

6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

- (a) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The curve crosses the y -axis at the origin and at the point P .

- (b) Find the equation of the normal to the curve at P , writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

7:

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

- (a) Show that the substitution $u = e^x - \frac{1}{2}$ transforms

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx$$

to

$$\int_a^b \frac{12u + 5}{u^2(2u + 1)} du$$

where a and b are constants to be found.

(4)

- (b) Find an expression for $\frac{12u + 5}{u^2(2u + 1)}$ in the form

$$\frac{A}{u^2} + \frac{B}{u} + \frac{C}{(2u + 1)}$$

where A , B and C are integers.

(3)

- (c) Hence show that

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2} dx = p + \ln q$$

where p and q are constants to be found.

(4)

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2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \quad y > 0$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of x and y

(6)

Curve C cuts the y -axis at the point P

- (b) Find the exact value of the gradient of the tangent to C at P

(2)

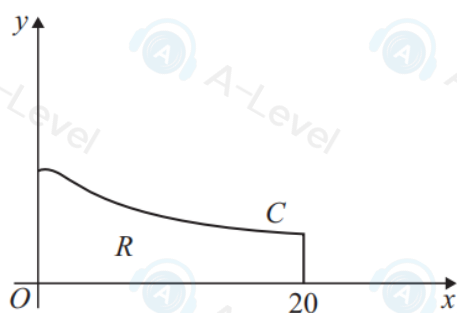


Figure 2

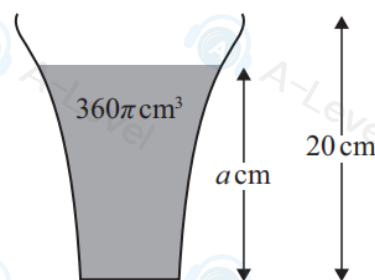


Figure 3

The curve C has equation

$$y = 12 \left(\frac{2x + 3}{x^2 + 3x + 5} \right)^{\frac{1}{2}}$$

The finite region, R , shown in Figure 2, is bounded by the curve C , the line with equation $x = 20$, the x -axis and the y -axis.

The region R is rotated through 2π radians about the x -axis to form a solid, S .

- (a) Use algebraic integration to find the exact volume of S .

Give your answer in the form $k \ln \lambda$ where k and λ are constants.

(4)

Figure 3 shows a hollow vase with identical shape to solid S .

The vase is made of glass, which has negligible thickness.

When the vase is full of water,

$$\text{volume of water in the vase} = \text{volume of } S$$

When the vase contains $360\pi \text{ cm}^3$ of water, the depth of the water is $a \text{ cm}$, as shown in Figure 3.

- (b) Use algebra to find the value of a , giving your answer to one decimal place.

(3)

1.
$$f(x) = \frac{5x + 10}{(1 - x)(2 + 3x)}$$

(a) Write $f(x)$ in partial fraction form.

(3)

(b) (i) Hence find, in ascending powers of x up to and including the terms in x^2 , the binomial series expansion of $f(x)$. Give each coefficient as a simplified fraction.

(5)

(ii) Find the range of values of x for which this expansion is valid.

(1)

6. Given that $n \in \mathbb{N}$, use algebra to prove by contradiction that

“if $n^2 - 4n + 5$ is even then n is odd”

(4)

6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos \theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

9.

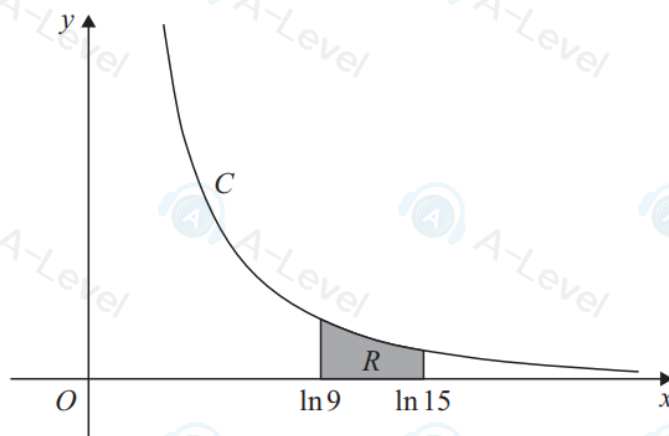


Figure 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t+1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

9. (i) Relative to a fixed origin O , the points A , B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points A , B and C lie in a straight line, with B lying between A and C .

Given $AB:AC = 1:3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a} \quad (3)$$

- (ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)

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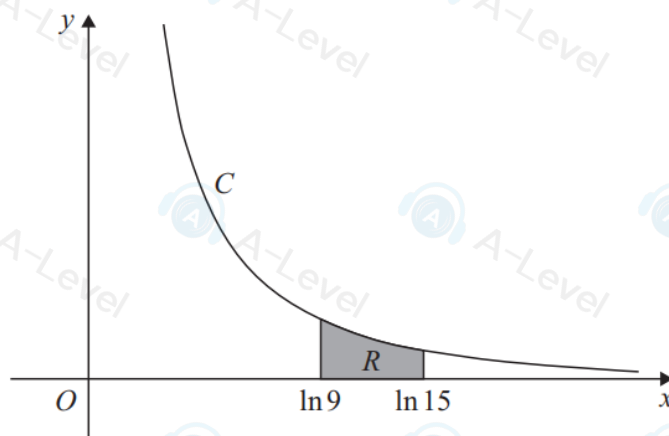


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Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t + 1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8 - 3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

- (b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y} \right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)

1.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$2y^2 - 6xy = 7e^{2x-1} + 13$$

The point P with x coordinate $\frac{1}{2}$ lies on C .

- (a) Find the two possible y coordinates of P .

(2)

Given that P lies above the x -axis,

- (b) find an equation for the tangent to C at P , giving your answer in the form $ax + by + c = 0$ where a , b and c are integers.

(6)

3. Given that the binomial expansion, in ascending powers of x , of

$$6(4 + Ax)^{-\frac{1}{2}}$$

is $B - \frac{1}{4}x + Cx^2 + \dots$

- (a) find the values of the constants A , B and C .

(5)

- (b) State the range of values of x for which this expansion is valid.

(1)

For the expansion,

- (c) find the coefficient of the term in x^3

(2)

7.

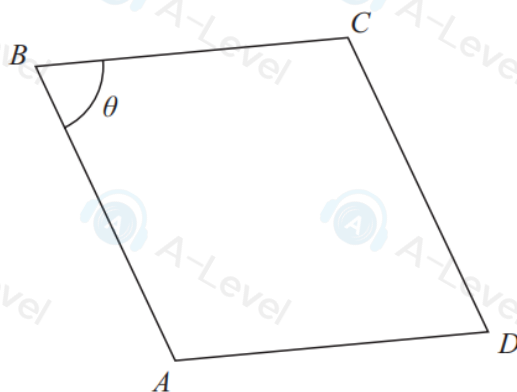


Figure 2

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\vec{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\vec{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90^\circ$

(a) find the exact value of $\cos \theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B .

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B ,

(b) find the value of p and the value of q .

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos \theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC , giving your answer as a surd.

(2)

4.

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y .

(5)

The point P lies on C .

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b .

(5)

6. Relative to a fixed origin O .

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B .

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

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4.

$$f(x) = \sqrt{1 - 4x^2} \quad |x| < \frac{1}{2}$$

(a) Find, in ascending powers of x , the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

(4)

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

(2)

2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, V cm³, is decreasing at a constant rate, k cm³ per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)

7. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3 \cos^2 t \quad y = 3 \sin t + 2 \cos t \quad 0 \leq t \leq \pi$$

- (a) Show that $\frac{dy}{dx} = 3$ where $t = \pi$ (4)

The point P lies on C where $t = \pi$

- (b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found. (3)

Given that the tangent to the curve at P cuts C at the point Q

- (c) show that the value of t at point Q satisfies the equation $9 \cos^2 t + 2 \cos t - 7 = 0$ (2)

- (d) Hence find the exact value of the y coordinate of Q (3)

5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \quad y = 1 - t^2 \quad t \neq 1$$

The point P , where $t = 2$, lies on C .

- (a) Use parametric differentiation to find the equation of the normal to C at P . Give your answer in the form $ax + by + c = 0$ where a , b and c are integers to be found. (5)

- (b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \quad x \neq k$$

where p , q , r and k are integers to be found. (4)

7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x , in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

- (a) Show that $k = 1.8$

(2)

- (b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x , in thousands, is modelled by the differential equation

$$3 \frac{dx}{dt} = x(9 - 2x)$$

- (c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

- (d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

- (e) Find the long-term population of goats predicted by this **second** model.

(1)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} \quad |x| < \frac{1}{2}$$

giving each term in simplest form.

(5)

Given that

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^n \left(\frac{1}{4} - \frac{1}{2}x\right)^{-\frac{3}{2}} = \left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}}$$

- (b) write down the value of n .

(1)

- (c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of x , of

$$\left(\frac{1}{4} - \frac{1}{2}x\right)^{\frac{1}{2}} \quad |x| < \frac{1}{2}$$

giving each term in simplest form.

(3)

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$(8 - 3x)^{-\frac{1}{3}} \quad |x| < \frac{8}{3}$$

giving each coefficient as a simplified fraction.

(4)

- (b) Use the answer from part (a) with $x = \frac{2}{3}$ to find a rational approximation to $\sqrt[3]{6}$

(2)

4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx$$

may be expressed in the form

$$\int_a^b k u^2 e^u du$$

where a , b and k are constants to be found.

(4)

- (b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx$$

giving your answer in simplest form.

(5)

9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, $\theta^\circ\text{C}$, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine

- is 85°C at the instant the engine is turned off
- is 40°C exactly 10 minutes after the engine is turned off

- (a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a , b , c and d are integers to be found.

(7)

- (b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20°C . Give your answer to the nearest minute.

(2)

2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, V cm³, is decreasing at a constant rate, k cm³ per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius. (4)

2. (a) Use the binomial expansion to expand

$$(4 - 5x)^{-\frac{1}{2}} \quad |x| < \frac{4}{5}$$

in ascending powers of x , up to and including the term in x^2 giving each coefficient as a fully simplified fraction. (4)

$$f(x) = \frac{2 + kx}{\sqrt{4 - 5x}} \quad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of $f(x)$, in ascending powers of x , is

$$1 + \frac{3}{10}x + mx^2 + \dots \quad \text{where } m \text{ is a constant}$$

- (b) find the value of k , (2)

- (c) find the value of m . (2)

blank

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2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x + 5}} \quad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$ (6)

9.

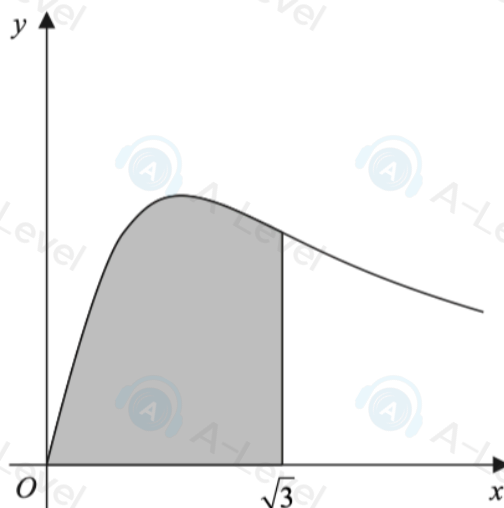
**Figure 3**

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan \theta \quad y = 2 \sin 2\theta \quad \theta \geq 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x -axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x -axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta) \, d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B .

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B ,

(b) find the value of p and the value of q .

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos \theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC , giving your answer as a surd.

(2)

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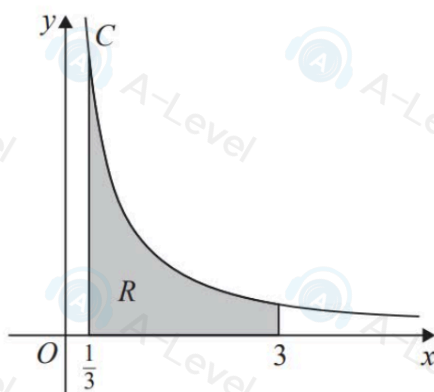


Figure 3

The curve C , shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x} (\arctan \sqrt{x})}$$

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = 3$, the x -axis and the line with equation $x = \frac{1}{3}$

The region R is rotated through 360° about the x -axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

- (a) show that the volume V of the solid formed is given by

$$k \int_a^b \frac{1}{u^2} du$$

where k , a and b are constants to be found.

(6)

- (b) Hence, using algebraic integration, find the value of V in simplest form.

(3)

5.

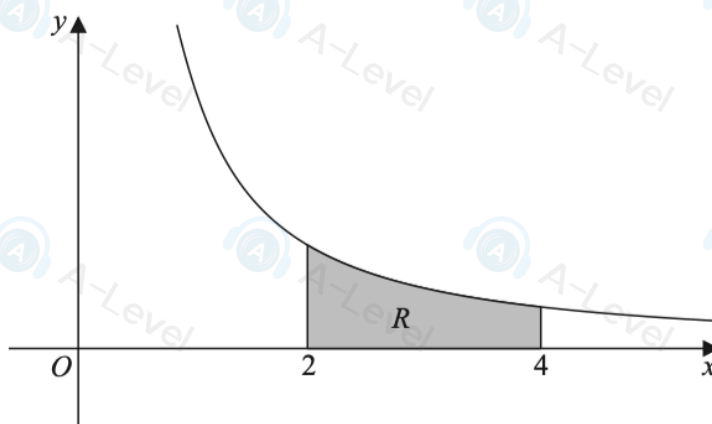


Figure 3

- (a) Find $\int \frac{\ln x}{x^2} dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \quad x > 0.5$$

The finite region R , shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x -axis and the line with equation $x = 4$

- (b) Use the answer to part (a) to find the exact area of R , writing your answer in simplest form.

(4)

7.

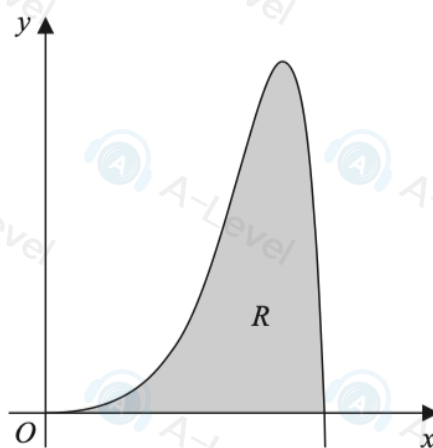


Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \quad x \geq 0$$

The finite region R is bounded by the curve and the x -axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\frac{e^{2\pi} + 1}{5}$

(Solutions relying on calculator technology are not acceptable.)

(2)

7. (a) Show that the substitution $u = \tan x$ transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, dx$$

to

$$\int \frac{u}{(k + u^2)^n} \, du$$

where k and n are integers to be found.

(4)

(b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, dx$$

(2)

7. The current, x amps, at time t seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where k is a constant.

Initially there is zero current in the circuit.

- (a) Solve the differential equation to find an equation, in terms of k , for the current in the circuit at time t seconds.

Give your answer in the form $x = f(t)$.

(6)

Given that in the long term the current in the circuit approaches 7 amps,

- (b) find the value of k .

(2)

- (c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

(3)

8.

$$f(x) = (8 - 3x)^{\frac{4}{3}} \quad 0 < x < \frac{8}{3}$$

- (a) Show that the binomial expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \dots$$

where A and B are constants to be found.

(4)

- (b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \quad 0 < x < \frac{8}{3}$$

where A and B are the constants found in part (a).

(Solutions relying on calculator technology are not acceptable.)

(4)

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4. Relative to a fixed origin O ,
- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
 - the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
 - the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B .

- (a) Find a vector equation for l .

(2)

The point C lies on l so that PC is perpendicular to l .

- (b) Find the coordinates of C .

(4)

The point P' is the reflection of P in the line l .

- (c) Find the coordinates of P'

(2)

- (d) Hence find $|\overrightarrow{PP'}|$, giving your answer as a simplified surd.

(2)

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

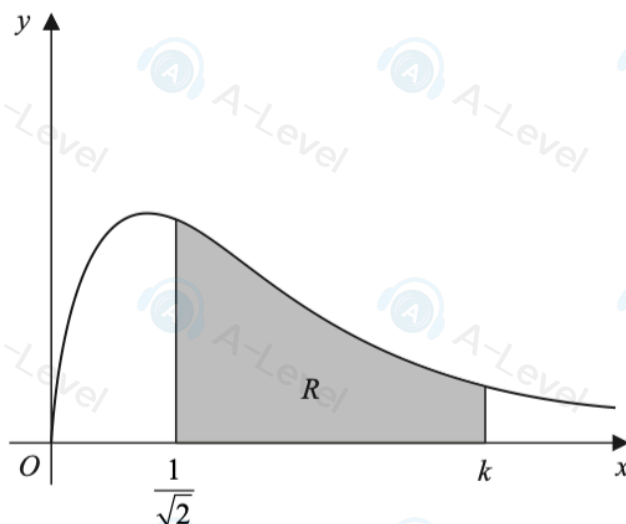


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R , shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x -axis and the line with equation $x = k$.

This region is rotated through 360° about the x -axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k .

(6)

1. Prove by contradiction that, if a and b are real numbers, $a > 0$ and $b > 0$, then

$$\frac{9a}{b} + \frac{4b}{a} \geq 12$$

(4)

(Total for Question 1 is 4 marks)

8.

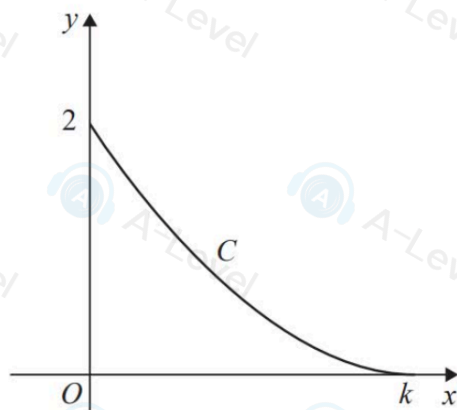


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \quad y = 2\cos t \quad 0 \leq t \leq \frac{\pi}{2}$$

The curve meets the y -axis at 2 and the x -axis at k , where k is a constant.

(a) State the value of k .

(1)

(b) Use parametric differentiation to show that

$$\frac{dy}{dx} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C .

The tangent to C at the point P cuts the y -axis at the point N .

(c) Find the exact y coordinate of N , giving your answer in simplest form.

(3)

The region bounded by the curve, the x -axis and the y -axis is rotated through 2π radians about the x -axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta(1 - \cos 4t) dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

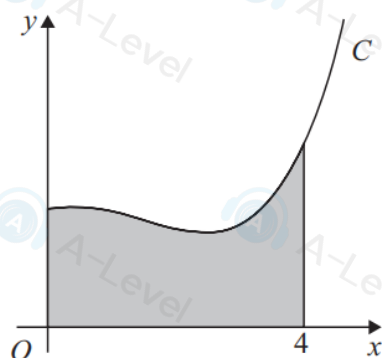


Figure 1

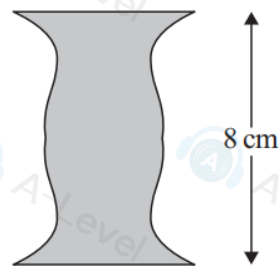


Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^x$$

The finite region, shown shaded in Figure 1, is bounded by C , the y -axis, the x -axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x -axis to form a solid S .

(a) Show that the exact volume of S is given by

$$\pi \int_0^4 (A + B \cos 2x + Ce^x \cos x + De^{2x}) dx$$

where A , B , C and D are constants to be found.

(4)

(b) Find $\int e^x \cos x \, dx$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together. The paperweight is 8 cm high.

(c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

9.

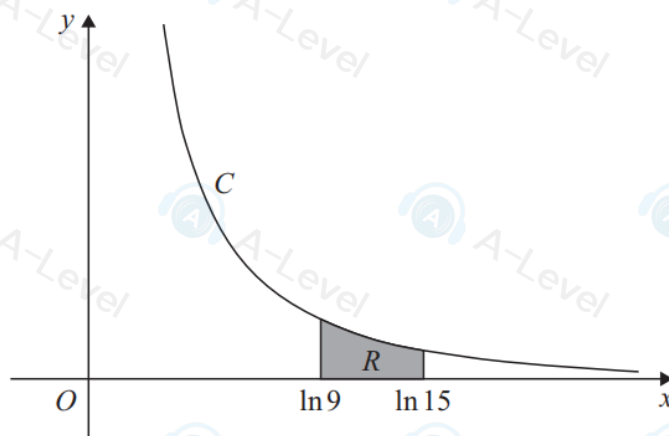


Figure 3

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \quad y = \frac{1}{t + 1} \quad t > -1$$

A point P lies on C .

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P .

(6)

The region R , shown shaded in Figure 3, is bounded by C , the line with equation $x = \ln 9$, the x -axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t+1)(2t+5)} dt$$

where a , b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R .

Write your answer in the form $\frac{1}{3} \ln \alpha$, where α is a rational number.

(9)

6. Given that $n \in \mathbb{N}$, use algebra to prove by contradiction that

“if $n^2 - 4n + 5$ is even then n is odd”

(4)

8.

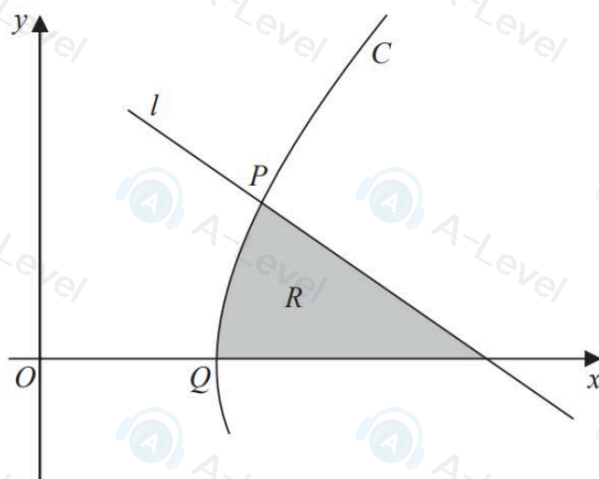


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \quad y = t - \frac{1}{t} \quad t > 0.7$$

The curve C intersects the x -axis at the point Q .

- (a) Find the x coordinate of Q .

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

- (b) write down the coordinates of P

(1)

- (c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R , shown shaded in Figure 2 is bounded by the curve C , the line l and the x -axis.

- (d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x -axis.

(7)

8. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \text{where } \mu \text{ is a scalar parameter}$$

Given that l_1 and l_2 meet at the point X ,

- (a) find the position vector of X .

(5)

The point $P(10, -7, 0)$ lies on l_1

The point Q lies on l_2

Given that $\vec{PQ}$ is perpendicular to l_2

- (b) calculate the coordinates of Q .

(5)

10.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

Use proof by contradiction to show that for all angles x , where $90^\circ < x < 180^\circ$

$$\left| \frac{\cos 2x}{\cos x - \sin x} \right| < 1$$

(4)

2.

**In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.**

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \quad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \quad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx$$

may be expressed in the form

$$\int_a^b k u^2 e^u du$$

where a , b and k are constants to be found.

(4)

- (b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4} e^{\sqrt{2x+1}} dx$$

giving your answer in simplest form.

(5)

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1. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \quad |x| < \frac{1}{20}$$

giving each coefficient in its simplest form.

(5)

By substituting $x = \frac{1}{100}$ into the answer for (a),

- (b) find an approximation for $\sqrt{5}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers to be found.

(2)

blank

8. Use proof by contradiction to prove that, for all positive real numbers x and y ,

$$\frac{9x}{y} + \frac{y}{x} \geq 6$$

(4)

2.

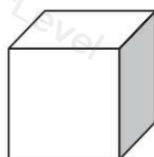
 $x \text{ cm}$ **Figure 1**

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is $x \text{ cm}$
- the surface area of the cube is $S \text{ cm}^2$
- the volume of the cube is $V \text{ cm}^3$

Given that the surface area of the cube is increasing at a constant rate of $4 \text{ cm}^2 \text{ s}^{-1}$

- (a) show that $\frac{dx}{dt} = \frac{k}{x}$ where k is a constant to be found,

(4)

- (b) show that $\frac{dV}{dt} = V^p$ where p is a constant to be found.

(3)

7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

7.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Use the substitution $x = 4 \sin \theta$ to find the exact value of

$$\int_2^{2\sqrt{3}} \frac{1}{(16-x^2)^{\frac{3}{2}}} dx$$

(6)

9. Relative to a fixed origin O , the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l , where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b .

(3)

(b) Find the vector $\vec{AB}$

(2)

The point C has coordinates $(4, -3, 2)$

(c) Find the size of angle CAB .

(3)

(d) Find the exact area of the triangle CAB .

(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB .

(e) Find the coordinates of the two possible positions of D .

(4)

(Total for Question 9 is 14 marks)

6. With respect to a fixed origin O , the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\vec{OA}| = 5\sqrt{10}$

- (a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

(3)

Hence

- (b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

- (ii) find the other possible position vector for A .

(3)

The line l_2 is parallel to l_1 and passes through O .

Given that

- $\vec{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\vec{OB}| = 4\sqrt{10}$

- (c) find the area of triangle OAB , giving your answer to one decimal place.

(4)

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8.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

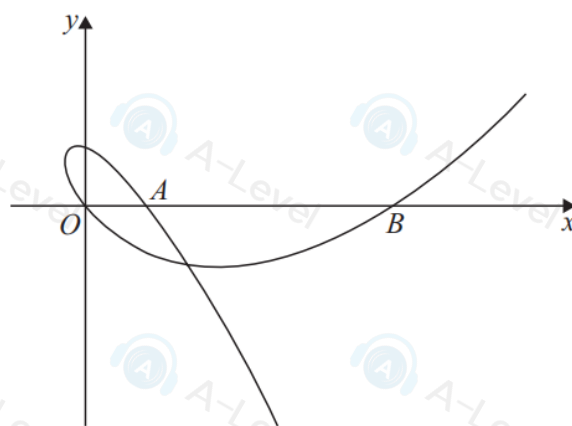


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \quad y = t^3 - 9t \quad t \in \mathbb{R}$$

The curve cuts the x -axis at the origin and at the points A and B as shown in Figure 3.

- (a) Find the coordinates of point A and show that point B has coordinates $(15, 0)$. (3)

- (b) Find the equation of the tangent to the curve at B , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)

The tangent to the curve at B cuts the curve again at point X .

- (c) Use algebra to find the coordinates of X , showing each stage of your working. (5)

(Total for Question 8 is 12 marks)

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{8}{(2 - 5x)^2}$$

writing each term in simplest form. (4)

- (b) Find the range of values of x for which this expansion is valid. (1)

DO NOT WRITE IN THIS

4.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2} \quad (4)$$

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

3.

$$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A , B , C and D .

(5)

A curve has equation

$$y = g(x) \quad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g .

(1)

1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{8}{(2-5x)^2}$$

writing each term in simplest form.

(4)

(b) Find the range of values of x for which this expansion is valid.

(1)

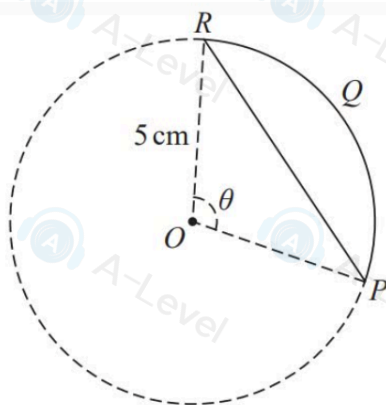


Figure 1

Figure 1 shows a sketch of a segment PQR of a circle with centre O and radius 5 cm.

Given that

- angle POR is θ radians
- θ is increasing, from 0 to π , at a constant rate of 0.1 radians per second
- the area of the segment PQR is $A \text{ cm}^2$

(a) show that

$$\frac{dA}{d\theta} = K(1 - \cos \theta)$$

where K is a constant to be found.

(2)

(b) Find, in $\text{cm}^2 \text{s}^{-1}$, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

(4)

5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2 \sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx = \int_0^p \left(\frac{3}{2} \sec u \tan u + \frac{1}{2} \sec^2 u \right) du$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}} dx$$

(4)

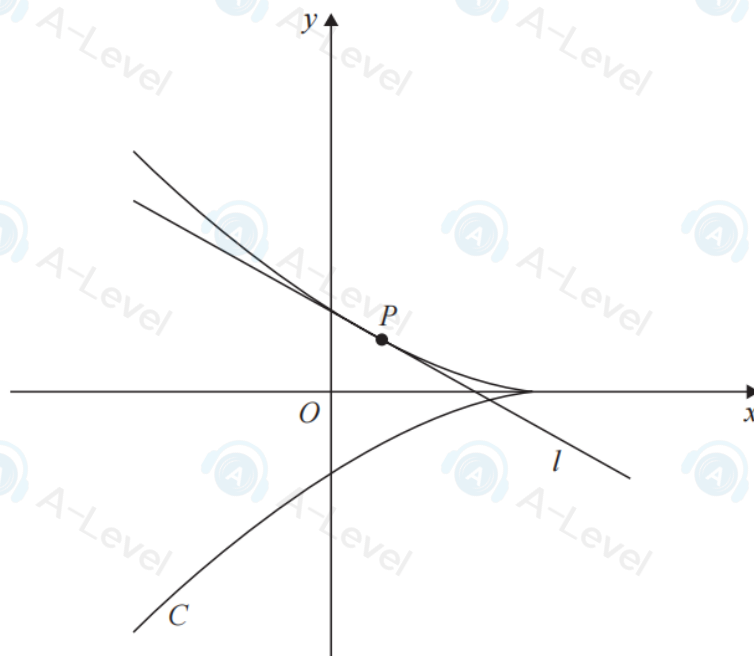


Figure 2

**In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2 \cos 2t \quad y = \sin^3 t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$.

The line l , shown in Figure 2, is the tangent to C at P .

(a) Use parametric differentiation to show that

(i) $\frac{dy}{dx} = k \sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q .

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q .

(6)

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \quad y = \frac{6}{2t+1} \quad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

1.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x \cos 3x \, dx$$

giving your answer in simplest form.

(5)

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9.

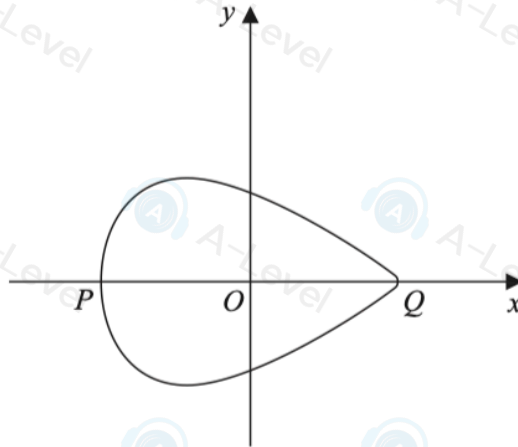


Figure 1

Figure 1 shows a sketch of a closed curve with parametric equations

$$x = 5 \cos \theta \quad y = 3 \sin \theta - \sin 2\theta \quad 0 \leq \theta < 2\pi$$

The region enclosed by the curve is rotated through π radians about the x -axis to form a solid of revolution.

(a) Show that the volume, V , of the solid of revolution is given by

$$V = 5\pi \int_{\alpha}^{\beta} \sin^3 \theta (3 - 2 \cos \theta)^2 \, d\theta$$

where α and β are constants to be found.

(4)

(b) Use the substitution $u = \cos \theta$ and algebraic integration to show that $V = k\pi$ where k is a rational number to be found.

(7)

3.

$$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \quad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x) dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x) dx = \frac{1}{2} \ln 20$$

(5)

Leave
blank1. Given that k is a constant and the binomial expansion of

$$\sqrt{1 + kx} \quad |kx| < 1$$

in ascending powers of x up to the term in x^3 is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of k ,(ii) find the value of the constant A and the constant B .

(5)

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

(2)

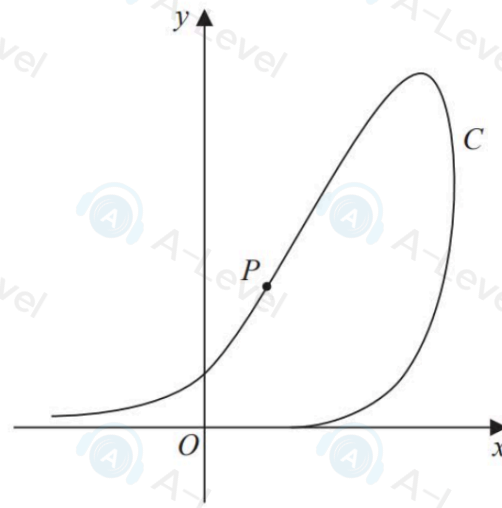


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \quad y \geq 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P .

(2)

(b) Find $\frac{dy}{dx}$ in terms of x and y .

(5)

The tangent to C at P crosses the x -axis at the point Q .

(c) Find the x coordinate of Q , giving your answer in the form $\frac{a \ln 2 + b}{c \ln 2 + d}$ where a, b, c and d are integers to be found.

3.

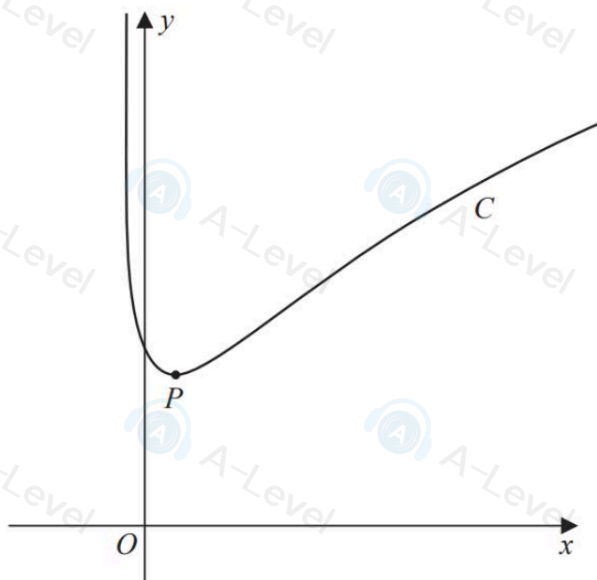


Figure 1

The curve C , shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \quad y > 0$$

where k is a constant.

- (a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

The point $P(p, 2)$, where p is a constant, lies on C .

Given that P is the minimum turning point on C ,

- (b) find
- (i) the value of p
 - (ii) the value of k

(4)

6. Three **consecutive** terms in a sequence of real numbers are given by

$$k, 1 + 2k \text{ and } 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(5)

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2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \quad y > 0$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of x and y

(6)

Curve C cuts the y -axis at the point P

- (b) Find the exact value of the gradient of the tangent to C at P

(2)

7. (a) Express $\frac{1}{(4-x)(2-x)}$ in partial fractions.

(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x) \quad t \geq 0 \quad 0 \leq x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

- (b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

(Solutions relying on calculator technology are not acceptable.)

(6)

Given that $k = 0.1$

- (c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.

(2)

(Total for Question 7 is 10 marks)

3.

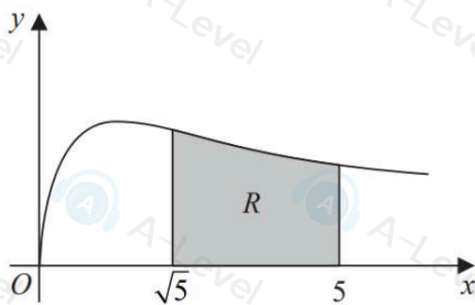


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \quad x \geq 0$$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x -axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

2:

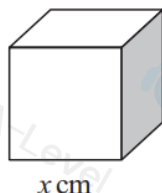


Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is $V \text{ cm}^3$

Given that the volume of the cube is increasing at a constant rate of $5 \text{ cm}^3 \text{ s}^{-1}$

(a) find $\frac{dx}{dt}$ giving your answer in terms of x .

(3)

(b) Hence find the value of $\frac{dx}{dt}$ when $V = 64$

(2)

5.

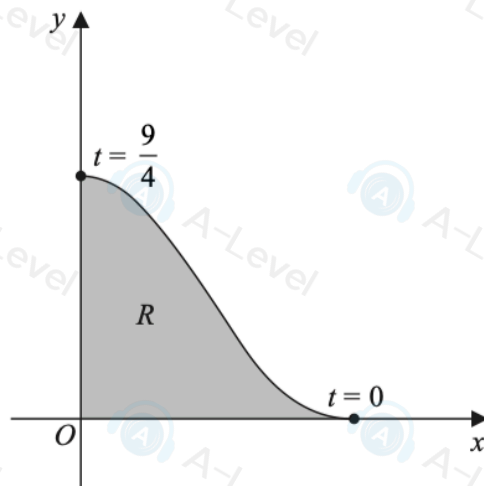


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \quad y = \frac{t^3}{\sqrt{9 + 4t}} \quad 0 \leq t \leq \frac{9}{4}$$

The curve touches the x -axis when $t = 0$ and meets the y -axis when $t = \frac{9}{4}$

The region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the y -axis.

(a) Show that the area of R is given by

$$K \int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}} dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R .

(Solutions relying on calculator technology are not acceptable.)

(6)

10: [In this question you may assume that if n is an integer and n^2 is even, then n is even.]

Given that p and q are integers, use proof by contradiction to show that

$$p^2 - 4q + 2 \neq 0$$

(5)

- 1: In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Find, by algebraic integration,

$$\int_1^3 2x \ln(3x) dx$$

giving the answer in the form $A \ln 3 + B$ where A and B are integers.

(5)

7. Water is flowing into a large container and is leaking from a hole at the base of the container.

At time t seconds after the water starts to flow, the volume, $V \text{ cm}^3$, of water in the container is modelled by the differential equation

$$\frac{dV}{dt} = 300 - kV$$

where k is a constant.

- (a) Solve the differential equation to show that, according to the model,

$$V = \frac{300}{k} + Ae^{-kt}$$

where A is a constant.

(5)

Given that the container is initially empty and that when $t = 10$, the volume of water is increasing at a rate of $200 \text{ cm}^3 \text{ s}^{-1}$

- (b) find the exact value of k .

(4)

- (c) Hence find, according to the model, the time taken for the volume of water in the container to reach 6 litres. Give your answer to the nearest second.

(2)

6. Use integration by parts to show that

$$\int e^{2x} \cos 3x dx = pe^{2x} \sin 3x + qe^{2x} \cos 3x + k$$

where p and q are rational numbers to be found and k is an arbitrary constant.

(6)

blank

3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \quad y = \frac{5}{t+2} \quad t \geq 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \quad e < x \leq f$$

and a, b, c, d, e and f are constants to be found.

(5)

(b) State the range of g .

(2)

10. A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking r and t ,

(5)

(b) find the time taken for the ball of ice to melt completely.

(2)

(c) On Diagram 1 on page 27, sketch a graph of r against t .

(1)

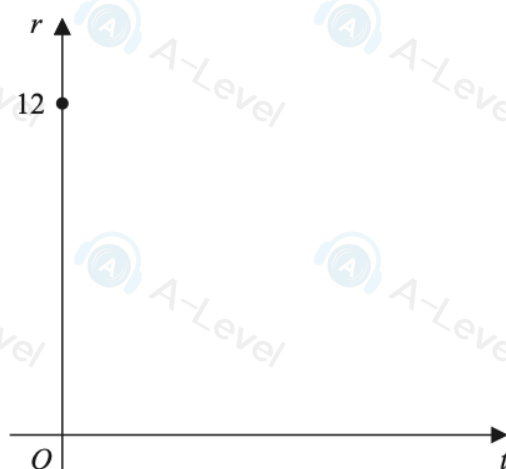


Diagram 1

6. With respect to a fixed origin O , the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\vec{OA}| = 5\sqrt{10}$

- (a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

(3)

Hence

- (b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

- (ii) find the other possible position vector for A .

(3)

The line l_2 is parallel to l_1 and passes through O .

Given that

- $\vec{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\vec{OB}| = 4\sqrt{10}$

- (c) find the area of triangle OAB , giving your answer to one decimal place.

(4)

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2. With respect to a fixed origin, O , the point A has position vector

$$\vec{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\vec{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

- (a) find the coordinates of the point B .

(2)

The point C has position vector

$$\vec{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\vec{OC}$ is perpendicular to $\vec{BC}$

- (b) find the possible values of a .

(4)

9. A student was asked to prove, for $p \in \mathbb{N}$, that

“if p^3 is a multiple of 3, then p must be a multiple of 3”

The start of the student's proof by contradiction is shown in the box below.

Assumption:

There exists a number $p, p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3

Let $p = 3k + 1, k \in \mathbb{N}$.

$$\begin{aligned} \text{Consider } p^3 &= (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1 \\ &= 3(9k^3 + 9k^2 + 3k) + 1 \quad \text{which is not a multiple of 3} \end{aligned}$$

- (a) Show the calculations and statements that are required to complete the proof.

(3)

- (b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

(5)

2. With respect to a fixed origin, O , the point A has position vector

$$\vec{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\vec{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

- (a) find the coordinates of the point B .

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The point C has position vector

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where a is a constant.

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- (b) find the possible values of a .

(4)