

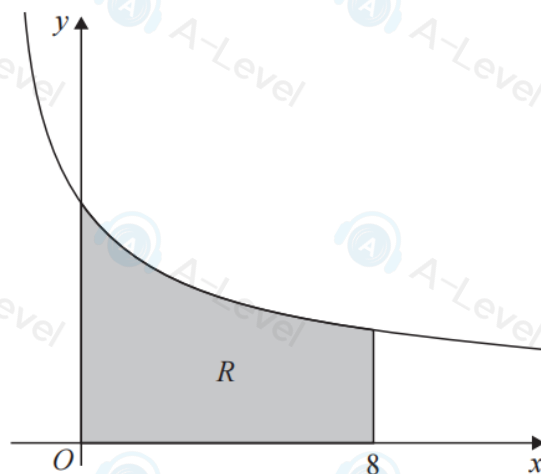


Figure 1

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

The curve shown in Figure 1 has equation

$$y = \frac{4}{x+2} \quad x > -2$$

The region R , bounded by the curve, the y -axis, the x -axis and the line with equation $x = 8$ is shown shaded in Figure 1

Region R is rotated through 360 degrees about the x -axis.

Use calculus to find the exact value of the volume of the solid generated, writing your answer in simplest form.

(5)

7.

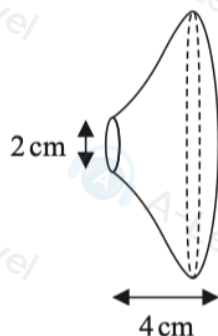


Figure 3

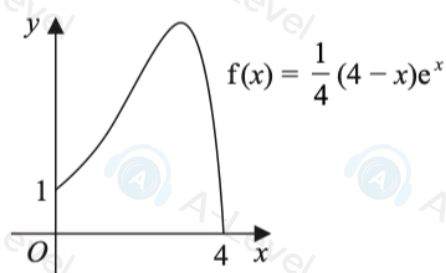


Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x -axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4 - x)e^x \quad 0 \leq x \leq 4$$

(a) Show that the volume, $V \text{ cm}^3$, of the doorknob is given by

$$V = K \int_0^4 (x^2 - 8x + 16)e^{2x} dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^q + r) \text{ cm}^3$ where p , q and r are simplified rational numbers to be found.

(5)

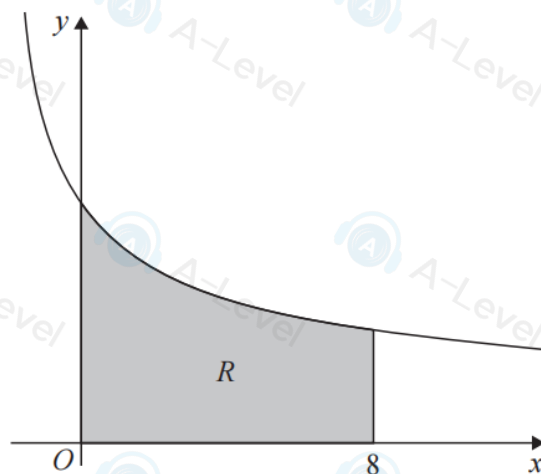


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Region R is rotated through 360 degrees about the x -axis.

Use calculus to find the exact value of the volume of the solid generated, writing your answer in simplest form.

(5)

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1.

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Find

$$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx$$

giving your answer in simplest form.

(5)

DO NOT WRITE IN THIS AREA

4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx$$

may be expressed in the form

$$\int_a^b k u^2 e^u du$$

where a , b and k are constants to be found.

(4)

- (b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx$$

giving your answer in simplest form.

(5)

3.

$$f(x) = \frac{8x-5}{(2x-1)(4x-3)} \quad x > 1$$

- (a) Express $f(x)$ in partial fractions.

(3)

- (b) Hence find $\int f(x) dx$

(3)

- (c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x) dx = \frac{1}{2} \ln 20$$

(5)

2.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \quad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \quad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

9.

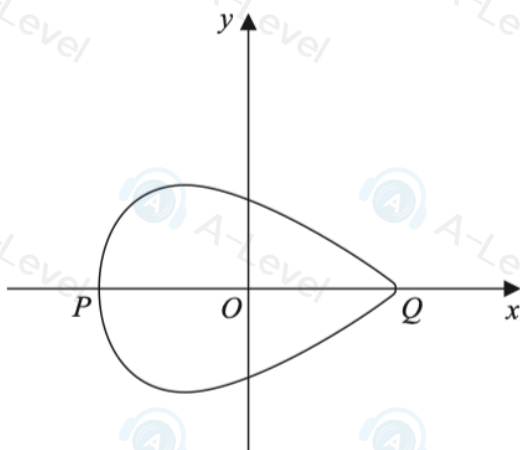


Figure 1

Figure 1 shows a sketch of a closed curve with parametric equations

$$x = 5 \cos \theta \quad y = 3 \sin \theta - \sin 2\theta \quad 0 \leq \theta < 2\pi$$

The region enclosed by the curve is rotated through π radians about the x -axis to form a solid of revolution.

- (a) Show that the volume, V , of the solid of revolution is given by

$$V = 5\pi \int_{\alpha}^{\beta} \sin^3 \theta (3 - 2 \cos \theta)^2 d\theta$$

where α and β are constants to be found.

(4)

- (b) Use the substitution $u = \cos \theta$ and algebraic integration to show that $V = k\pi$ where k is a rational number to be found.

(7)

4.

$$f(x) = \frac{4 - 4x}{x(x - 2)^2} \quad x > 2$$

- (a) Express $f(x)$ in partial fractions.

(4)

- (b) Hence find $\int f(x) dx$

(3)

- (c) Find

$$\int_3^5 f(x) dx$$

giving your answer in the form $a + \ln b$, where a and b are rational numbers to be found.

(2)

2.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of $\int_1^e \frac{\ln x}{x^2} dx$

Write your answer in the form $a + \frac{b}{e}$, where a and b are integers.

(6)

(Total for Question 2 is 6 marks)

5.

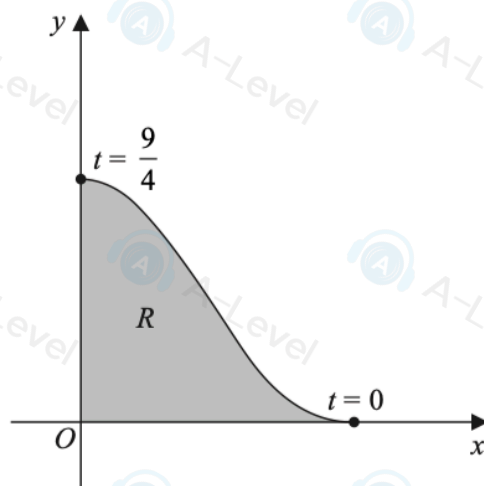


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \quad y = \frac{t^3}{\sqrt{9 + 4t}} \quad 0 \leq t \leq \frac{9}{4}$$

The curve touches the x -axis when $t = 0$ and meets the y -axis when $t = \frac{9}{4}$

The region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the y -axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R .

(Solutions relying on calculator technology are not acceptable.)

(6)

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5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y} \right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)

3.

$$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \quad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x) \, dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x) \, dx = \frac{1}{2} \ln 20$$

(5)

7.

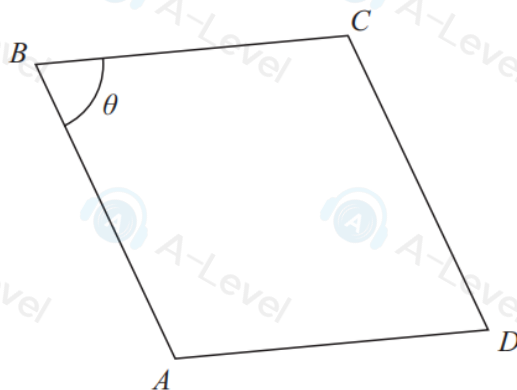


Figure 2

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\vec{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\vec{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90^\circ$

(a) find the exact value of $\cos \theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

9.

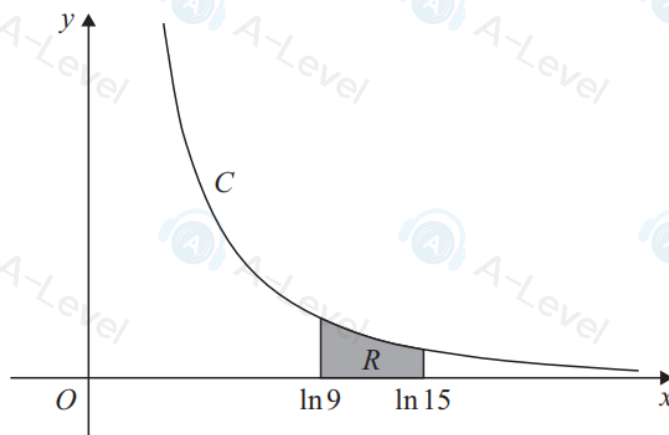


Figure 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t+1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

6.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta^\circ\text{C}$, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85°C at the instant the engine is turned off
- is 40°C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a , b , c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20°C . Give your answer to the nearest minute.

(2)

7. (a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

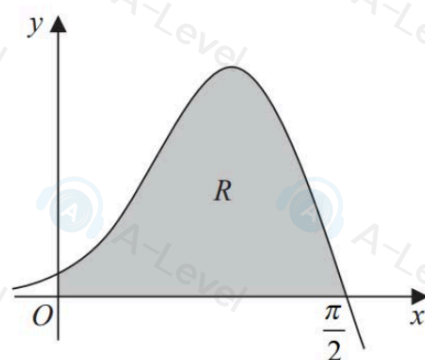


Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x} \cos x$$

The region R , shown shaded in Figure 3, is bounded by the positive x -axis, the positive y -axis and the curve.

The region R is rotated through 2π radians about the x -axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

7.

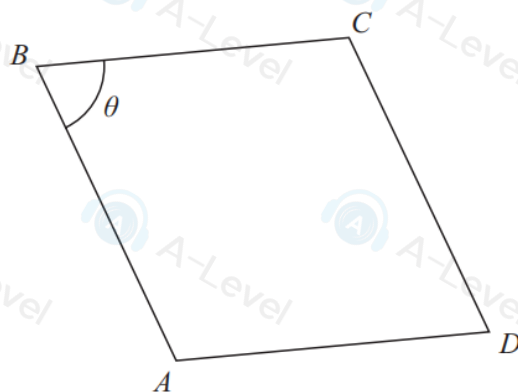


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- $\vec{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90^\circ$

(a) find the exact value of $\cos \theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

6.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

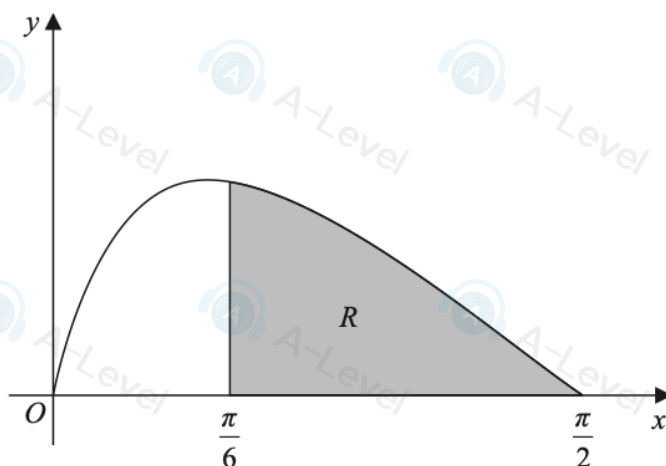


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16 \sin 2x}{(3 + 4 \sin x)^2} \quad 0 \leq x \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4 \sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

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7:

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

- (a) Show that the substitution $u = e^x - \frac{1}{2}$ transforms

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx$$

to

$$\int_a^b \frac{12u + 5}{u^2(2u + 1)} du$$

where a and b are constants to be found.

(4)

- (b) Find an expression for $\frac{12u + 5}{u^2(2u + 1)}$ in the form

$$\frac{A}{u^2} + \frac{B}{u} + \frac{C}{(2u + 1)}$$

where A , B and C are integers.

(3)

- (c) Hence show that

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx = p + \ln q$$

where p and q are constants to be found.

(4)

6. Use integration by parts to show that

$$\int e^{2x} \cos 3x dx = pe^{2x} \sin 3x + qe^{2x} \cos 3x + k$$

where p and q are rational numbers to be found and k is an arbitrary constant.

(6)

blank

5. (i) Find

$$\int x^2 e^x dx$$

(4)

(ii) Use the substitution $u = \sqrt{1 - 3x}$ to show that

$$\int \frac{27x}{\sqrt{1 - 3x}} dx = -2(1 - 3x)^{\frac{1}{2}}(Ax + B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

7. The current, x amps, at time t seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where k is a constant.

Initially there is zero current in the circuit.

- (a) Solve the differential equation to find an equation, in terms of k , for the current in the circuit at time t seconds.
Give your answer in the form $x = f(t)$.

(6)

Given that in the long term the current in the circuit approaches 7 amps,

- (b) find the value of k .

(2)

- (c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

(3)

7.

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Use the substitution $x = 4 \sin \theta$ to find the exact value of

$$\int_2^{2\sqrt{3}} \frac{1}{(16 - x^2)^{\frac{3}{2}}} dx$$

(6)

6.

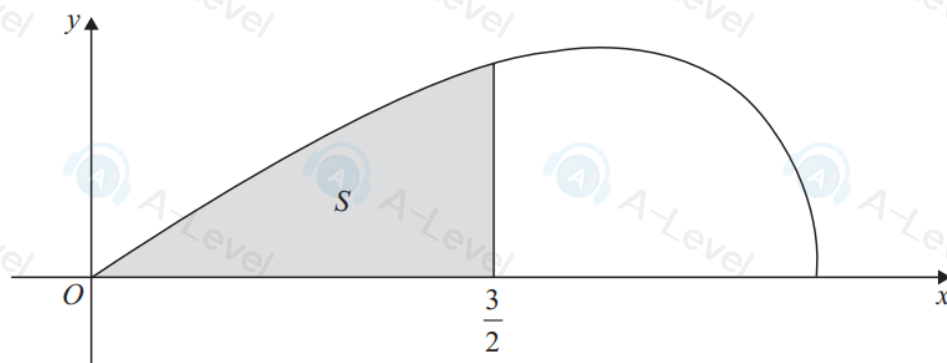


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 3 \sin t \quad y = 2 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The finite region S , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line with the equation $x = \frac{3}{2}$

The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution is given by

$$k \int_0^a \sin^2 t \cos^3 t \, dt$$

where k and a are constants to be given in terms of π .

(4)

(b) Use the substitution $u = \sin t$, or otherwise, to find the exact value of this volume, giving your answer in the form $\frac{p\pi}{q}$ where p and q are integers.

(Solutions relying entirely on calculator technology are not acceptable.)

(5)

(Total for Question 6 is 9 marks)

5. (i) Find

$$\int x^2 e^{4x} \, dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} \, dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

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9. Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, $A \text{ cm}^2$, covered by the bacteria, t days after the bacteria start to grow, is modelled by the differential equation

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{5t^2} \quad t > 0$$

Given that $A = 2.25$ when $t = 3$

- (a) show that

$$A = \left(\frac{pt}{qt + r} \right)^2$$

where p , q and r are integers to be found.

(7)

According to the model, there is a limit to the area that will be covered by the bacteria.

- (b) Find the value of this limit.

(2)

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4.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{5 + 17x - 10x^2}{x(1 - x)(2x + 1)} \quad x > 1$$

- (a) Write $f(x)$ in the form

$$\frac{A}{x} + \frac{B}{1 - x} + \frac{C}{2x + 1}$$

where A , B and C are constants to be found.

(3)

- (b) Hence, use algebraic integration to find the exact value of

$$\int_2^4 f(x) \, dx$$

Write your answer in the form $p \ln 2 + q \ln \left(\frac{3}{5} \right)$, where p and q are integers to be found.

(5)

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6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = 2(x^3 + 1)^k (x^3 - A) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

7. The volume $V \text{ cm}^3$ of a spherical balloon with radius $r \text{ cm}$ is given by the formula

$$V = \frac{4}{3} \pi r^3$$

- (a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t + 3)^2} \quad t \geq 0$$

Given that $V = 0$ when $t = 0$

- (b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t + 3}$$

- (ii) Hence find the upper limit to the volume of the balloon.

(5)

- (c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

- (d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

7. Water is flowing into a large container and is leaking from a hole at the base of the container.

At time t seconds after the water starts to flow, the volume, $V \text{ cm}^3$, of water in the container is modelled by the differential equation

$$\frac{dV}{dt} = 300 - kV$$

where k is a constant.

- (a) Solve the differential equation to show that, according to the model,

$$V = \frac{300}{k} + Ae^{-kt}$$

where A is a constant.

(5)

Given that the container is initially empty and that when $t = 10$, the volume of water is increasing at a rate of $200 \text{ cm}^3 \text{ s}^{-1}$

- (b) find the exact value of k .

(4)

- (c) Hence find, according to the model, the time taken for the volume of water in the container to reach 6 litres. Give your answer to the nearest second.

(2)

9.

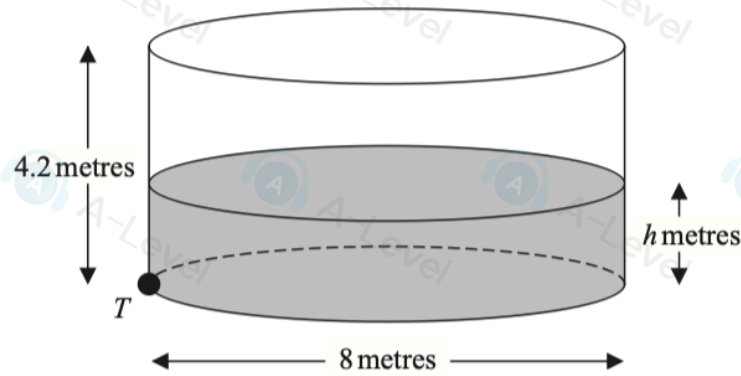
**Figure 4**

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\text{ m}^3$ per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)\text{ m}^3$ per minute

(a) Show that

$$\frac{dh}{dt} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

7. (a) Express $\frac{1}{(4-x)(2-x)}$ in partial fractions. (2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x) \quad t \geq 0 \quad 0 \leq x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

- (b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

(Solutions relying on calculator technology are not acceptable.)

(6)

Given that $k = 0.1$

- (c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures. (2)

(Total for Question 7 is 10 marks)

2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \quad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

(6)

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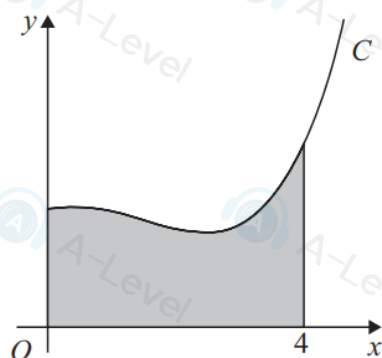


Figure 1

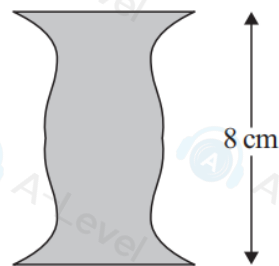


Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^x$$

The finite region, shown shaded in Figure 1, is bounded by C , the y -axis, the x -axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x -axis to form a solid S .

(a) Show that the exact volume of S is given by

$$\pi \int_0^4 \left(A + B \cos 2x + Ce^x \cos x + De^{2x} \right) dx$$

where A , B , C and D are constants to be found.

(4)

(b) Find $\int e^x \cos x \, dx$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together. The paperweight is 8 cm high.

(c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

6.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta^{\circ}\text{C}$, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85°C at the instant the engine is turned off
- is 40°C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a , b , c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20°C . Give your answer to the nearest minute.

(2)

6.

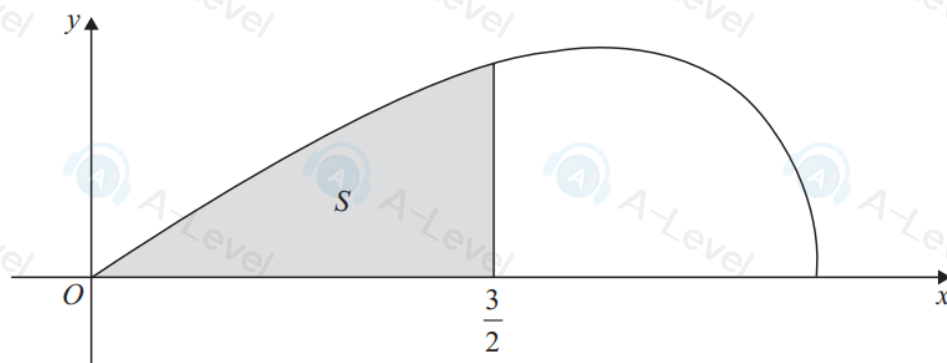


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 3 \sin t \quad y = 2 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The finite region S , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line with the equation $x = \frac{3}{2}$

The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution is given by

$$k \int_0^a \sin^2 t \cos^3 t \, dt$$

where k and a are constants to be given in terms of π .

(4)

(b) Use the substitution $u = \sin t$, or otherwise, to find the exact value of this volume, giving your answer in the form $\frac{p\pi}{q}$ where p and q are integers.

(Solutions relying entirely on calculator technology are not acceptable.)

(5)

(Total for Question 6 is 9 marks)

7. (a) Show that the substitution $u = \tan x$ transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} dx$$

to

$$\int \frac{u}{(k + u^2)^n} du$$

where k and n are integers to be found.

(4)

- (b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} dx$$

(2)

5.

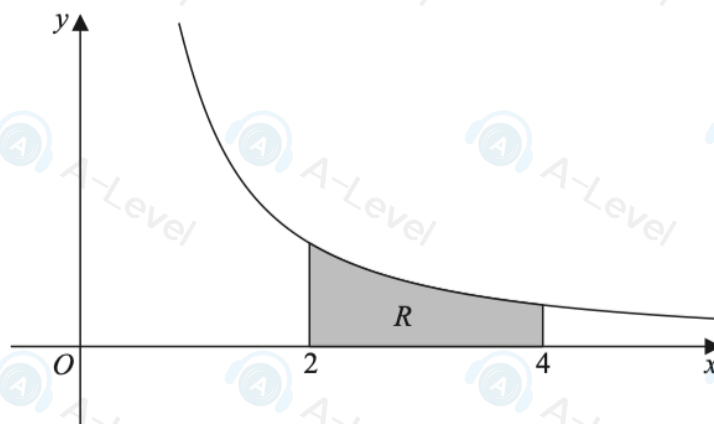


Figure 3

- (a) Find $\int \frac{\ln x}{x^2} dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \quad x > 0.5$$

The finite region R , shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$

- (b) Use the answer to part (a) to find the exact area of R , writing your answer in simplest form.

(4)

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3.

In this question you must show all stages of your working.

Solutions based on calculator technology are not acceptable.

- (i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x} dx$$

giving your answer in simplest form.

(5)

- (ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2} dx = a + \ln b$$

where a and b are constants to be found.

(7)

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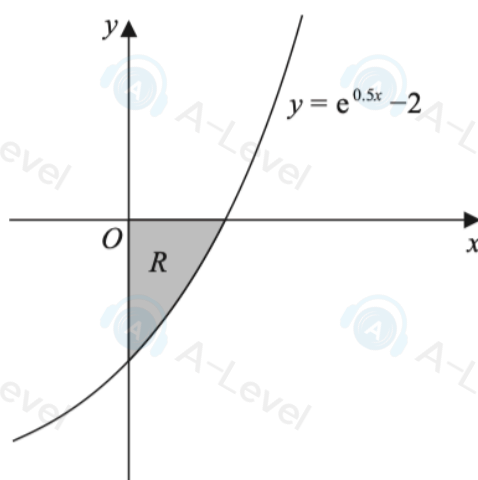


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{0.5x} - 2$ The region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the y -axis.The region R is rotated 360° about the x -axis to form a solid of revolution.Show that the volume of this solid can be written in the form $a \ln 2 + b$, where a and b are constants to be found.

(6)

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7:

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

- (a) Show that the substitution $u = e^x - \frac{1}{2}$ transforms

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx$$

to

$$\int_a^b \frac{12u + 5}{u^2(2u + 1)} du$$

where a and b are constants to be found.

(4)

- (b) Find an expression for $\frac{12u + 5}{u^2(2u + 1)}$ in the form

$$\frac{A}{u^2} + \frac{B}{u} + \frac{C}{(2u + 1)}$$

where A , B and C are integers.

(3)

- (c) Hence show that

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx = p + \ln q$$

where p and q are constants to be found.

(4)

9.

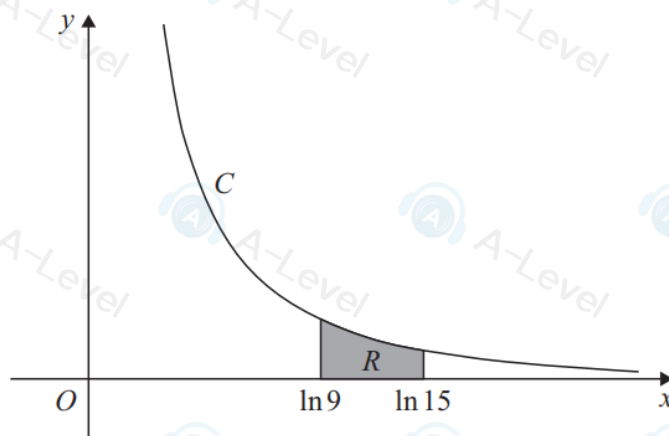


Figure 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t+1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P .

(6)

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = \ln 9$, the x -axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t+1)(2t+5)} dt$$

where a , b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R .

Write your answer in the form $\frac{1}{3} \ln \alpha$, where α is a rational number.

(9)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B .

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B ,

(b) find the value of p and the value of q .

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos \theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC , giving your answer as a surd.

(2)

7. The volume $V \text{ cm}^3$ of a spherical balloon with radius $r \text{ cm}$ is given by the formula

$$V = \frac{4}{3} \pi r^3$$

- (a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t + 3)^2} \quad t \geq 0$$

Given that $V = 0$ when $t = 0$

- (b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t + 3}$$

- (ii) Hence find the upper limit to the volume of the balloon.

(5)

- (c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

- (d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

7. (a) Using the substitution $u = 4x + 2 \sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x} \cos^2 x \, dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

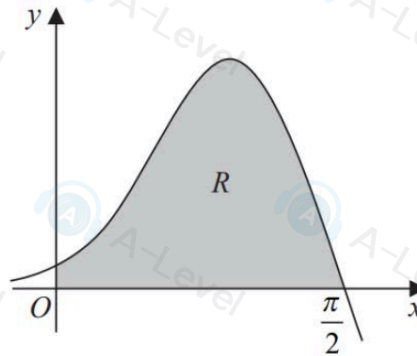


Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x} \cos x$$

The region R , shown shaded in Figure 3, is bounded by the positive x -axis, the positive y -axis and the curve.

The region R is rotated through 2π radians about the x -axis to form a solid.

- (b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x , in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

- (a) Show that $k = 1.8$

(2)

- (b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x , in thousands, is modelled by the differential equation

$$3 \frac{dx}{dt} = x(9 - 2x)$$

- (c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

2. (a) Express $\frac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)} dx = \ln k$$

where k is a fully simplified fraction to be found.

(Solutions relying entirely on calculator technology are not acceptable.)

(4)

9.

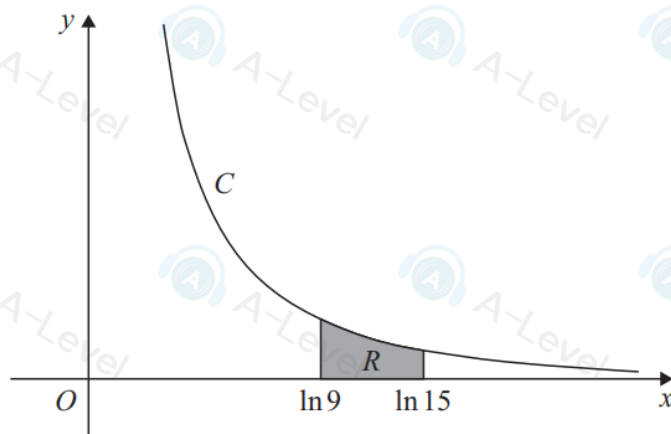


Figure 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t+1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

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- (a) use calculus to find the exact y coordinate of P .

(6)

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = \ln 9$, the x -axis and the line with equation $x = \ln 15$

- (b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t+1)(2t+5)} dt$$

where a , b and k are constants to be found.

- (ii) Hence, using algebraic integration, find the exact area of R .

Write your answer in the form $\frac{1}{3} \ln \alpha$, where α is a rational number.

(9)

8.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

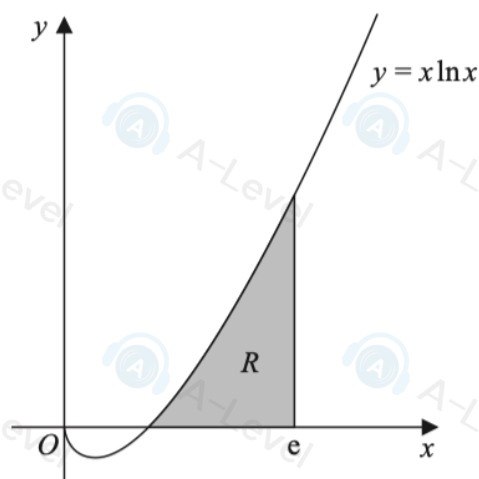


Figure 3

- (a) Find $\int x^2 \ln x dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \quad x > 0$$

The region R , shown shaded in Figure 3, lies entirely above the x -axis and is bounded by the curve, the x -axis and the line with equation $x = e$.

This region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (b) Find the exact volume of the solid formed, giving your answer in simplest form.

(4)

9.

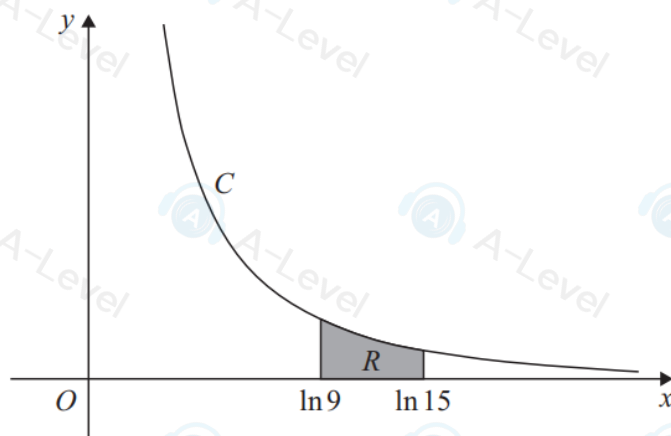


Figure 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t+1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y} \right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)

9. (a) Find the derivative with respect to y of

$$\frac{1}{(1 + 2\ln y)^2}$$

(2)

- (b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x) \frac{dy}{dx} = y(1 + 2\ln y)^3 \quad y > 0 \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

- (c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = e^{A \sec x - \frac{1}{2}}$$

where A is an irrational number to be found.

(5)

10.

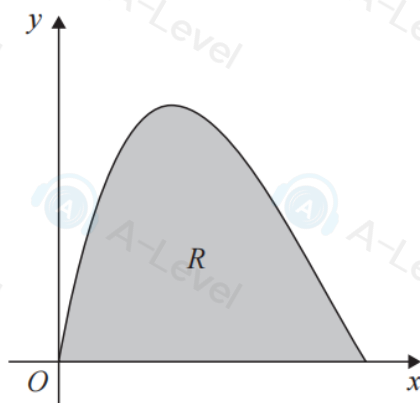


Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \quad y = \sin t \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 5, is bounded by the curve and the x -axis.

(a) Show that the area of R is

$$k \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R , giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

2. Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants A , B and C .

(4)

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} \, dx$$

giving your answer in the form $p \ln q + r$ where p , q and r are rational numbers.

(6)

7.

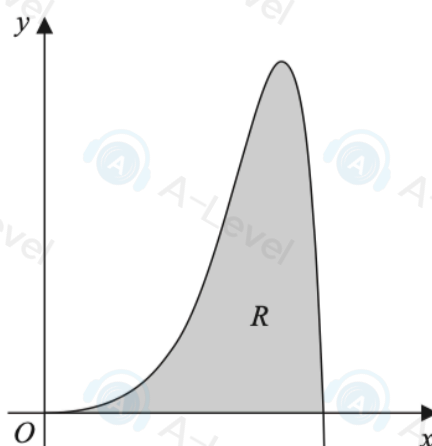


Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \quad x \geq 0$$

The finite region R is bounded by the curve and the x -axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\frac{e^{2\pi} + 1}{5}$

(Solutions relying on calculator technology are not acceptable.)

(2)

7.

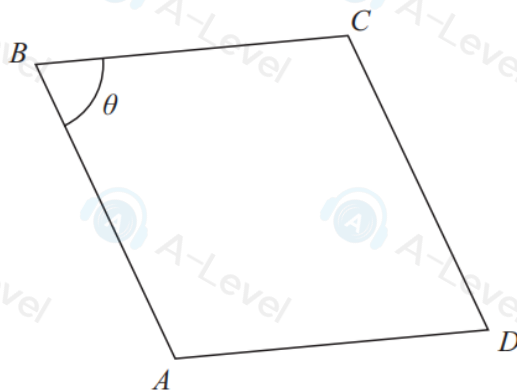


Figure 2

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\vec{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\vec{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90^\circ$

(a) find the exact value of $\cos \theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

5. (i) Find

$$\int x^2 e^{4x} dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

2.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of $\int_1^e \frac{\ln x}{x^2} dx$

Write your answer in the form $a + \frac{b}{e}$, where a and b are integers.

(6)

(Total for Question 2 is 6 marks)

3.

In this question you must show all stages of your working.
Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x} dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2} dx = a + \ln b$$

where a and b are constants to be found.

(7)

5.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2 \sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx = \int_0^p \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx$$

(4)

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8. (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^3}{e^{2x}} \quad y \geq 0$$

giving your answer in the form $y^2 = g(x)$.

(7)

- (b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

(2)

3. Given that $y = 4$ at $x = \frac{\pi}{6}$, solve the differential equation

$$y \cos^2(2x) \frac{dy}{dx} = 3 \sin(2x) \quad y > 0 \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

giving your answer in the form $y^2 = g(x)$

(6)

7. (a) Express $\frac{1}{(4-x)(2-x)}$ in partial fractions.

(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x) \quad t \geq 0 \quad 0 \leq x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

- (b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

(Solutions relying on calculator technology are not acceptable.)

(6)

Given that $k = 0.1$

- (c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.

(2)

(Total for Question 7 is 10 marks)

2.

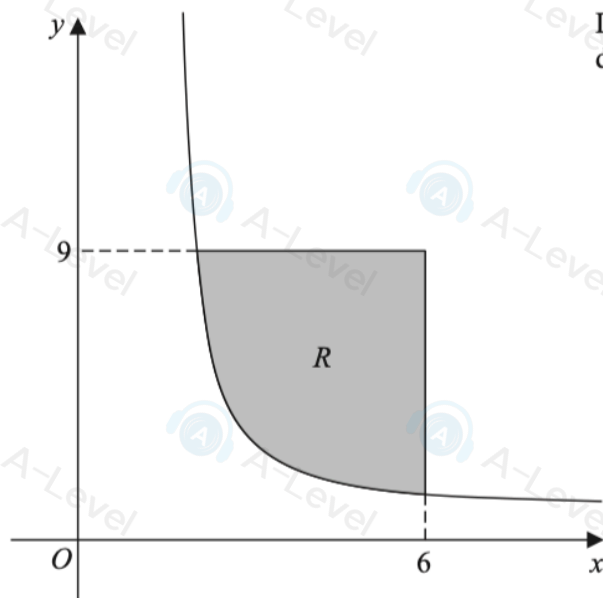
Diagram **NOT**
drawn to scale**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x - 3)^{1.25}} \quad x > \frac{3}{2}$$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through 2π radians about the x -axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

(7)

3.

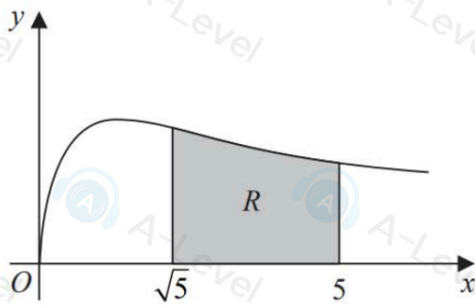


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \quad x \geq 0$$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x -axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = 2(x^3 + 1)^k (x^3 - A) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

4.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{5 + 17x - 10x^2}{x(1-x)(2x+1)} \quad x > 1$$

(a) Write $f(x)$ in the form

$$\frac{A}{x} + \frac{B}{1-x} + \frac{C}{2x+1}$$

where A , B and C are constants to be found.

(3)

(b) Hence, use algebraic integration to find the exact value of

$$\int_2^4 f(x) \, dx$$

Write your answer in the form $p \ln 2 + q \ln \left(\frac{3}{5} \right)$, where p and q are integers to be found.

(5)

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4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}} \, dx$$

Write your answer in the form $4 \ln \left(\frac{a}{b} \right)$, where a and b are integers to be found.

(Solutions relying entirely on calculator technology are not acceptable.)

(8)

9.

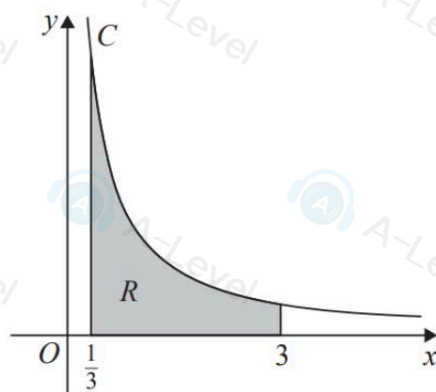


Figure 3

The curve C , shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x} (\arctan \sqrt{x})}$$

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = 3$, the x -axis and the line with equation $x = \frac{1}{3}$.

The region R is rotated through 360° about the x -axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume V of the solid formed is given by

$$k \int_a^b \frac{1}{u^2} du$$

where k , a and b are constants to be found.

(6)

(b) Hence, using algebraic integration, find the value of V in simplest form.

(3)

1:

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Find, by algebraic integration,

$$\int_1^3 2x \ln(3x) dx$$

giving the answer in the form $A \ln 3 + B$ where A and B are integers.

(5)

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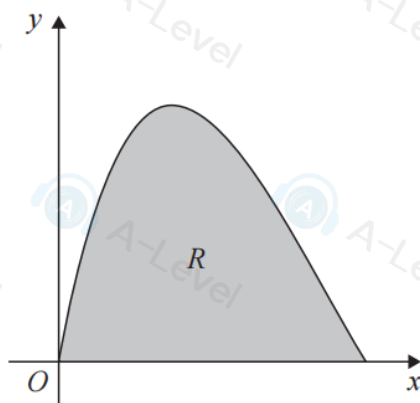


Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \quad y = \sin t \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 5, is bounded by the curve and the x -axis.

(a) Show that the area of R is

$$k \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R , giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

8.

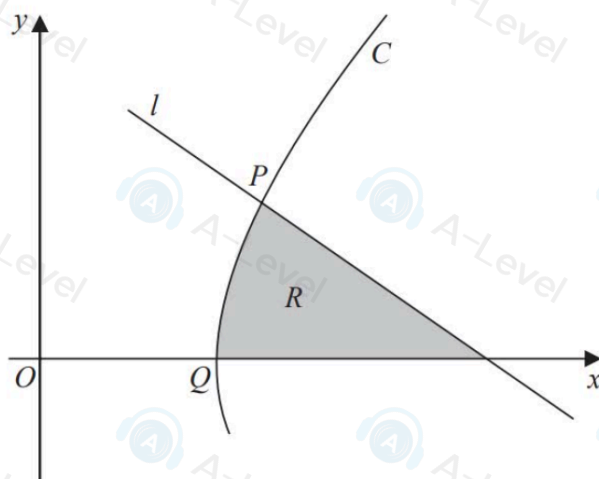


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \quad y = t - \frac{1}{t} \quad t > 0.7$$

The curve C intersects the x -axis at the point Q .

- (a) Find the x coordinate of Q .

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

- (b) write down the coordinates of P

(1)

- (c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R , shown shaded in Figure 2 is bounded by the curve C , the line l and the x -axis.

- (d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x -axis.

(7)

5. (i) Find

$$\int x^2 e^x dx$$

(4)

(ii) Use the substitution $u = \sqrt{1 - 3x}$ to show that

$$\int \frac{27x}{\sqrt{1 - 3x}} dx = -2(1 - 3x)^{\frac{1}{2}}(Ax + B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

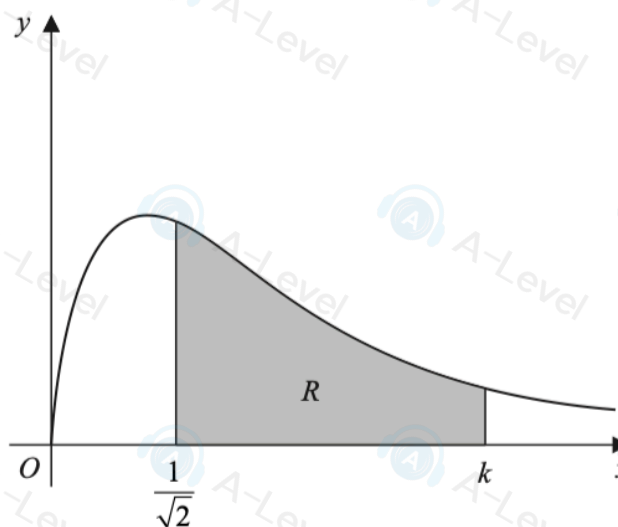


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R , shown shaded in Figure 2, is bounded by the curve, the line with

equation $x = \frac{1}{\sqrt{2}}$, the x -axis and the line with equation $x = k$.

This region is rotated through 360° about the x -axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the

exact value of the constant k .

(6)

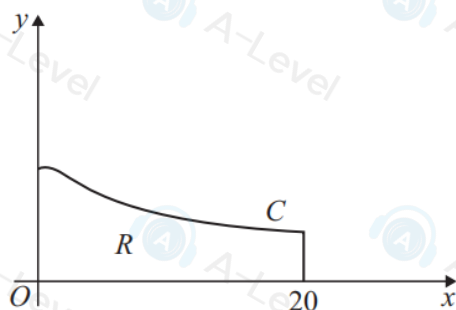


Figure 2

The curve C has equation

$$y = 12 \left(\frac{2x+3}{x^2+3x+5} \right)^{\frac{1}{2}}$$

The finite region, R , shown in Figure 2, is bounded by the curve C , the line with equation $x = 20$, the x -axis and the y -axis.

The region R is rotated through 2π radians about the x -axis to form a solid, S .

(a) Use algebraic integration to find the exact volume of S .

Give your answer in the form $k \ln \lambda$ where k and λ are constants.

(4)

Figure 3 shows a hollow vase with identical shape to solid S .

The vase is made of glass, which has negligible thickness.

When the vase is full of water,

$$\text{volume of water in the vase} = \text{volume of } S$$

When the vase contains $360\pi \text{ cm}^3$ of water, the depth of the water is $a \text{ cm}$, as shown in Figure 3.

(b) Use algebra to find the value of a , giving your answer to one decimal place.

(3)

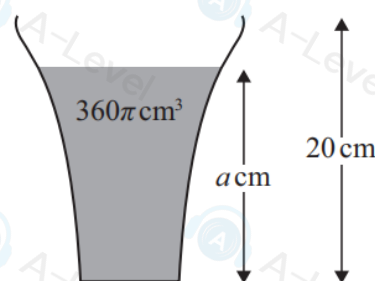


Figure 3

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7.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Use the substitution $x = 4 \sin \theta$ to find the exact value of

$$\int_2^{2\sqrt{3}} \frac{1}{(16-x^2)^{\frac{3}{2}}} dx$$

(6)

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = 2(x^3 + 1)^k (x^3 - A) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

6.

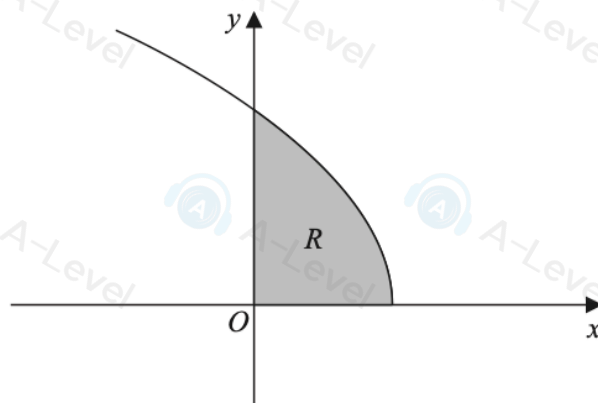


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = 4 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve, the x -axis and the y -axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{2}} 32 \sin^2 t \cos t \, dt$

(ii) Hence find, by algebraic integration, the exact value of the area of R . (6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found. (3)

The curve C has equation $y = f(x)$ where f is the function

$$f(x) = \sqrt{ax + b} \quad -2 \leq x \leq 2$$

and a and b are the constants found in part (b).

(c) State the range of f . (1)

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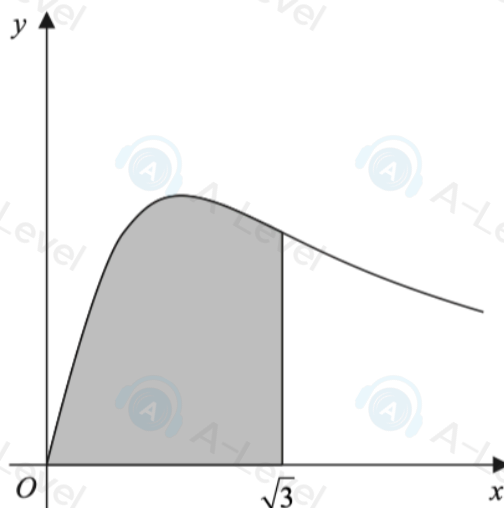
**Figure 3**

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan \theta \quad y = 2 \sin 2\theta \quad \theta \geq 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x -axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta) d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

2. Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants A , B and C .

(4)

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2} dx$$

giving your answer in the form $p \ln q + r$ where p , q and r are rational numbers.

(6)

5. In this question you should show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_1^{13} \frac{4}{3 + \sqrt{2x - 1}} dx$$

giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(8)

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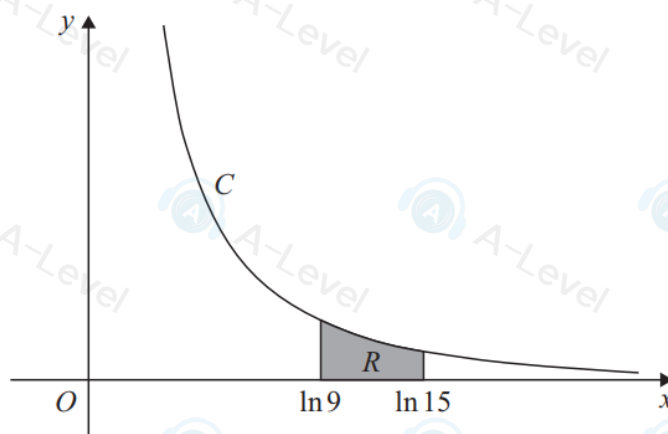


Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t + 1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

7. (i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x - 1}} dx$$

(Solutions relying entirely on calculator technology are not acceptable.)

(6)

- (ii) Find

$$\int \frac{6x^2 - 16}{(x + 1)(2x - 3)} dx$$

(6)

1.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx$$

giving your answer in simplest form.

(5)

7. (a) Show that the substitution
- $u = \tan x$
- transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, dx$$

to

$$\int \frac{u}{(k + u^2)^n} \, du$$

where k and n are integers to be found.

(4)

- (b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, dx$$

(2)

5.

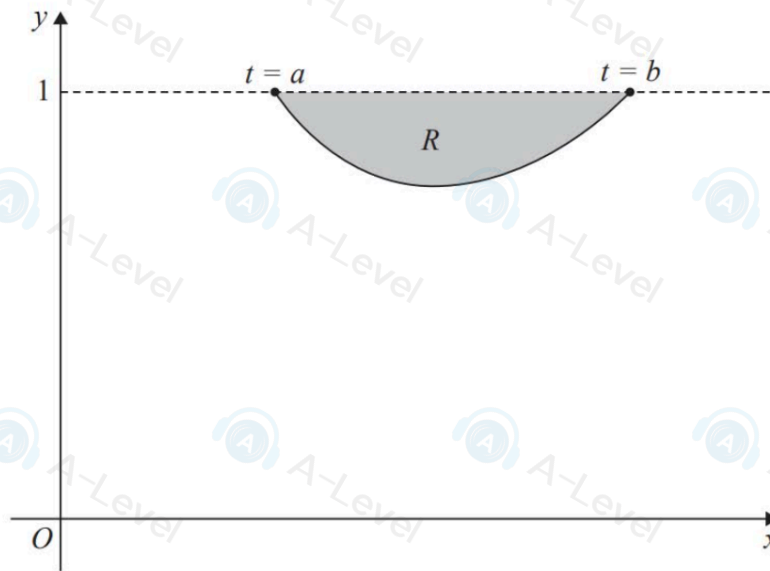
**Figure 2**

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \quad y = \frac{2}{t(3-t)} \quad a \leq t \leq b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R , shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k \int_a^b \frac{t+1}{t(3-t)} dt$$

where M and k are constants to be found.

(5)

(c) (i) Write $\frac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R , giving your answer in simplest form.

(6)

10.

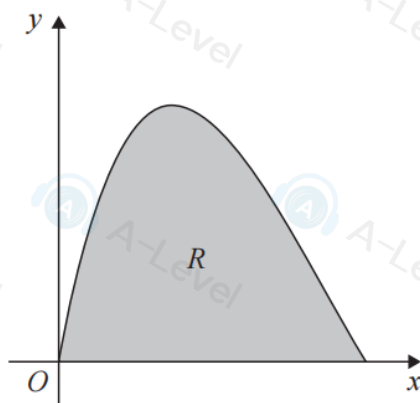


Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \quad y = \sin t \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 5, is bounded by the curve and the x -axis.

(a) Show that the area of R is

$$k \int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, dt$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R , giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)