)**

The point $D$ lies on the line $l$ so that the area of triangle $CAD$ is twice the area of triangle $CAB$.

(d) Find the coordinates of the two possible positions of $D$.

**(4)**

blank

**9.** (a) Find the derivative with respect to $y$ of

$$\frac{1}{(1 + 2\ln y)^2}$$

**(2)**

(b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x)\frac{\mathrm{d}y}{\mathrm{d}x} = y(1 + 2\ln y)^3 \qquad y > 0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

**(4)**

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = \mathrm{e}^{A\sec x - \frac{1}{2}}$$

where $A$ is an irrational number to be found.

**(5)**

**1.** (a) Find the first 4 terms of the binomial expansion, in ascending powers of $x$, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} \qquad |x| < \frac{1}{2}$$

giving each term in simplest form.

**(5)**

Given that

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{n}\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} = \left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$$

(b) write down the value of $n$.

**(1)**

(c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of $x$, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}} \qquad |x| < \frac{1}{2}$$

giving each term in simplest form.

**(3)**

**4.**

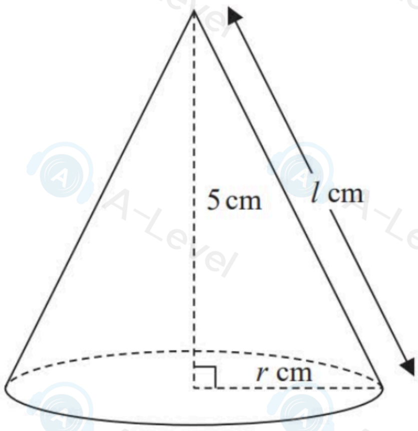


**Figure 2**

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius $r$ cm
- slant height $l$ cm

(a) Find an expression for $l$ in terms of $r$

**(1)**

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in $cm^2$ per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi r l$]

**(4)**