6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is 45°

(b) find the exact possible values of α

(6)

8: Relative to a fixed origin O,

- the point A has position vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
- the point B has position vector $3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

Given that the line l passes through A and B,

(a) (i) find $\overrightarrow{AB}$

(ii) find a vector equation for l.

(3)

The point C has position vector $p\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, where p is a constant.

Given that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$,

(b) find the possible values of p.

(4)

The points A, B and C form a triangle ABC.

Given also that $p > 0$

(c) find the area of triangle ABC, giving your answer to 3 significant figures.

(4)

6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point C has position vector $3\mathbf{i} + p\mathbf{j} - \mathbf{k}$

where p is a constant.

The line l passes through A and B.

(a) Find a vector equation for the line l

(3)

Given that $\overrightarrow{AC}$ is perpendicular to l

(b) find the value of p

(3)

(c) Hence find the area of triangle ABC, giving your answer as a surd in simplest form.

(3)

6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

7. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to l,

(a) find $\overrightarrow{OA}$

(2)

The point X lies on l.

Given that X is the point on l that is closest to the origin,

(b) find the coordinates of X.

(5)

The points O, X and A form the triangle OXA.

(c) Find the exact area of triangle OXA.

(3)

6. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\overrightarrow{OA}| = 5\sqrt{10}$

(a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

(3)

Hence

(b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for A.

(3)

The line l_2 is parallel to l_1 and passes through O.

Given that

- $\overrightarrow{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\overrightarrow{OB}| = 4\sqrt{10}$

(c) find the area of triangle OAB, giving your answer to one decimal place.

(4)

Leave blank

8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ b \end{pmatrix}$$

where λ and μ are scalar parameters and b is a constant.

Prove that for all values of $b \neq 7$, the lines l_1 and l_2 are skew.

(6)

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6. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\overrightarrow{OA}| = 5\sqrt{10}$

(a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

(3)

Hence

(b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for A.

(3)

The line l_2 is parallel to l_1 and passes through O.

Given that

- $\overrightarrow{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\overrightarrow{OB}| = 4\sqrt{10}$

(c) find the area of triangle OAB, giving your answer to one decimal place.

(4)

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9. With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)

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6. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

(a) find the value of p,

(4)

(b) find the position vector of the point of intersection.

(2)

(c) Find the acute angle between l_1 and l_2

Give your answer in degrees to one decimal place.

(3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with $\overrightarrow{AB}$ perpendicular to l_2

(d) Find the coordinates of B

(5)

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6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

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2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

9. Relative to a fixed origin O, the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l, where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b.

(3)

(b) Find the vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, -3, 2)$

(c) Find the size of angle CAB.

(3)

(d) Find the exact area of the triangle CAB.

(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB.

(e) Find the coordinates of the two possible positions of D.

(4)

(Total for Question 9 is 14 marks)

6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is 45°

(b) find the exact possible values of α

(6)

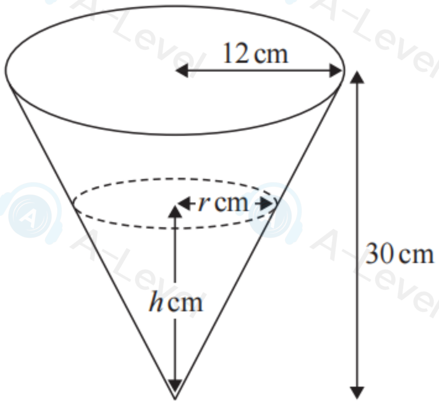

Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of 2π cm^3 s^{-1}

(b) find, in cm s^{-1}, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

4. Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $\left|\overrightarrow{PP'}\right|$, giving your answer as a simplified surd.

(2)

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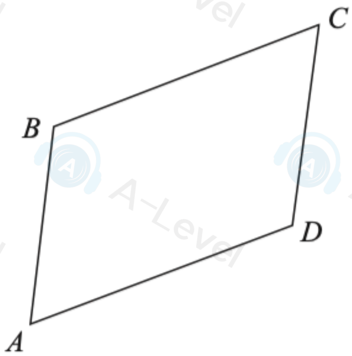

Figure 1

Figure 1 shows a sketch of parallelogram $ABCD$.

Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle ABC, giving your answer in degrees, to 2 decimal places.

(3)

(b) Find the area of parallelogram $ABCD$, giving your answer to one decimal place.

(2)

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6. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

(a) find the value of p,

(4)

(b) find the position vector of the point of intersection.

(2)

(c) Find the acute angle between l_1 and l_2

Give your answer in degrees to one decimal place.

(3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with $\overrightarrow{AB}$ perpendicular to l_2

(d) Find the coordinates of B

(5)

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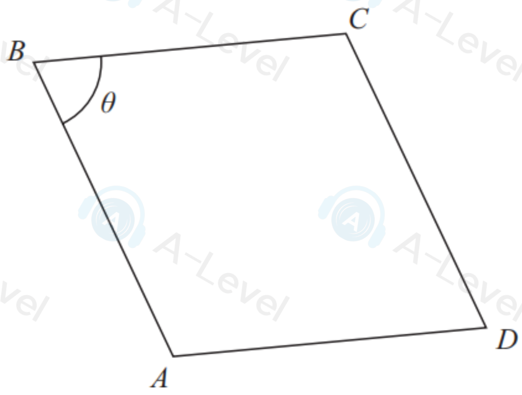

Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\overrightarrow{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90°$

(a) find the exact value of $\cos\theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

4. Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $\left|\overrightarrow{PP'}\right|$, giving your answer as a simplified surd.

(2)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

8. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

The point A and the point B lie on line l

Given that

- A has coordinates $(-2, a, 4)$
- B has coordinates $(b, 3, 1)$

(a) find the value of the constant a and the value of the constant b.

(2)

(b) Hence find vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, 7, -2)$.

(c) Find the size of angle CAB, giving your answer in degrees to one decimal place.

(4)

The point D lies on the line l so that the area of triangle CAD is twice the area of triangle CAB.

(d) Find the coordinates of the two possible positions of D.

(4)

8. With respect to a fixed origin O the points A and B have position vectors

$$\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix}$$

respectively.

The line l_1 passes through the points A and B.

(a) Write down an equation for l_1

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where λ is a scalar parameter.

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

where μ is a scalar parameter.

(b) Show that l_1 and l_2 do **not** meet.

(4)

The point C is on l_2 where $\mu = -1$

(c) Find the acute angle between AC and l_2

Give your answer in degrees to one decimal place.

(5)

3.

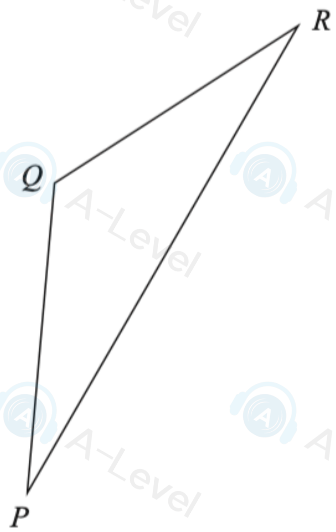

Figure 1

Figure 1 shows a sketch of triangle PQR.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$

(2)

(b) Find the size of angle PQR, in degrees, to three significant figures.

(3)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

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5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 13 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to one decimal place.

(3)

A circle with centre A and radius 35 cuts the line l_1 at the points P and Q.

Given that the x coordinate of P is greater than the x coordinate of Q,

(c) find the coordinates of P and the coordinates of Q.

(4)

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9. Relative to a fixed origin O, the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l, where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b.

(3)

(b) Find the vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, -3, 2)$

(c) Find the size of angle CAB.

(3)

(d) Find the exact area of the triangle CAB.

(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB.

(e) Find the coordinates of the two possible positions of D.

(4)

(Total for Question 9 is 14 marks)

8: Relative to a fixed origin O,

- the point A has position vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
- the point B has position vector $3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

Given that the line l passes through A and B,

(a) (i) find $\overrightarrow{AB}$

(ii) find a vector equation for l.

(3)

The point C has position vector $p\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, where p is a constant.

Given that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$,

(b) find the possible values of p.

(4)

The points A, B and C form a triangle ABC.

Given also that $p > 0$

(c) find the area of triangle ABC, giving your answer to 3 significant figures.

(4)

Leave blank

8. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \text{where } \mu \text{ is a scalar parameter}$$

Given that l_1 and l_2 meet at the point X,

(a) find the position vector of X.

(5)

The point $P(10, -7, 0)$ lies on l_1

The point Q lies on l_2

Given that $\overrightarrow{PQ}$ is perpendicular to l_2

(b) calculate the coordinates of Q.

(5)

Leave blank

7. With respect to a fixed origin O,

- the line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where λ is a scalar constant
- the point A has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that X is the point on l nearest to A,

(a) find

(i) the coordinates of X

(ii) the shortest distance from A to l.
Give your answer in the form $\sqrt{d}$, where d is an integer.

(7)

The point B is the image of A after reflection in l.

(b) Find the position vector of B.

(2)

8. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

The point A and the point B lie on line l

Given that

- A has coordinates $(-2, a, 4)$
- B has coordinates $(b, 3, 1)$

(a) find the value of the constant a and the value of the constant b.

(2)

(b) Hence find vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, 7, -2)$.

(c) Find the size of angle CAB, giving your answer in degrees to one decimal place.

(4)

The point D lies on the line l so that the area of triangle CAD is twice the area of triangle CAB.

(d) Find the coordinates of the two possible positions of D.

(4)