5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \qquad y = 1 - t^2 \qquad t \neq 1$$

The point P, where $t = 2$, lies on C.

(a) Use parametric differentiation to find the equation of the normal to C at P. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(5)

(b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \qquad x \neq k$$

where p, q, r and k are integers to be found.

(4)

9.

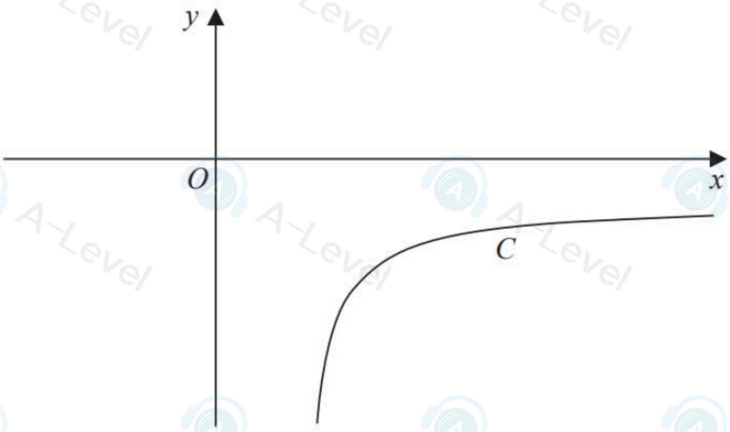

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \qquad y = \sqrt{3}\tan\left(t + \frac{\pi}{3}\right) \qquad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t

(3)

(b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are constants.

(4)

(c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found.

(5)

3.

$$f(x) = \frac{8x - 5}{(2x-1)(4x-3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

1. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$2y^2 - 6xy = 7e^{2x-1} + 13$$

The point P with x coordinate $\frac{1}{2}$ lies on C.

(a) Find the two possible y coordinates of P.

(2)

Given that P lies above the x-axis,

(b) find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

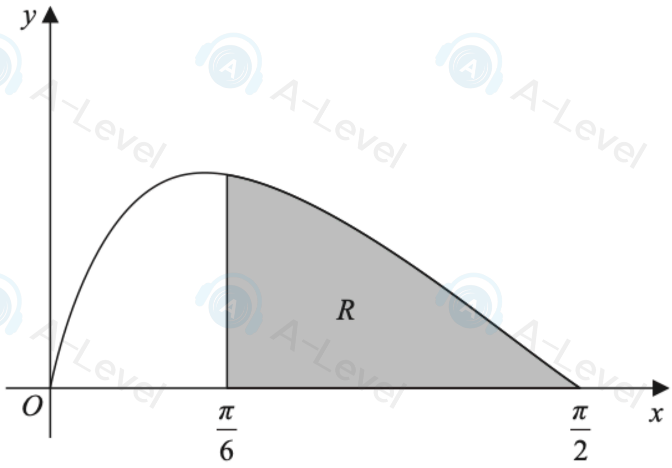

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

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7. (a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x \, dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

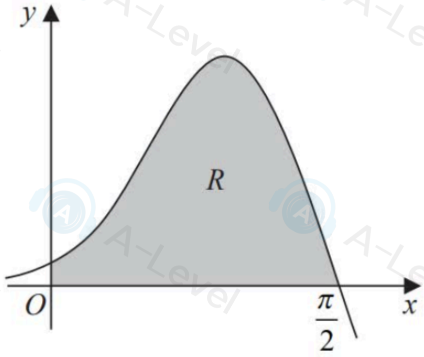

Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x}\cos x$$

The region R, shown shaded in Figure 3, is bounded by the positive x-axis, the positive y-axis and the curve.

The region R is rotated through 2π radians about the x-axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

9.

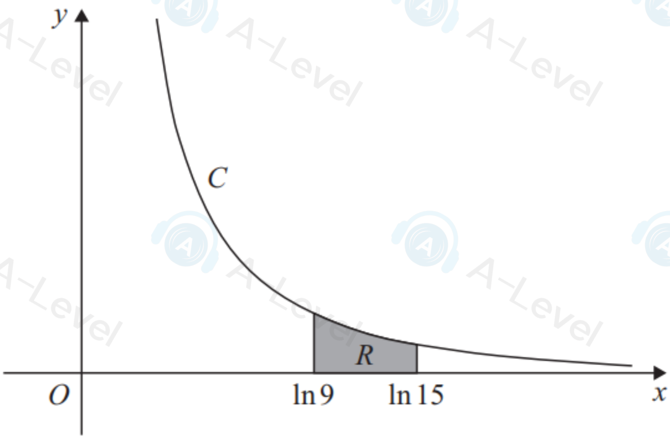

Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t+1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \qquad y = \frac{5}{t+2} \qquad t \geqslant 0$$

(a) Show that a Cartesian equation of the curve is $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \frac{ax+b}{cx+d} \qquad e < x \leqslant f$$

and a, b, c, d, e and f are constants to be found.

(5)

(b) State the range of g.

(2)

4.

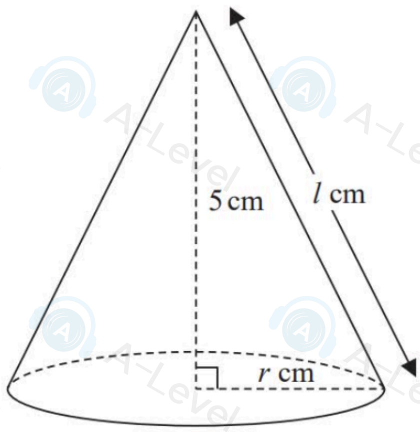

Figure 2

A cone, shown in Figure 2, has
- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r

(1)

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi r l$]

(4)

3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

(Total for Question 3 is 7 marks)

Leave blank

5. A curve has equation

$$y^2 = y\mathrm{e}^{-2x} - 3x$$

(a) Show that

$$\frac{dy}{dx} = \frac{2y\mathrm{e}^{-2x} + 3}{\mathrm{e}^{-2x} - 2y}$$

(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

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