7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

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(4)

8. Use proof by contradiction to prove that the curve with equation

$$y = 2x + x^3 + \cos x$$

has no stationary points.

(4)

4. (a) Prove by contradiction that for all positive numbers k

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

(1)

9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use proof by contradiction to show that for all angles x, where $90° < x < 180°$

$$\left|\frac{\cos 2x}{\cos x - \sin x}\right| < 1$$

(4)

10: [*In this question you may assume that if n is an integer and n^2 is even, then n is even.*]

Given that p and q are integers, use proof by contradiction to show that

$$p^2 - 4q + 2 \neq 0$$

(5)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$
$\Rightarrow (3x - y)(x + y) = 25$
If $(3x - y) = 1$ and $(x + y) = 25$
$\left.\begin{array}{l} 3x - y = 1 \\ x + y = 25 \end{array}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5$ Not integers

Show the calculations and statements that are needed to complete the proof.

(4)

Leave blank

10. (a) A student's attempt to answer the question

"Prove by contradiction that if n^3 is even, then n is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number n such that n^3 is even, but n is odd.
If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$
So $n^3 = (2p+1)^3$
$= 8p^3 + 12p^2 + 6p + 1$
$=$
This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.
(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.
(5)

6. Given that $n \in \mathbb{N}$, use algebra to prove by contradiction that

" if $n^2 - 4n + 5$ is even then n is odd"

(4)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.
(4)

6. Given that $n \in \mathbb{N}$, use algebra to prove by contradiction that

" if $n^2 - 4n + 5$ is even then n is odd"

(4)

8. $$f(x) = (8 - 3x)^{\frac{4}{3}} \qquad 0 < x < \frac{8}{3}$$

(a) Show that the binomial expansion of f(x) in ascending powers of x up to and including the term in x^3 is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \ldots$$

where A and B are constants to be found.

(4)

(b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \qquad 0 < x < \frac{8}{3}$$

where A and B are the constants found in part (a).

(*Solutions relying on calculator technology are not acceptable.*)

(4)

Leave blank

1. Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(4)

10: [*In this question you may assume that if n is an integer and n^2 is even, then n is even.*]

Given that p and q are integers, use proof by contradiction to show that

$$p^2 - 4q + 2 \neq 0$$

(5)

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9. A student was asked to prove, for $p \in \mathbb{N}$, that

"if p^3 is a multiple of 3, then p must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

> Assumption:
>
> There exists a number p, $p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3
>
> Let $p = 3k + 1$, $k \in \mathbb{N}$.
>
> Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$
>
> $= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof. **(3)**

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number. **(5)**

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6. Three **consecutive** terms in a sequence of real numbers are given by

$$k, \ 1 + 2k \ \text{ and } \ 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence. **(5)**

10.

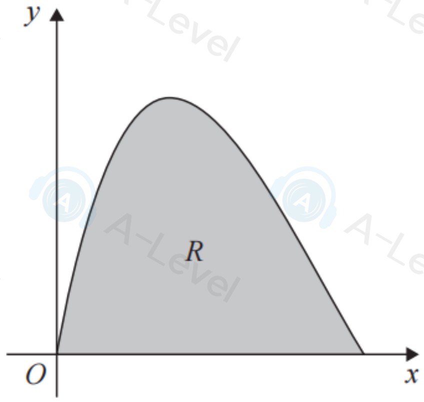

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t\,\mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

3. Prove by contradiction that there is no greatest odd integer.

(2)

blank

1. Prove by contradiction that, if a and b are real numbers, $a > 0$ and $b > 0$, then

$$\frac{9a}{b} + \frac{4b}{a} \geqslant 12$$

(4)

(Total for Question 1 is 4 marks)

1. Prove by contradiction that, if a and b are real numbers, $a > 0$ and $b > 0$, then

$$\frac{9a}{b} + \frac{4b}{a} \geqslant 12$$

(4)

(Total for Question 1 is 4 marks)

8. Use proof by contradiction to prove that, for all positive real numbers x and y,

$$\frac{9x}{y} + \frac{y}{x} \geqslant 6$$

(4)

4. (a) Prove by contradiction that for all positive numbers k

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

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9. A student was asked to prove, for $p \in \mathbb{N}$, that

"if p^3 is a multiple of 3, then p must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

Assumption: There exists a number p, $p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3 Let $p = 3k + 1$, $k \in \mathbb{N}$. Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$ $= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

(3)

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

(5)

8. $$\mathrm{f}(x) = (8 - 3x)^{\frac{4}{3}} \qquad 0 < x < \frac{8}{3}$$

(a) Show that the binomial expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3 is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \ldots$$

where A and B are constants to be found.

(4)

(b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \qquad 0 < x < \frac{8}{3}$$

where A and B are the constants found in part (a).

(*Solutions relying on calculator technology are not acceptable.*)

(4)

9. (i) Relative to a fixed origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points A, B and C lie in a straight line, with B lying between A and C.

Given $AB:AC = 1:3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

(3)

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)

8. Use proof by contradiction to prove that the curve with equation

$$y = 2x + x^3 + \cos x$$

has no stationary points.

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10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use proof by contradiction to show that for all angles x, where $90° < x < 180°$

$$\left|\frac{\cos 2x}{\cos x - \sin x}\right| < 1$$

(4)