

- 5: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$2x + 4y^2 = 6 + 3e^{xy}$$

The point $P\left(\frac{9}{2}, 0\right)$ lies on C .

Find an equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(6)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3 \cos^2 t \quad y = 3 \sin t + 2 \cos t \quad 0 \leq t \leq 5$$

(a) Show that $\frac{dy}{dx} = 3$ where $t = \pi$

(4)

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found.

(3)

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9 \cos^2 t + 2 \cos t - 7 = 0$$

(2)

(d) Hence find the exact value of the y coordinate of Q

(3)

8.

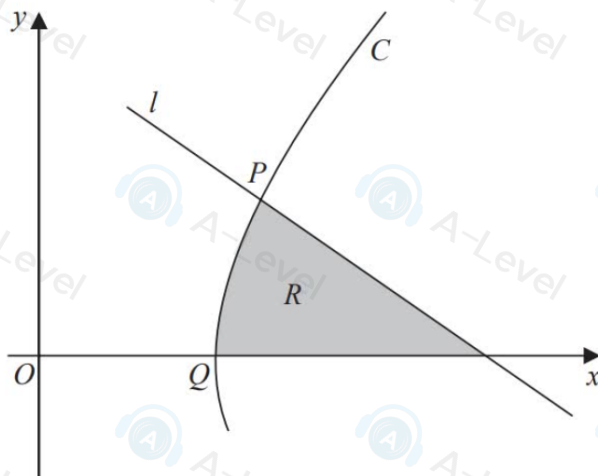


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \quad y = t - \frac{1}{t} \quad t > 0.7$$

The curve C intersects the x -axis at the point Q .

- (a) Find the x coordinate of Q .

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

- (b) write down the coordinates of P

(1)

- (c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R , shown shaded in Figure 2 is bounded by the curve C , the line l and the x -axis.

- (d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x -axis.

(7)

4.

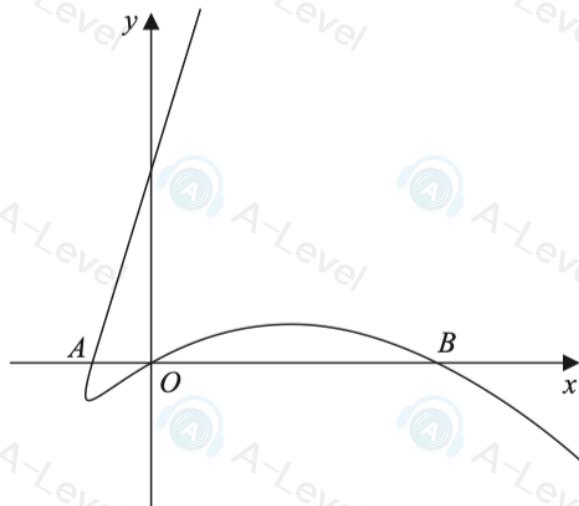
**Figure 2**

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \quad y = t^3 - 4t, \quad t \in \mathbb{R}$$

The curve cuts the x -axis at the origin and at the points A and B , as shown in Figure 2.

- (a) Find the coordinates of A and show that B has coordinates $(20, 0)$.

(3)

- (b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P .

- (c) Find, using algebra, the x coordinate of P .

(4)

1. A curve C has parametric equations

$$x = \frac{t}{t-3} \quad y = \frac{1}{t} + 2 \quad t \in \mathbb{R} \quad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax - 1}{bx}$$

where a and b are constants to be found.

(3)

9.

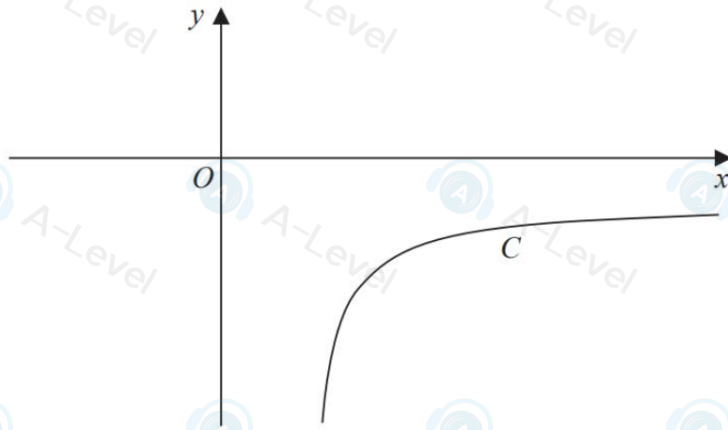


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \quad y = \sqrt{3} \tan\left(t + \frac{\pi}{3}\right) \quad \frac{\pi}{6} < t < \frac{\pi}{2}$$

- (a) Find $\frac{dy}{dx}$ in terms of t (3)

- (b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$.
Give your answer in the form $y = mx + c$, where m and c are constants. (4)

- (c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found. (5)

- 6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = 8 \cos t + 2 \quad y = 6 \sin t - 3 \quad 0 < t \leq 2\pi$$

The point P lies on C where $t = \frac{\pi}{3}$

- (a) Use parametric differentiation to find the exact gradient of the **tangent** to C at P . (3)

The **normal** to C at P intersects the x -axis at the point R .

- (b) Find, using algebra, the exact coordinates of R . (6)

- (c) Find a Cartesian equation of C . The equation must not involve trigonometric expressions. (2)

- 6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = 8 \cos t + 2 \quad y = 6 \sin t - 3 \quad 0 < t \leq 2\pi$$

The point P lies on C where $t = \frac{\pi}{3}$

- (a) Use parametric differentiation to find the exact gradient of the **tangent** to C at P . (3)

The **normal** to C at P intersects the x -axis at the point R .

- (b) Find, using algebra, the exact coordinates of R . (6)

- (c) Find a Cartesian equation of C . The equation must not involve trigonometric expressions. (2)

5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \quad y = 1 - t^2 \quad t \neq 1$$

The point P , where $t = 2$, lies on C .

- (a) Use parametric differentiation to find the equation of the normal to C at P . Give your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(5)

- (b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \quad x \neq k$$

where p , q , r and k are integers to be found.

(4)

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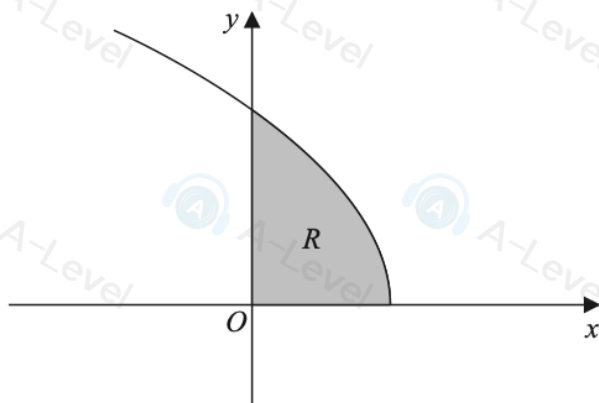


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = 4 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the y -axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32 \sin^2 t \cos t \, dt$

(ii) Hence find, by algebraic integration, the exact value of the area of R . (6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found. (3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \sqrt{ax + b} \quad -2 \leq x \leq 2$$

and a and b are the constants found in part (b).

(c) State the range of f . (1)

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \quad y = \frac{5}{t+2} \quad t \geq 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax + b}{cx + d} \quad e < x \leq f$$

and a, b, c, d, e and f are constants to be found. (5)

(b) State the range of g . (2)

8.

In this question you must show all stages of your working.
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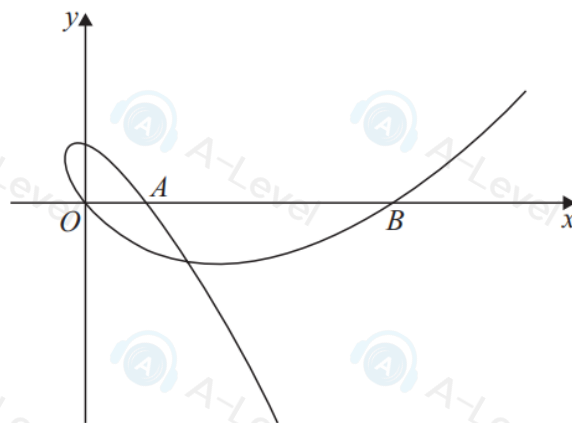


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \quad y = t^3 - 9t \quad t \in \mathbb{R}$$

The curve cuts the x -axis at the origin and at the points A and B as shown in Figure 3.

- (a) Find the coordinates of point A and show that point B has coordinates $(15, 0)$. (3)

- (b) Find the equation of the tangent to the curve at B , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

The tangent to the curve at B cuts the curve again at point X .

- (c) Use algebra to find the coordinates of X , showing each stage of your working. (5)

(Total for Question 8 is 12 marks)

10.

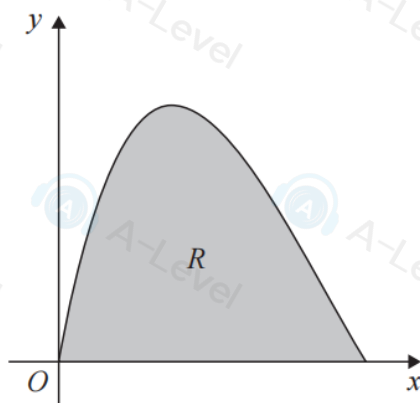


Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \quad y = \sin t \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 5, is bounded by the curve and the x -axis.

(a) Show that the area of R is

$$k \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R , giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The curve crosses the y -axis at the origin and at the point P .

(b) Find the equation of the normal to the curve at P , writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

- 5: In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$2x + 4y^2 = 6 + 3e^{xy}$$

The point $P\left(\frac{9}{2}, 0\right)$ lies on C .

Find an equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(6)

5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \quad y = 1 - t^2 \quad t \neq 1$$

The point P , where $t = 2$, lies on C .

- (a) Use parametric differentiation to find the equation of the normal to C at P . Give your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found.

(5)

- (b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \quad x \neq k$$

where p , q , r and k are integers to be found.

(4)

5.

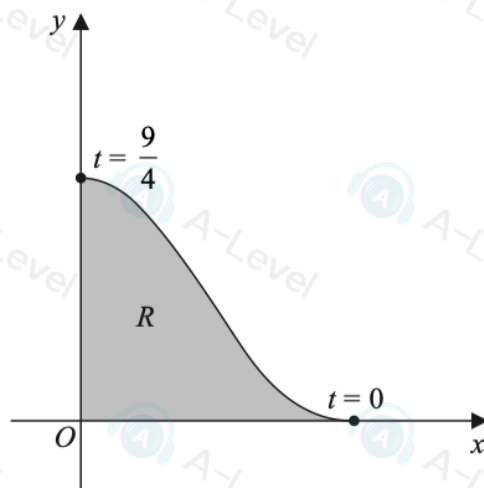


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \quad y = \frac{t^3}{\sqrt{9 + 4t}} \quad 0 \leq t \leq \frac{9}{4}$$

The curve touches the x -axis when $t = 0$ and meets the y -axis when $t = \frac{9}{4}$

The region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the y -axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R .

(Solutions relying on calculator technology are not acceptable.)

(6)

9.

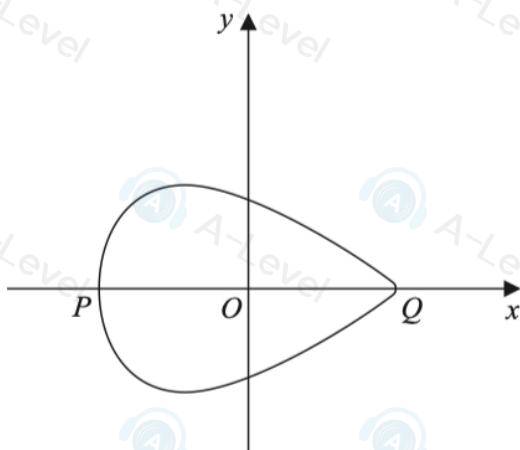


Figure 1

Figure 1 shows a sketch of a closed curve with parametric equations

$$x = 5 \cos \theta \quad y = 3 \sin \theta - \sin 2\theta \quad 0 \leq \theta < 2\pi$$

The region enclosed by the curve is rotated through π radians about the x -axis to form a solid of revolution.

- (a) Show that the volume, V , of the solid of revolution is given by

$$V = 5\pi \int_{\alpha}^{\beta} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$$

where α and β are constants to be found.

(4)

- (b) Use the substitution $u = \cos \theta$ and algebraic integration to show that $V = k\pi$ where k is a rational number to be found.

(7)

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \quad y = \frac{5}{t+2} \quad t \geq 0$$

- (a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \quad e < x \leq f$$

and a, b, c, d, e and f are constants to be found.

(5)

- (b) State the range of g .

(2)

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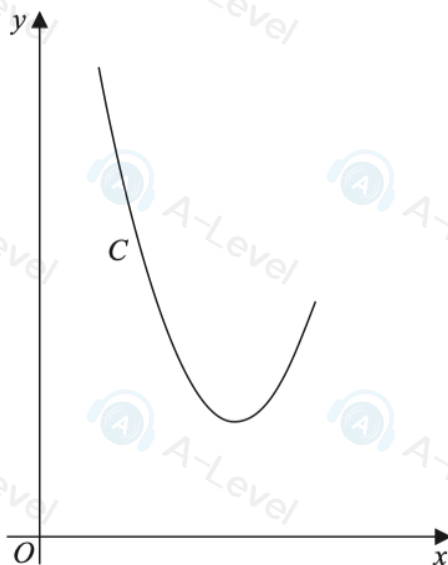
**Figure 1**

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2 \tan t \quad y = 8 \sec^2 t \quad -\frac{\pi}{3} \leq t \leq \frac{\pi}{4}$$

- (a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = f(x)$, where f is a quadratic function.

- (b) Find $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants to be found.

(3)

- (c) Find the range of f .

(2)

4. The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \quad y = \frac{1 - 2t}{3 + t} \quad t > 0$$

- (a) Show that the equation of C can be written in the form $y = g(x)$ where g is the function

$$g(x) = \frac{ax + b}{cx + d} \quad x > k$$

where a , b , c , d and k are integers to be found.

(5)

- (b) Hence, or otherwise, state the range of g .

(2)

9.

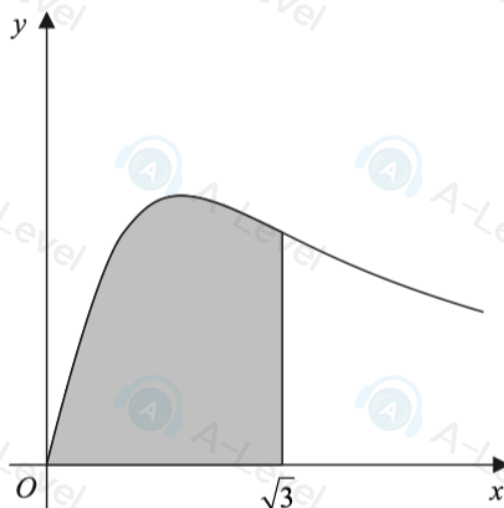
**Figure 3**

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan \theta \quad y = 2 \sin 2\theta \quad \theta \geq 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x -axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta) \, d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \quad y = \frac{6}{2t+1} \quad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

9.

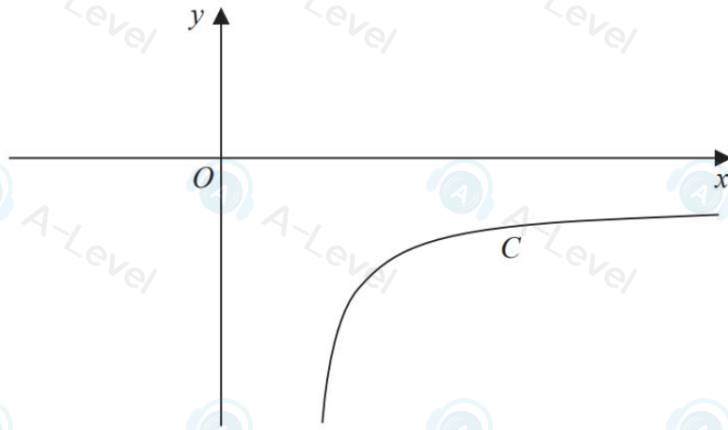


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \quad y = \sqrt{3} \tan\left(t + \frac{\pi}{3}\right) \quad \frac{\pi}{6} < t < \frac{\pi}{2}$$

- (a) Find $\frac{dy}{dx}$ in terms of t (3)

- (b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$.
Give your answer in the form $y = mx + c$, where m and c are constants. (4)

- (c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found. (5)

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \quad y = \frac{6}{2t+1} \quad t \neq -\frac{1}{2}$$

- (a) Show that all points $P(x, y)$ lie on a straight line. (4)
- (b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$ (2)

2. The curve C has parametric equations

$$x = \frac{t^4}{2t+1} \quad y = \frac{t^3}{2t+1} \quad t > 0$$

- (a) Write down $\frac{x}{y}$ in terms of t , giving your answer in simplest form.

(1)

- (b) Hence show that all points on C satisfy the equation

$$x^3 - 2xy^3 - y^4 = 0$$

(3)

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \quad y = \frac{5}{t+2} \quad t \geq 0$$

- (a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \quad e < x \leq f$$

and a, b, c, d, e and f are constants to be found.

(5)

- (b) State the range of g .

(2)

3. The curve C has parametric equations

$$x = 3 + 2 \sin t \quad y = \frac{6}{7 + \cos 2t} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

- (a) Show that C has Cartesian equation

$$y = \frac{12}{(7-x)(1+x)} \quad p \leq x \leq q$$

where p and q are constants to be found.

(6)

- (b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x+b} + \frac{c}{x+d} \quad p \leq x \leq q$$

where a, b, c and d are constants.

(3)

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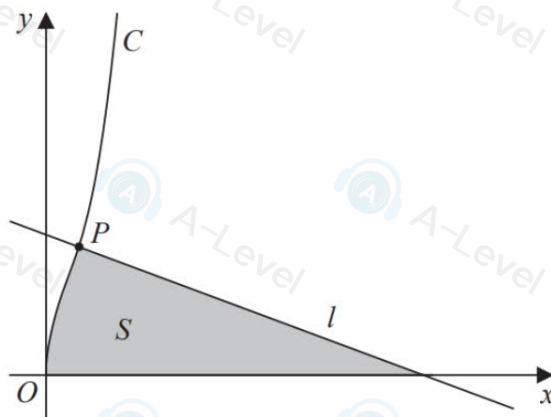


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \quad y = 2 \tan t \quad 0 \leq t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The line l is the normal to C at P , as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

The region S , shown shaded in Figure 3, is bounded by C , l and the x -axis.

(b) Find, using calculus, the exact area of S .

(6)

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \quad y = \frac{5}{t+2} \quad t \geq 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \quad e < x \leq f$$

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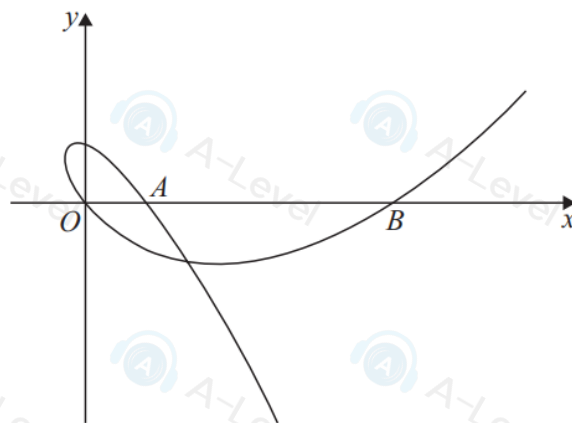


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \quad y = t^3 - 9t \quad t \in \mathbb{R}$$

The curve cuts the x -axis at the origin and at the points A and B as shown in Figure 3.

- (a) Find the coordinates of point A and show that point B has coordinates $(15, 0)$. (3)

- (b) Find the equation of the tangent to the curve at B , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

The tangent to the curve at B cuts the curve again at point X .

- (c) Use algebra to find the coordinates of X , showing each stage of your working. (5)

(Total for Question 8 is 12 marks)

8.

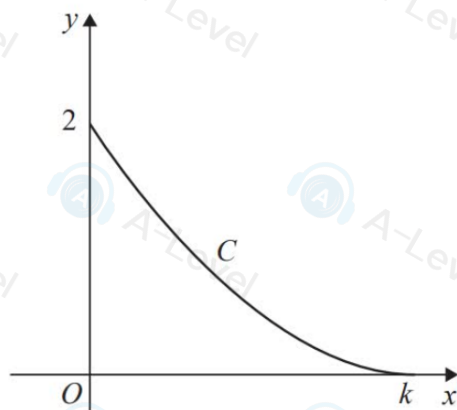


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3 \sin 2t \quad y = 2 \cos t \quad 0 \leq t \leq \frac{\pi}{2}$$

The curve meets the y -axis at 2 and the x -axis at k , where k is a constant.

(a) State the value of k .

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The tangent to C at the point P cuts the y -axis at the point N .

(c) Find the exact y coordinate of N , giving your answer in simplest form.

(3)

The region bounded by the curve, the x -axis and the y -axis is rotated through 2π radians about the x -axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta (1 - \cos 4t) dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

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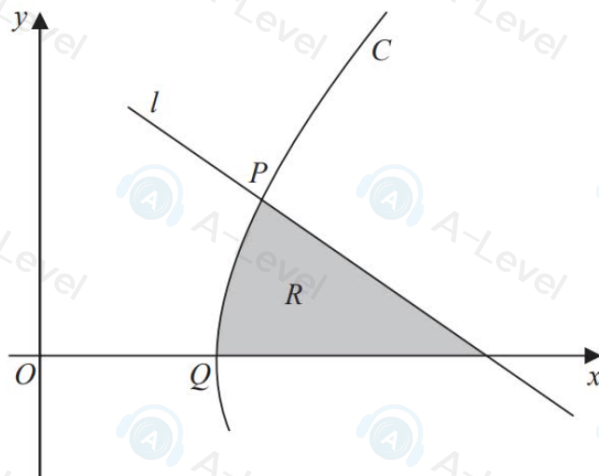


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \quad y = t - \frac{1}{t} \quad t > 0.7$$

The curve C intersects the x -axis at the point Q .

- (a) Find the x coordinate of Q .

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

- (b) write down the coordinates of P

(1)

- (c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R , shown shaded in Figure 2 is bounded by the curve C , the line l and the x -axis.

- (d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x -axis.

(7)

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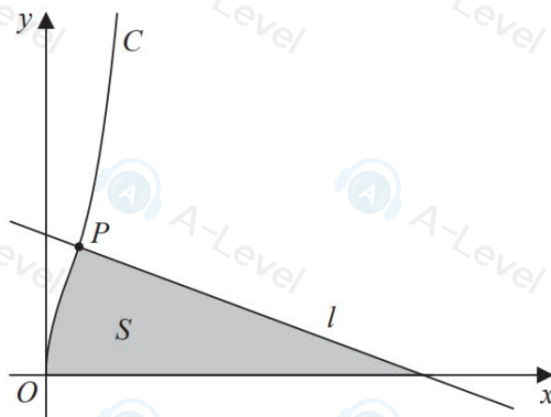


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$$x = \sin^2 t \quad y = 2 \tan t \quad 0 \leq t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The line l is the normal to C at P , as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

The region S , shown shaded in Figure 3, is bounded by C , l and the x -axis.

(b) Find, using calculus, the exact area of S .

(6)

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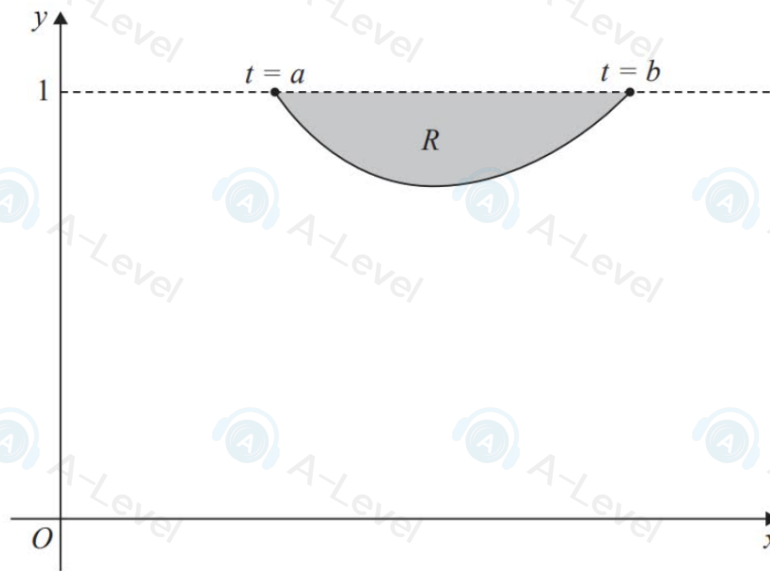
**Figure 2**

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \quad y = \frac{2}{t(3-t)} \quad a \leq t \leq b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R , shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k \int_a^b \frac{t+1}{t(3-t)} dt$$

where M and k are constants to be found.

(5)

(c) (i) Write $\frac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R , giving your answer in simplest form.

(6)

5.

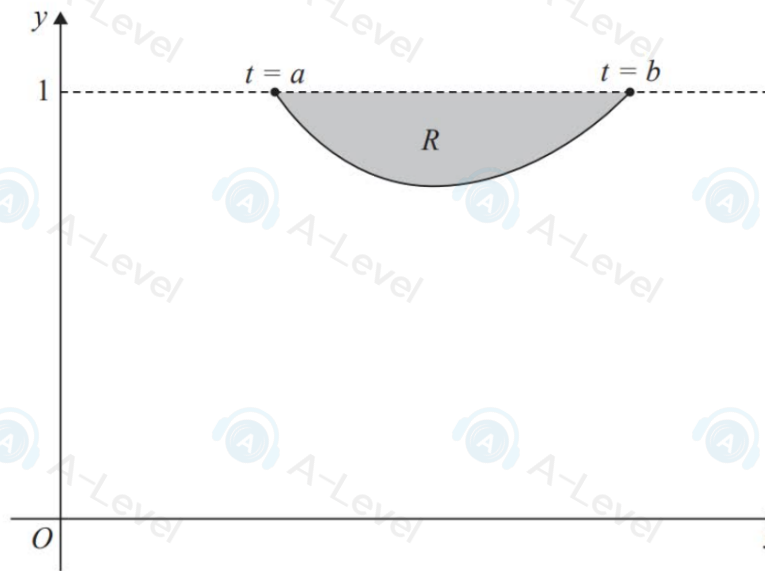
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Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \quad y = \frac{2}{t(3-t)} \quad a \leq t \leq b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R , shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k \int_a^b \frac{t+1}{t(3-t)} dt$$

where M and k are constants to be found.

(5)

(c) (i) Write $\frac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R , giving your answer in simplest form.

(6)

6.

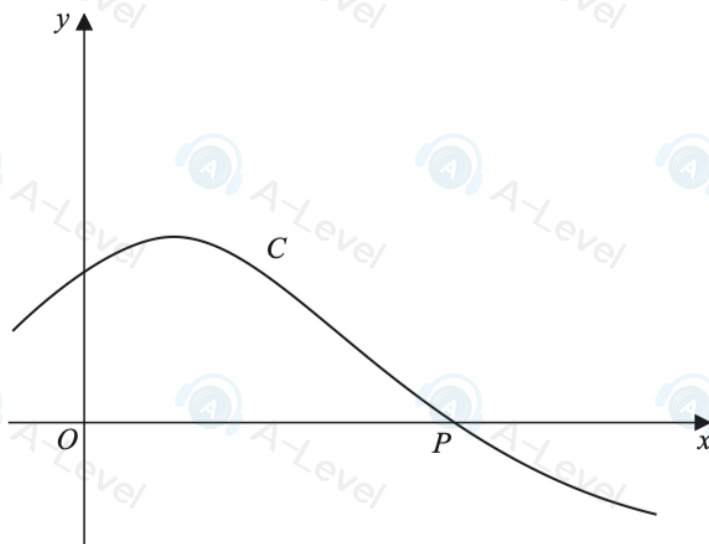
**Figure 3**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3 \tan t \quad y = 2 \cos 2t \quad -\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$$

The curve crosses the x -axis at point P , as shown in Figure 3.

- (a) Find the equation of the tangent to C at P , writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

The curve C has equation $y = f(x)$, where f is a function with domain $\left[k, 1 + 3\sqrt{3} \right]$

- (b) Find the exact value of the constant k .

(1)

- (c) Find the range of f .

(2)

8.

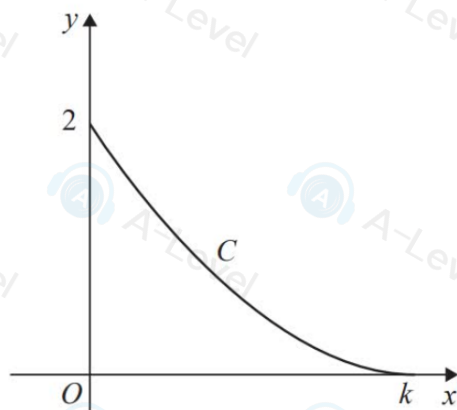


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3 \sin 2t \quad y = 2 \cos t \quad 0 \leq t \leq \frac{\pi}{2}$$

The curve meets the y -axis at 2 and the x -axis at k , where k is a constant.

(a) State the value of k .

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The tangent to C at the point P cuts the y -axis at the point N .

(c) Find the exact y coordinate of N , giving your answer in simplest form.

(3)

The region bounded by the curve, the x -axis and the y -axis is rotated through 2π radians about the x -axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta (1 - \cos 4t) dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

10.

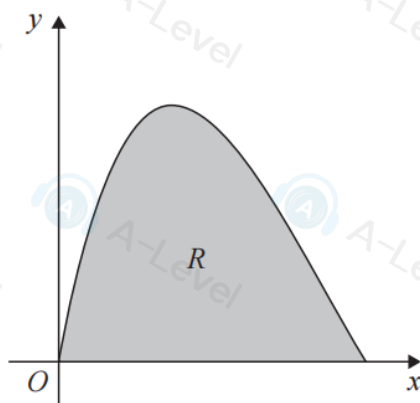


Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \quad y = \sin t \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 5, is bounded by the curve and the x -axis.

(a) Show that the area of R is

$$k \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R , giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)