

5.

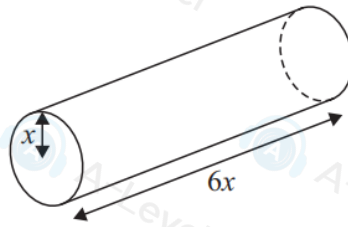


Figure 1

Figure 1 shows a right circular cylindrical rod which is expanding as it is heated.

At time t seconds the radius of the rod is x cm and the length of the rod is $6x$ cm.

Given that the **cross-sectional area** of the rod is increasing at a constant rate of

$\frac{\pi}{20} \text{ cm}^2 \text{ s}^{-1}$, find the rate of increase of the volume of the rod when $x = 2$

Write your answer in the form $k\pi \text{ cm}^3 \text{ s}^{-1}$ where k is a rational number.

(6)

(Total for Question 5 is 6 marks)

10. (a) A student's attempt to answer the question

“Prove by contradiction that if n^3 is even, then n is even”

is shown below. Line 5 of the proof is missing.

Assume that there exists a number n such that n^3 is even, but n is odd.

If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$

$$\begin{aligned} \text{So } n^3 &= (2p + 1)^3 \\ &= 8p^3 + 12p^2 + 6p + 1 \end{aligned}$$

=

This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.

(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

(5)

Leave blank

DO NOT WRITE IN THIS AREA

DO NOT WRITE!

6. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta^\circ\text{C}$, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85°C at the instant the engine is turned off
- is 40°C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a , b , c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20°C . Give your answer to the nearest minute.

(2)

Leave
blank

1. Given that k is a constant and the binomial expansion of

$$\sqrt{1 + kx} \quad |kx| < 1$$

in ascending powers of x up to the term in x^3 is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of k ,

(ii) find the value of the constant A and the constant B .

(5)

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

(2)

4. (a) Use the binomial expansion, in ascending powers of x , of $\frac{1}{\sqrt{1-2x}}$ to show that

$$\frac{2+3x}{\sqrt{1-2x}} \approx 2+5x+6x^2 \quad |x| < 0.5 \quad (4)$$

- (b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{1-2x}} = 2+5x+6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

(Total for Question 4 is 7 marks)

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

(4)

- (b) Find the range of values of x for which this expansion is valid.

(1)

DO NOT WRITE IN THIS

2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, V cm³, is decreasing at a constant rate, k cm³ per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)

DO NOT WRITE IN

5. (i) Find

$$\int x^2 e^x dx$$

(4)

- (ii) Use the substitution $u = \sqrt{1-3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} dx = -2(1-3x)^{\frac{1}{2}}(Ax+B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

DO NOT WRITE IN THIS AREA

9.

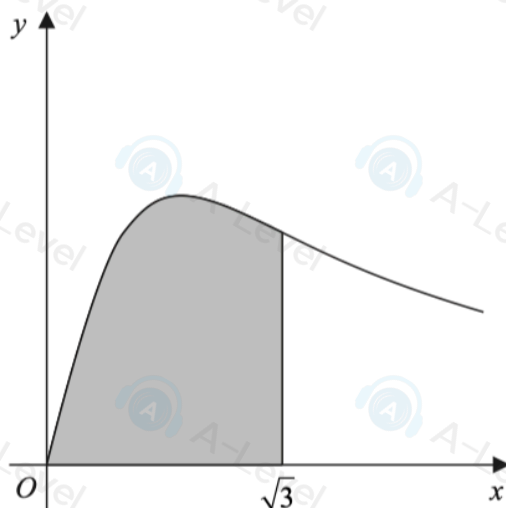


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan \theta \quad y = 2 \sin 2\theta \quad \theta \geq 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x -axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta) \, d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

7.

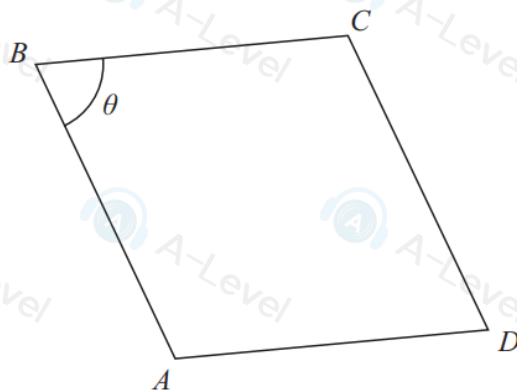


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\vec{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\vec{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90^\circ$

(a) find the exact value of $\cos\theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = 2(x^3 + 1)^k (x^3 - A) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)