9. With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)

6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is 45°

(b) find the exact possible values of α

(6)

2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is 45°

(b) find the exact possible values of α

(6)

6. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

(a) find the value of p,

(4)

(b) find the position vector of the point of intersection.

(2)

(c) Find the acute angle between l_1 and l_2

Give your answer in degrees to one decimal place.

(3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with $\overrightarrow{AB}$ perpendicular to l_2

(d) Find the coordinates of B

(5)

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2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

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8. With respect to a fixed origin O the points A and B have position vectors

$$\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix}$$

respectively.

The line l_1 passes through the points A and B.

(a) Write down an equation for l_1

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where λ is a scalar parameter.

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

where μ is a scalar parameter.

(b) Show that l_1 and l_2 do **not** meet.

(4)

The point C is on l_2 where $\mu = -1$

(c) Find the acute angle between AC and l_2

Give your answer in degrees to one decimal place.

(5)

9. Relative to a fixed origin O, the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l, where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b.

(3)

(b) Find the vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, -3, 2)$

(c) Find the size of angle CAB.

(3)

(d) Find the exact area of the triangle CAB.

(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB.

(e) Find the coordinates of the two possible positions of D.

(4)

(Total for Question 9 is 14 marks)

6. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\overrightarrow{OA}| = 5\sqrt{10}$

(a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

(3)

Hence

(b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for A.

(3)

The line l_2 is parallel to l_1 and passes through O.

Given that

- $\overrightarrow{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\overrightarrow{OB}| = 4\sqrt{10}$

(c) find the area of triangle OAB, giving your answer to one decimal place.

(4)

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5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 13 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to one decimal place.

(3)

A circle with centre A and radius 35 cuts the line l_1 at the points P and Q.

Given that the x coordinate of P is greater than the x coordinate of Q,

(c) find the coordinates of P and the coordinates of Q.

(4)

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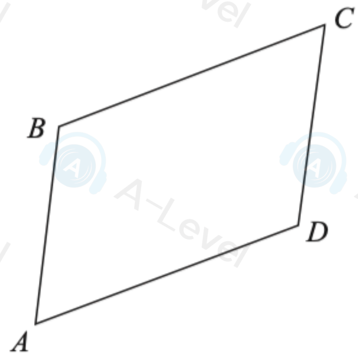

Figure 1

Figure 1 shows a sketch of parallelogram *ABCD*.

Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle *ABC*, giving your answer in degrees, to 2 decimal places.

(3)

(b) Find the area of parallelogram *ABCD*, giving your answer to one decimal place.

(2)