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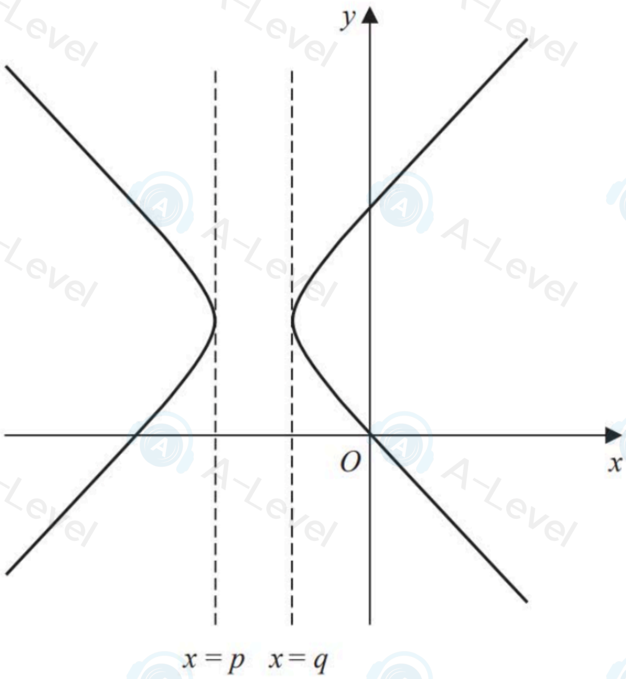

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

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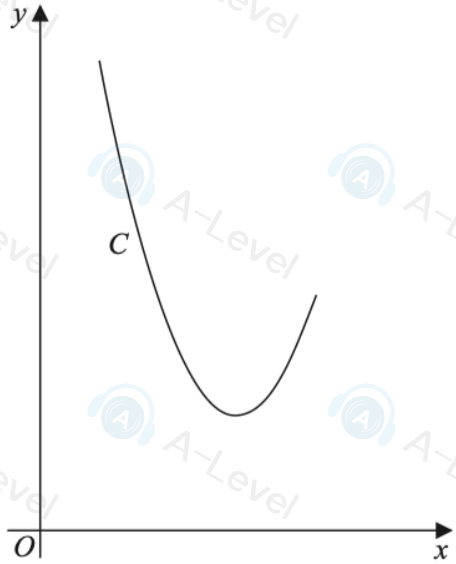

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

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6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

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1. The curve C has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C.

Find the equation of the normal to C at the point P, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(7)

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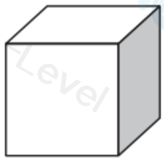

x cm

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is $V\,\text{cm}^3$

Given that the volume of the cube is increasing at a constant rate of $5\,\text{cm}^3\text{s}^{-1}$

(a) find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ giving your answer in terms of x.

(3)

(b) Hence find the value of $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ when $V = 64$

(2)

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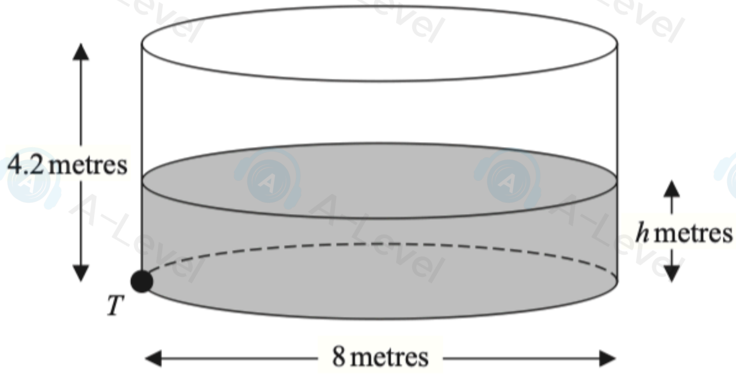

Figure 4

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\,\text{m}^3$ per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)\,\text{m}^3$ per minute

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

1. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$2y^2 - 6xy = 7\mathrm{e}^{2x-1} + 13$$

The point P with x coordinate $\frac{1}{2}$ lies on C.

(a) Find the two possible y coordinates of P.

(2)

Given that P lies above the x-axis,

(b) find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

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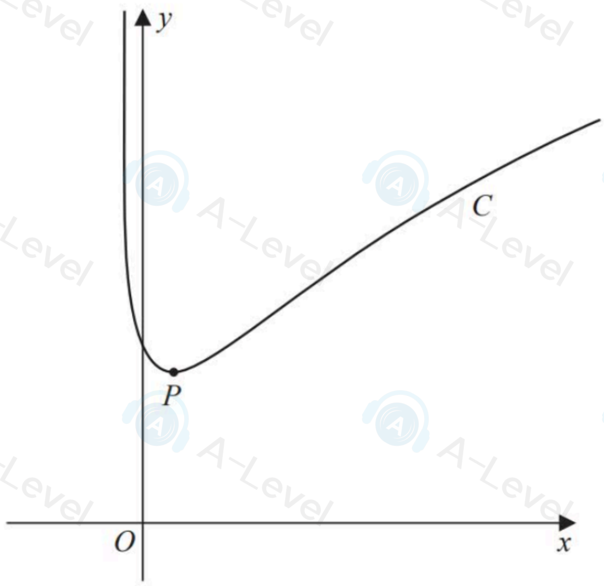

Figure 1

The curve C, shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \qquad y > 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

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The point $P(p, 2)$, where p is a constant, lies on C.

Given that P is the minimum turning point on C,

(b) find

(i) the value of p

(ii) the value of k

(4)

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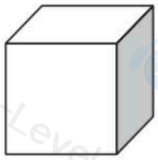

x cm

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the surface area of the cube is $S\,\text{cm}^2$
- the volume of the cube is $V\,\text{cm}^3$

Given that the surface area of the cube is increasing at a constant rate of $4\,\text{cm}^2\,\text{s}^{-1}$

(a) show that $\dfrac{dx}{dt} = \dfrac{k}{x}$ where k is a constant to be found,

(4)

(b) show that $\dfrac{dV}{dt} = V^p$ where p is a constant to be found.

(3)

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5. A curve has equation

$$y^2 = y\mathrm{e}^{-2x} - 3x$$

(a) Show that

$$\frac{dy}{dx} = \frac{2y\mathrm{e}^{-2x} + 3}{\mathrm{e}^{-2x} - 2y}$$

(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

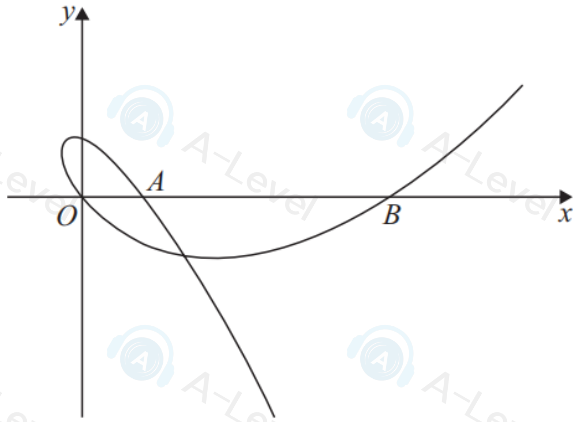

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \qquad y = t^3 - 9t \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Find the equation of the tangent to the curve at B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The tangent to the curve at B cuts the curve again at point X.

(c) Use algebra to find the coordinates of X, showing each stage of your working.

(5)

(Total for Question 8 is 12 marks)

2.

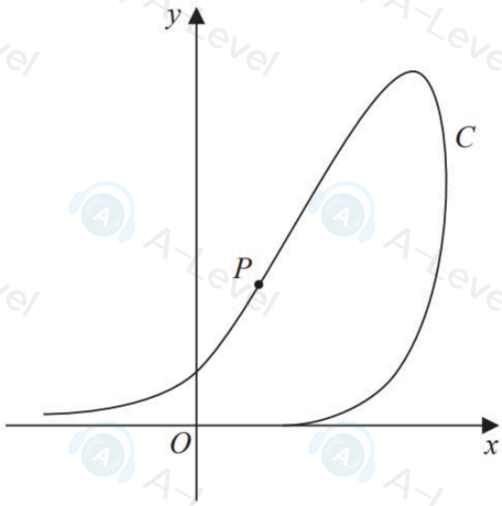

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P.

(2)

(b) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\frac{a \ln 2 + b}{c \ln 2 + d}$ where a, b, c and d are integers to be found.

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3. The curve C is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to C at the point (2, 5), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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3: Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y\frac{dy}{dx} = xe^{-y^2} \qquad y \geqslant 0 \quad x \geqslant 5$$

giving your answer in the form $y^2 = f(x)$

(5)

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2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \qquad y > 0$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y

(6)

Curve C cuts the y-axis at the point P

(b) Find the exact value of the gradient of the tangent to C at P

(2)

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leqslant t \leqslant 5$$

(a) Show that $\frac{dy}{dx} = 3$ where $t = \pi$

(4)

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found.

(3)

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$

(2)

(d) Hence find the exact value of the y coordinate of Q

(3)

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3. The curve C is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to C at the point $(2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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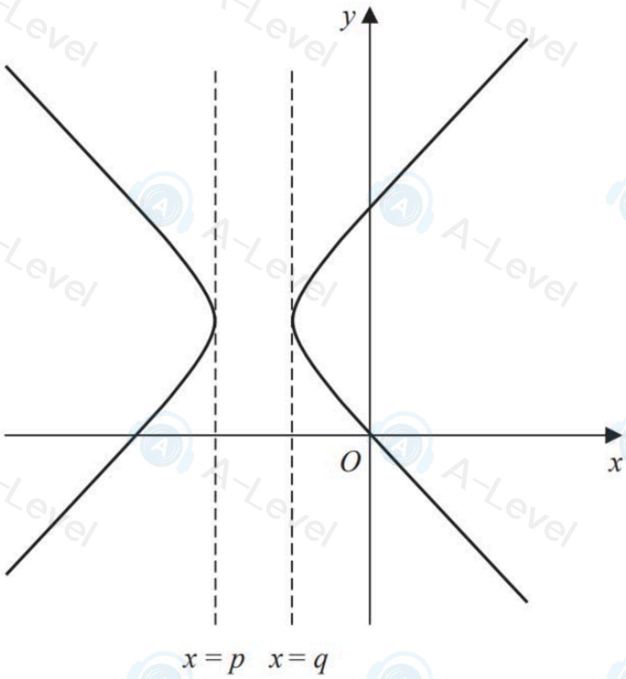

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

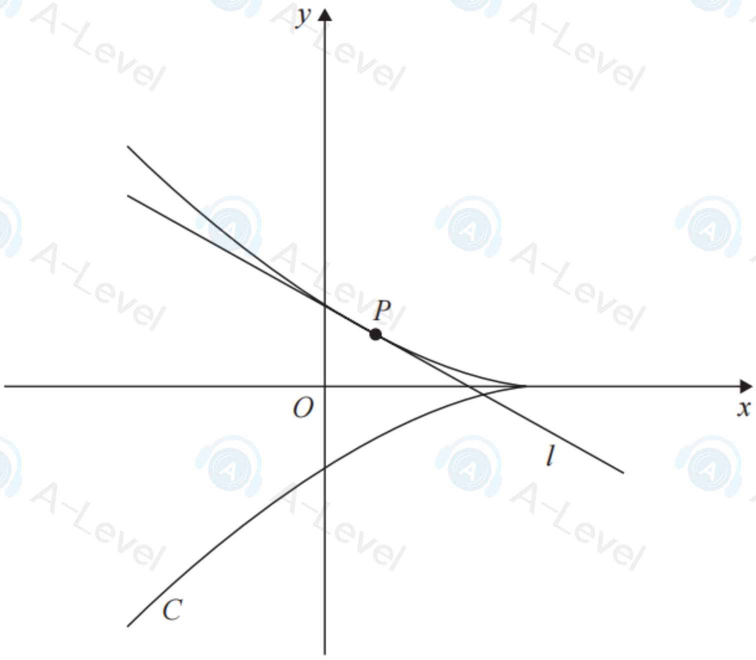

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

1. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$2y^2 - 6xy = 7e^{2x-1} + 13$$

The point P with x coordinate $\frac{1}{2}$ lies on C.

(a) Find the two possible y coordinates of P.

(2)

Given that P lies above the x-axis,

(b) find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

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2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, V cm^3, is decreasing at a constant rate, k cm^3 per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)

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8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

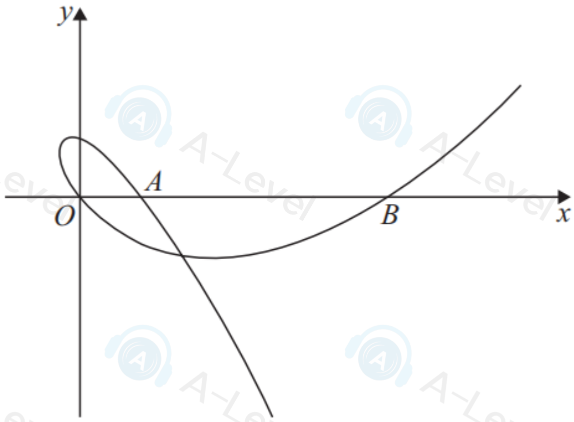

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \qquad y = t^3 - 9t \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Find the equation of the tangent to the curve at B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The tangent to the curve at B cuts the curve again at point X.

(c) Use algebra to find the coordinates of X, showing each stage of your working.

(5)

(Total for Question 8 is 12 marks)

5.

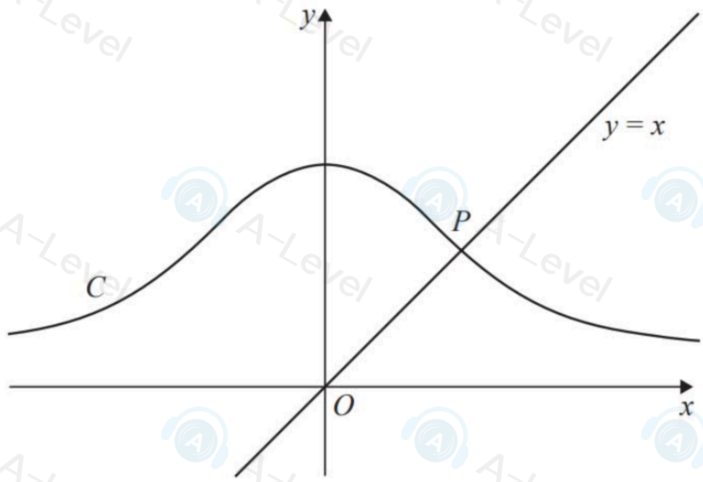

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

The point P lies on C.

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

(b) find the value of k.

(5)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\dfrac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

3. $$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A, B, C and D.

(5)

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g.

(1)

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4. **In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.**

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

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1. The curve C has equation

$$xy^2 = x^2y + 6 \qquad x \neq 0 \quad y \neq 0$$

Find an equation for the tangent to C at the point $P(2, 3)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

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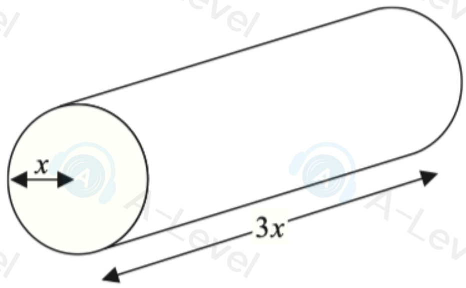

Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5\,\text{mm}^2\,\text{s}^{-1}$

(a) Find $\dfrac{dx}{dt}$ when $x = 7$ **(4)**

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$ **(4)**

4.

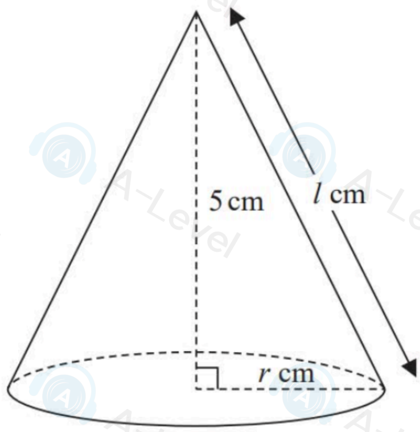

Figure 2

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r **(1)**

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi r l$]

(4)

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6. A curve C has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x \ln x = 0$$

(2)

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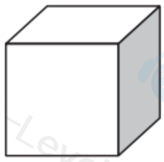

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is V cm^3

Given that the volume of the cube is increasing at a constant rate of 5 cm^{3}s^{-1}

(a) find $\frac{dx}{dt}$ giving your answer in terms of x.

(3)

(b) Hence find the value of $\frac{dx}{dt}$ when $V = 64$

(2)

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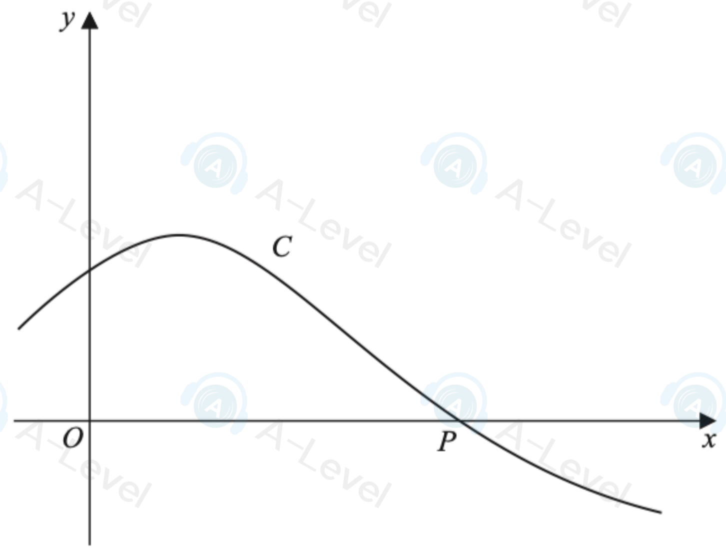

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the x-axis at point P, as shown in Figure 3.

(a) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

The curve C has equation $y = \mathrm{f}(x)$, where f is a function with domain $\left[k,\ 1 + 3\sqrt{3}\right]$

(b) Find the exact value of the constant k.

(1)

(c) Find the range of f.

(2)

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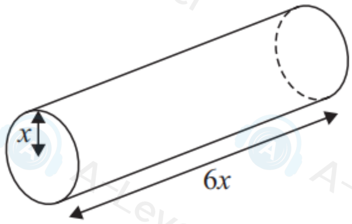

Figure 1

Figure 1 shows a right circular cylindrical rod which is expanding as it is heated.

At time t seconds the radius of the rod is x cm and the length of the rod is $6x$ cm.

Given that the **cross-sectional area** of the rod is increasing at a constant rate of $\frac{\pi}{20}\text{cm}^2\,\text{s}^{-1}$, find the rate of increase of the volume of the rod when $x = 2$

Write your answer in the form $k\pi\,\text{cm}^3\,\text{s}^{-1}$ where k is a rational number.

(6)

(Total for Question 5 is 6 marks)

5.

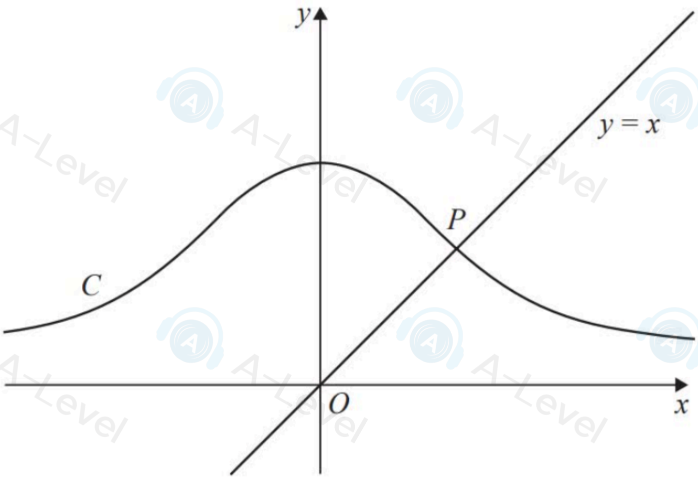

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P lies on C.

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

(b) find the value of k.

(5)

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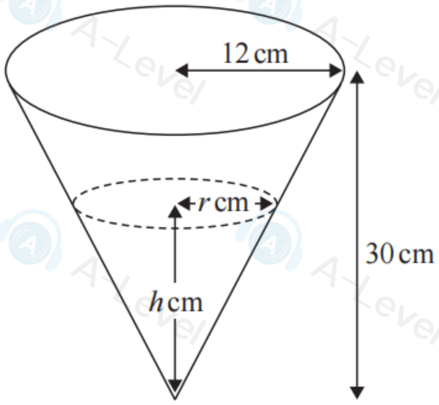

Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is $V\,\text{cm}^3$

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of $2\pi\,\text{cm}^3\,\text{s}^{-1}$

(b) find, in $\text{cm}\,\text{s}^{-1}$, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

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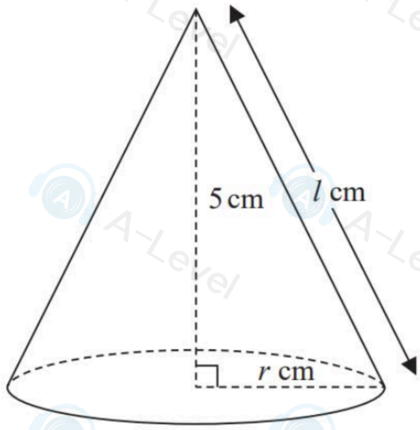

Figure 2

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r

(1)

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi rl$]

(4)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

4.

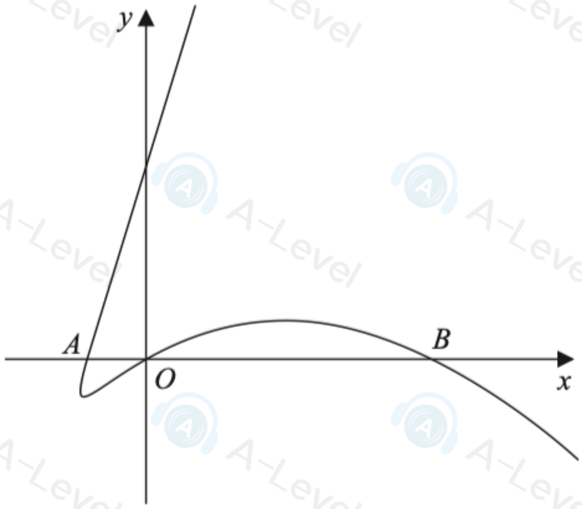

Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

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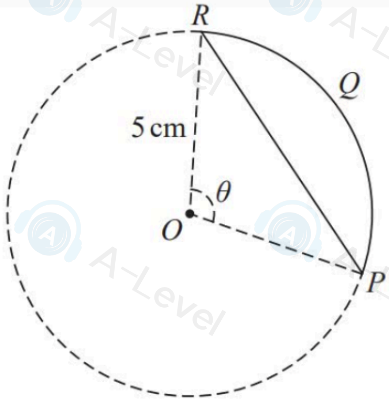

Figure 1

Figure 1 shows a sketch of a segment $PQRP$ of a circle with centre O and radius 5 cm.

Given that

- angle POR is θ radians
- θ is increasing, from 0 to π, at a constant rate of 0.1 radians per second
- the area of the segment $PQRP$ is A cm^2

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K(1 - \cos\theta)$$

where K is a constant to be found.

(2)

(b) Find, in cm^2 s^{-1}, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

(4)

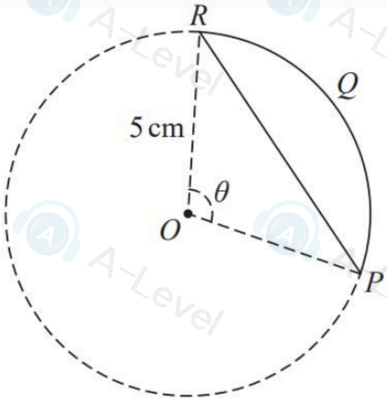

Figure 1

Figure 1 shows a sketch of a segment $PQRP$ of a circle with centre O and radius 5 cm.

Given that

- angle POR is θ radians
- θ is increasing, from 0 to π, at a constant rate of 0.1 radians per second
- the area of the segment $PQRP$ is $A\text{ cm}^2$

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K\left(1 - \cos\theta\right)$$

where K is a constant to be found.

(2)

(b) Find, in $\text{cm}^2\,\text{s}^{-1}$, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

(4)

9.

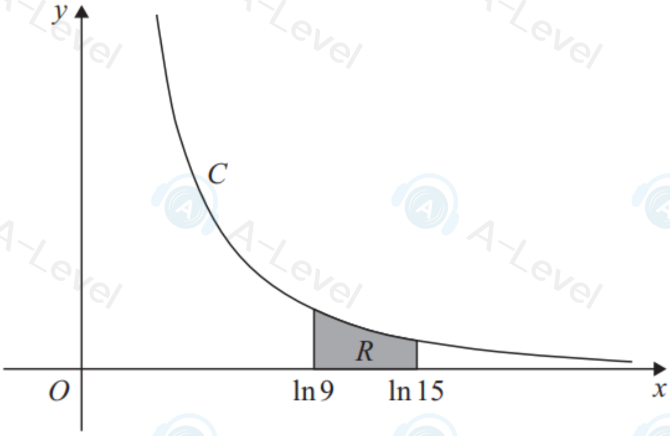

Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t+1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P.

(6)

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t+1)(2t+5)}\,\mathrm{d}t$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number.

(9)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

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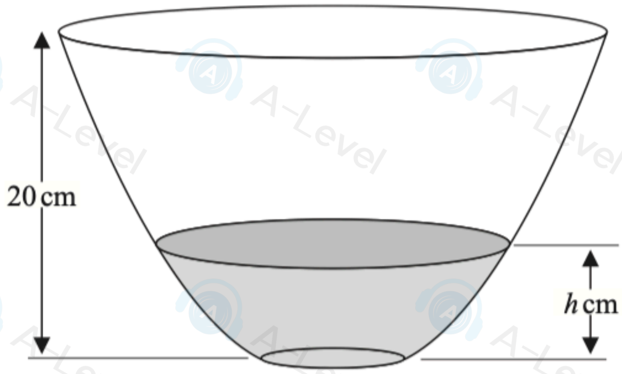

Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2(h + 4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm^3 s^{-1}, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

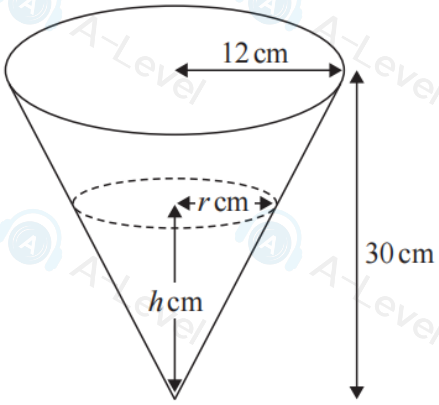

Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of 2π cm^3 s^{-1}

(b) find, in cm s^{-1}, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

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3. The curve C has equation

$$3y^2 - 11x^2 + 11xy = 20y - 36x + 28$$

(a) Find, in simplest form, $\frac{dy}{dx}$ in terms of x and y.

(5)

The point $P(4, k)$, where k is a constant, lies on C.

Given that $k < 0$

(b) find the value of the gradient of C at P

(5)

5.

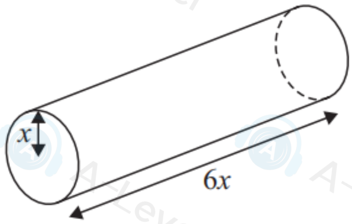

Figure 1

Figure 1 shows a right circular cylindrical rod which is expanding as it is heated.

At time t seconds the radius of the rod is x cm and the length of the rod is $6x$ cm.

Given that the **cross-sectional area** of the rod is increasing at a constant rate of $\frac{\pi}{20}\text{cm}^2\,\text{s}^{-1}$, find the rate of increase of the volume of the rod when $x = 2$

Write your answer in the form $k\pi\,\text{cm}^3\,\text{s}^{-1}$ where k is a rational number.

(6)

(Total for Question 5 is 6 marks)

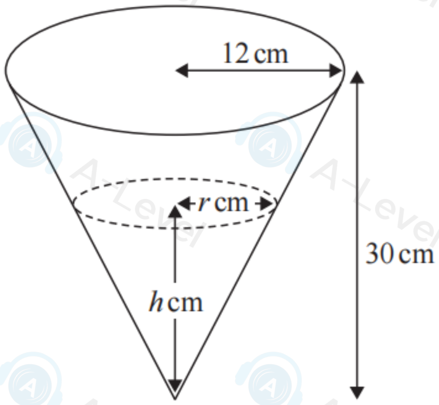

Figure 3

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The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is $V\,\text{cm}^3$

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of $2\pi\,\text{cm}^3\,\text{s}^{-1}$

(b) find, in $\text{cm}\,\text{s}^{-1}$, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, V cm^3, is decreasing at a constant rate, k cm^3 per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)

2.

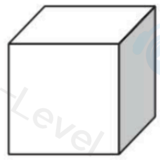

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the surface area of the cube is S cm^2
- the volume of the cube is V cm^3

Given that the surface area of the cube is increasing at a constant rate of 4 cm^2 s^{-1}

(a) show that $\frac{dx}{dt} = \frac{k}{x}$ where k is a constant to be found,

(4)

(b) show that $\frac{dV}{dt} = V^p$ where p is a constant to be found.

(3)

3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

(Total for Question 3 is 7 marks)

2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \qquad y > 0$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y **(6)**

Curve C cuts the y-axis at the point P

(b) Find the exact value of the gradient of the tangent to C at P **(2)**

9.

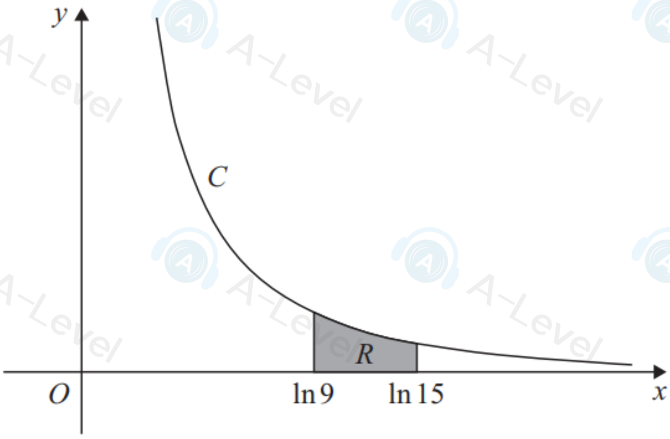

Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t + 1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P. **(6)**

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t + 1)(2t + 5)}\,dt$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number. **(9)**

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9. (a) Find the derivative with respect to y of

$$\frac{1}{(1+2\ln y)^2}$$

(2)

(b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3 \qquad y > 0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = e^{A\sec x - \frac{1}{2}}$$

where A is an irrational number to be found.

(5)

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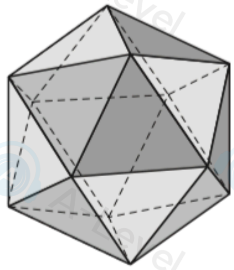

Figure 1

A regular icosahedron of side length x cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length x cm.

(a) Show that the surface area, A cm^2, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

(2)

Given that the volume, V cm^3, of the icosahedron is given by

$$V = \frac{5}{12}\left(3 + \sqrt{5}\right)x^3$$

(b) show that $\dfrac{dV}{dA} = \dfrac{\left(3 + \sqrt{5}\right)x}{8\sqrt{3}}$

(3)

The surface area of the icosahedron is increasing at a constant rate of $0.025\,\text{cm}^2\,\text{s}^{-1}$

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

(3)

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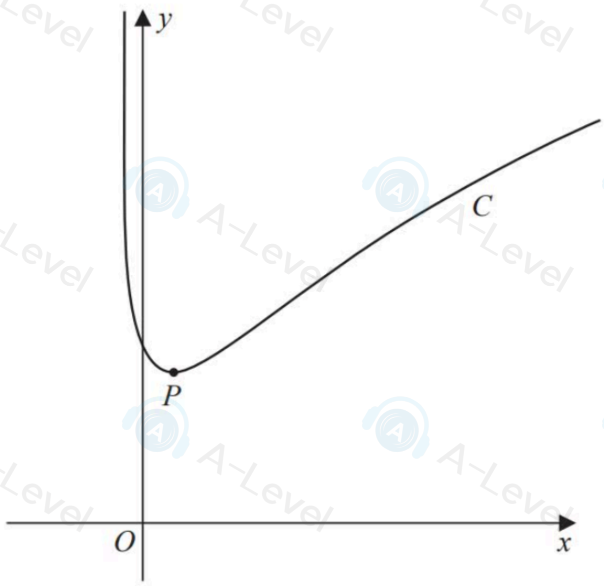

Figure 1

The curve C, shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \qquad y > 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

The point $P(p, 2)$, where p is a constant, lies on C.

Given that P is the minimum turning point on C,

(b) find

(i) the value of p

(ii) the value of k

(4)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

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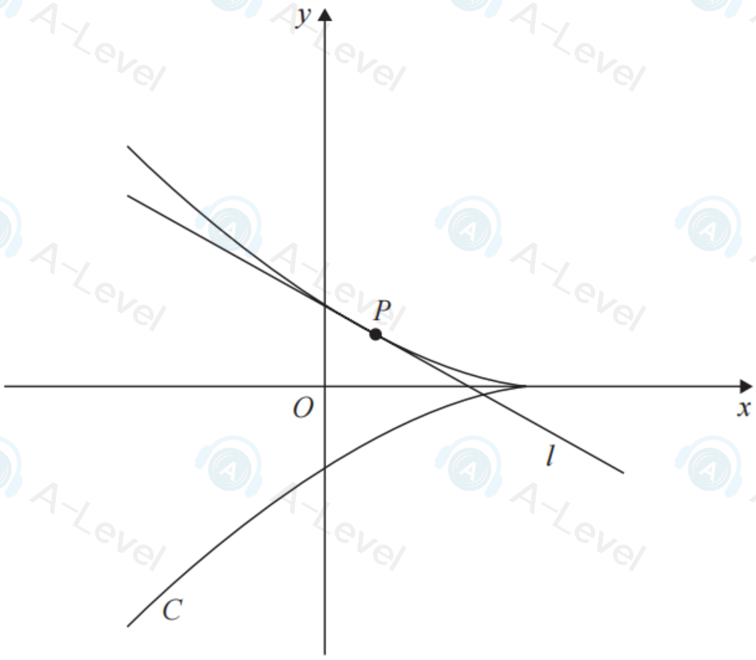

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \dfrac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\dfrac{dy}{dx} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

3: Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y\frac{dy}{dx} = xe^{-y^2} \qquad y \geqslant 0 \quad x \geqslant 5$$

giving your answer in the form $y^2 = f(x)$

(5)

11.

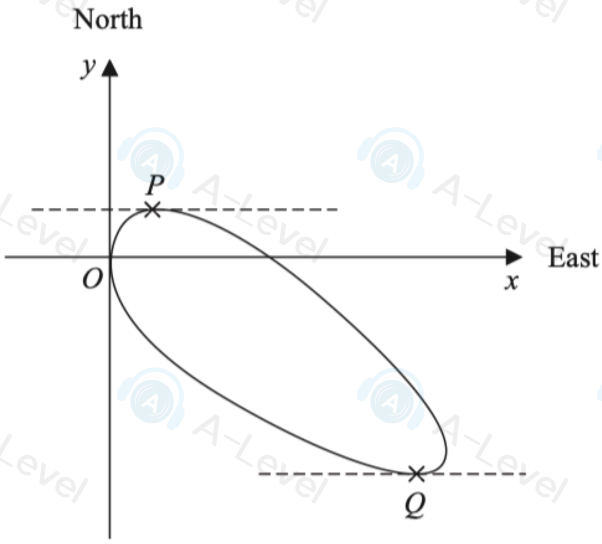

Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x+y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$$

(5)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest north and furthest south of the origin O, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point Q is south of O. Give your answer to the nearest 100 m.

(4)

2.

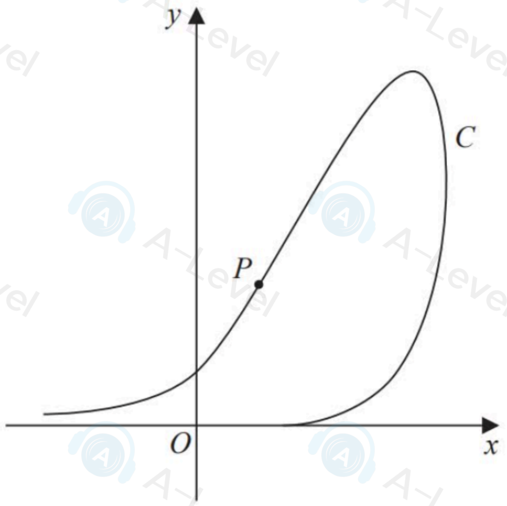

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P.

(2)

(b) Find $\dfrac{dy}{dx}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\dfrac{a\ln 2 + b}{c\ln 2 + d}$ where a, b, c and d are integers to be found.

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3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\dfrac{dy}{dx}$ at the point on C with coordinates (2, 3). Give your answer in the form $\dfrac{a + \ln b}{8}$, where a and b are integers.

(7)

(Total for Question 3 is 7 marks)