

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8 - 3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

1.

$$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

- (a) Write $f(x)$ in partial fraction form.

(3)

- (b) (i) Hence find, in ascending powers of x up to and including the terms in x^2 , the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.

(5)

- (ii) Find the range of values of x for which this expansion is valid.

(1)

1. The binomial expansion of

$$(3 + kx)^{-2} \quad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \dots$$

where A , B , C and D are constants.

- (a) Find the value of A

(1)

Given that $C = 3B$

- (b) show that

$$k^2 + 6k = 0$$

(3)

- (c) Hence (i) find the value of k

- (ii) find the value of D

(3)

4. (a) Use the binomial expansion, in ascending powers of x , of $\frac{1}{\sqrt{1-2x}}$ to show that

$$\frac{2+3x}{\sqrt{1-2x}} \approx 2 + 5x + 6x^2 \quad |x| < 0.5 \quad (4)$$

- (b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{1-2x}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

(Total for Question 4 is 7 marks)

8. (a) Find, in ascending powers of x , the first three non-zero terms of the binomial series expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient as a simplified fraction.

(4)

Using the expansion from part (a),

- (b) state the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{4-x}}$

(1)

- (c) Hence, or otherwise, show that

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} \approx a + bx^2$$

where a and b are fully simplified fractions to be found.

(2)

- (d) Use

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} = a + bx^2$$

with $x = 1$ and the values of a and b to find a fully simplified rational approximation for $\sqrt{135}$

Show your working and make your method clear.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

4. $f(x) = \sqrt{1 - 4x^2} \quad |x| < \frac{1}{2}$

- (a) Find, in ascending powers of x , the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

(4)

- (b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

(2)

DO NOT WRITE IN THIS AREA

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{8}{(2 - 5x)^2}$$

writing each term in simplest form.

(4)

- (b) Find the range of values of x for which this expansion is valid.

(1)

DO NOT WRITE IN THIS AREA

3. Given that the binomial expansion, in ascending powers of x , of

$$6(4 + Ax)^{-\frac{1}{2}}$$

is $B - \frac{1}{4}x + Cx^2 + \dots$

- (a) find the values of the constants A , B and C .

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

For the expansion,

- (c) find the coefficient of the term in x^3

(2)

DO NOT WRITE IN THIS AREA

4. (a) Use the binomial expansion, in ascending powers of x , of $\frac{1}{\sqrt{1-2x}}$ to show that

$$\frac{2+3x}{\sqrt{1-2x}} \approx 2 + 5x + 6x^2 \quad |x| < 0.5 \quad (4)$$

- (b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{1-2x}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

(Total for Question 4 is 7 marks)

8. (a) Find, in ascending powers of x , the first three non-zero terms of the binomial series expansion of

$$\frac{1}{\sqrt{4+x}}$$

giving each coefficient as a simplified fraction.

(4)

Using the expansion from part (a),

- (b) state the first three non-zero terms of the binomial series expansion of $\frac{1}{\sqrt{4-x}}$

(1)

- (c) Hence, or otherwise, show that

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} \approx a + bx^2$$

where a and b are fully simplified fractions to be found.

(2)

- (d) Use

$$\frac{1}{\sqrt{4+x}} \times \frac{1}{\sqrt{4-x}} = a + bx^2$$

with $x = 1$ and the values of a and b to find a fully simplified rational approximation for $\sqrt{135}$

Show your working and make your method clear.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{8}{(2 - 5x)^2}$$

writing each term in simplest form.

(4)

- (b) Find the range of values of x for which this expansion is valid.

(1)

Leave
blank

1. Given that k is a constant and the binomial expansion of

$$\sqrt{1 + kx} \quad |kx| < 1$$

in ascending powers of x up to the term in x^3 is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

- (a) (i) find the value of k ,

(ii) find the value of the constant A and the constant B .

(5)

- (b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

(2)

- 4: (a) Find, in ascending powers of x , the first 4 terms of the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}$$

giving each term in simplest form.

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

Using the expansion from part (a),

- (c) (i) state the first 4 terms of the binomial series expansion of $(4 - 5x)^{\frac{1}{2}}$

(ii) show that, if x is sufficiently small,

$$(4 + 5x)^{\frac{1}{2}} + (4 - 5x)^{\frac{1}{2}} \approx a + bx^2$$

where a and b are constants to be found.

(3)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} \quad |x| < \frac{1}{2}$$

giving each term in simplest form.

(5)

Given that

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^n \left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = \left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$$

- (b) write down the value of n .

(1)

- (c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of x , of

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} \quad |x| < \frac{1}{2}$$

giving each term in simplest form.

(3)

3. Given that the binomial expansion, in ascending powers of x , of

$$6(4 + Ax)^{-\frac{1}{2}}$$

is $B - \frac{1}{4}x + Cx^2 + \dots$

- (a) find the values of the constants A , B and C .

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

For the expansion,

- (c) find the coefficient of the term in x^3

(2)

1. Find, in ascending powers of x up to and including the term in x^3 , the binomial expansion of

$$(1 - 4x)^{-3} \quad |x| < \frac{1}{4}$$

fully simplifying each term.

(4)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} \quad |x| < \frac{1}{2}$$

giving each term in simplest form.

(5)

Given that

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^n \left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = \left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$$

- (b) write down the value of n .

(1)

- (c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of x , of

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} \quad |x| < \frac{1}{2}$$

giving each term in simplest form.

(3)

- 4: (a) Find, in ascending powers of x , the first 4 terms of the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}$$

giving each term in simplest form.

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

Using the expansion from part (a),

- (c) (i) state the first 4 terms of the binomial series expansion of $(4 - 5x)^{\frac{1}{2}}$

- (ii) show that, if x is sufficiently small,

$$(4 + 5x)^{\frac{1}{2}} + (4 - 5x)^{\frac{1}{2}} \approx a + bx^2$$

where a and b are constants to be found.

(3)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\frac{2}{\sqrt{9-2x}} \quad |x| < \frac{9}{2}$$

giving each coefficient as a simplified fraction.

(5)

By substituting $x = 1$ into the answer to part (a),

- (b) find an approximation for $\sqrt{7}$, giving your answer to 4 decimal places.

(2)

1. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \quad |x| < \frac{1}{20}$$

giving each coefficient in its simplest form.

(5)

By substituting $x = \frac{1}{100}$ into the answer for (a),

- (b) find an approximation for $\sqrt{5}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers to be found.

(2)

2. (a) Find, in ascending powers of x , the first three non-zero terms of the binomial series expansion of

$$\sqrt[3]{1+4x^3} \quad |x| < \sqrt[3]{4}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the expansion from part (a) with $x = \frac{1}{3}$ to find a rational approximation to $\sqrt[3]{31}$

(3)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8-3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8 - 3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

2. (a) Use the binomial expansion to expand

$$(4 - 5x)^{-\frac{1}{2}} \quad |x| < \frac{4}{5}$$

in ascending powers of x , up to and including the term in x^2 giving each coefficient as a fully simplified fraction.

(4)

$$f(x) = \frac{2 + kx}{\sqrt{4 - 5x}} \quad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of $f(x)$, in ascending powers of x , is

$$1 + \frac{3}{10}x + mx^2 + \dots \quad \text{where } m \text{ is a constant}$$

- (b) find the value of k ,

(2)

- (c) find the value of m .

(2)

1. Find, in ascending powers of x up to and including the term in x^3 , the binomial expansion of

$$(1 - 4x)^{-3} \quad |x| < \frac{1}{4}$$

fully simplifying each term.

(4)

1.

$$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

- (a) Write $f(x)$ in partial fraction form.

(3)

- (b) (i) Hence find, in ascending powers of x up to and including the terms in x^2 , the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.

(5)

- (ii) Find the range of values of x for which this expansion is valid.

(1)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8 - 3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

4.

$$g(x) = \frac{1}{\sqrt{4 - x^2}}$$

- (a) Find, in ascending powers of x , the first four non-zero terms of the binomial expansion of $g(x)$. Give each coefficient in simplest form.

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

- (c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear.

(2)