3.

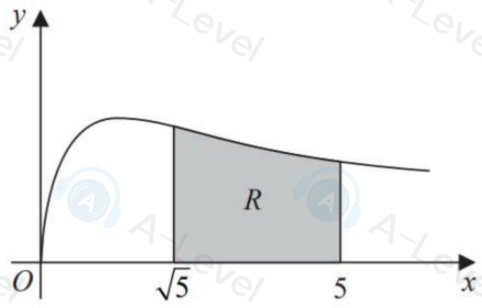

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

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8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

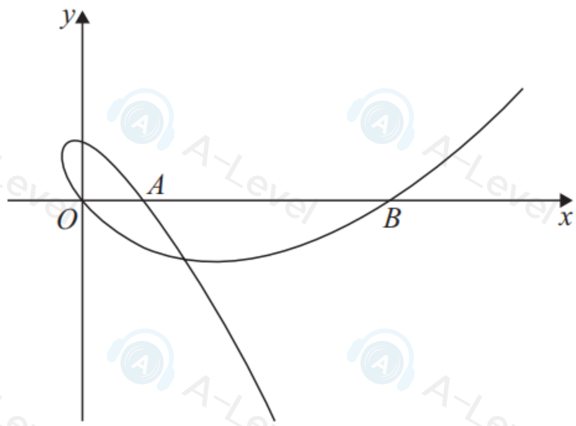

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \qquad y = t^3 - 9t \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Find the equation of the tangent to the curve at B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The tangent to the curve at B cuts the curve again at point X.

(c) Use algebra to find the coordinates of X, showing each stage of your working.

(5)

(Total for Question 8 is 12 marks)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi \text{ cm}^3 \text{ s}^{-1}$

Find the rate of increase of the radius of the balloon, in cm s^{-1}, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

10. A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking r and t,

(5)

(b) find the time taken for the ball of ice to melt completely.

(2)

(c) On Diagram 1 on page 27, sketch a graph of r against t.

(1)

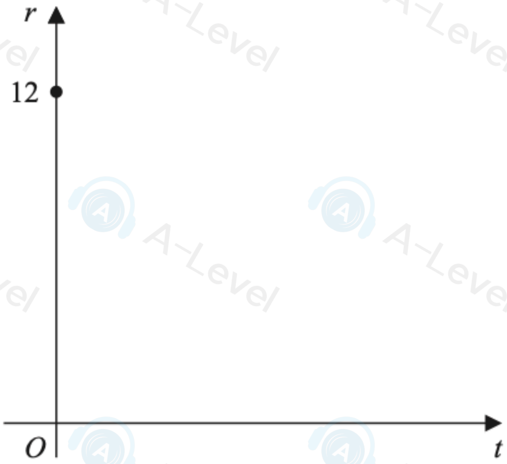

Diagram 1

5. (i) Find

$$\int x^2 e^x \, dx$$

(4)

(ii) Use the substitution $u = \sqrt{1 - 3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} \, dx = -2(1-3x)^{\frac{1}{2}}(Ax + B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

2.

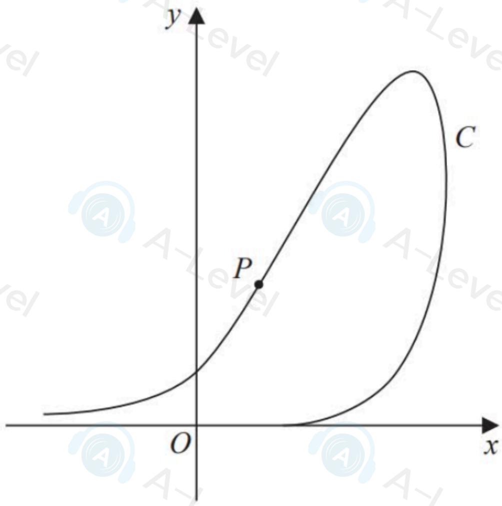

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P.

(2)

(b) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\frac{a\ln 2 + b}{c\ln 2 + d}$ where a, b, c and d are integers to be found.

3. **In this question you must show all stages of your working.**

Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x}\, \mathrm{d}x$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2}\, \mathrm{d}x = a + \ln b$$

where a and b are constants to be found.

(7)

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

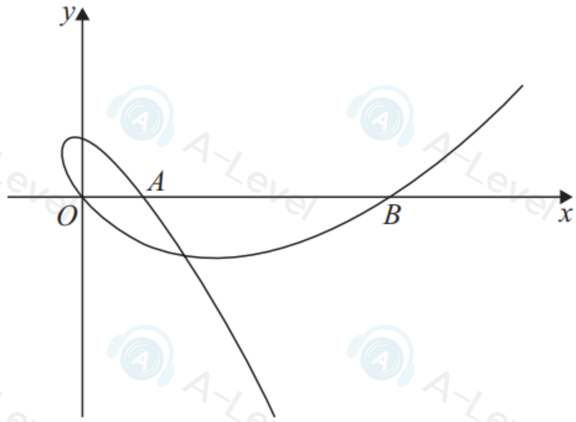

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \qquad y = t^3 - 9t \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Find the equation of the tangent to the curve at B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The tangent to the curve at B cuts the curve again at point X.

(c) Use algebra to find the coordinates of X, showing each stage of your working.

(5)

(Total for Question 8 is 12 marks)

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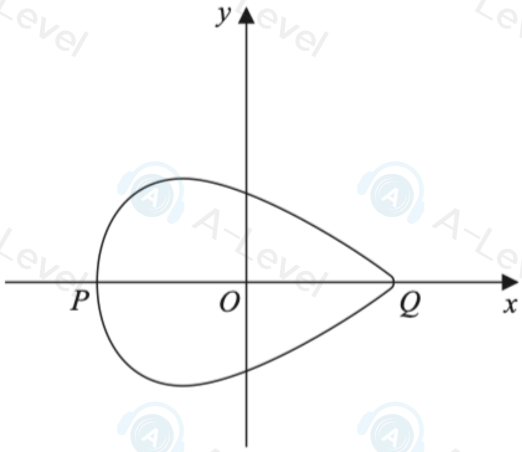

Figure 1

Figure 1 shows a sketch of a closed curve with parametric equations

$$x = 5\cos\theta \qquad y = 3\sin\theta - \sin 2\theta \qquad 0 \leqslant \theta < 2\pi$$

The region enclosed by the curve is rotated through π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of the solid of revolution is given by

$$V = 5\pi\int_{\alpha}^{\beta} \sin^3\theta\,(3 - 2\cos\theta)^2\,\mathrm{d}\theta$$

where α and β are constants to be found.

(4)

(b) Use the substitution $u = \cos\theta$ and algebraic integration to show that $V = k\pi$ where k is a rational number to be found.

(7)

3.

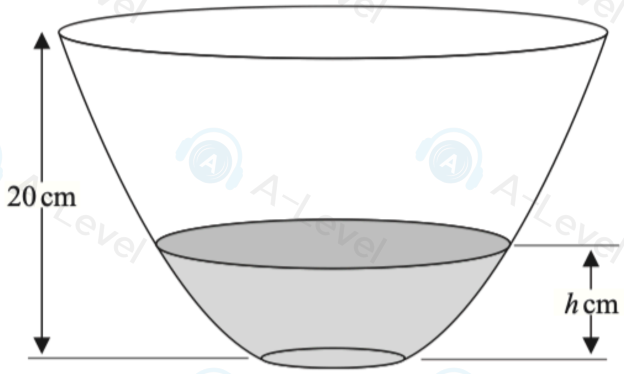

Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2(h+4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm^3 s^{-1}, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

6. Use the substitution $u = \sqrt{x^3+1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3+1}}\,dx = 2\left(x^3+1\right)^k\left(x^3 - A\right) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)