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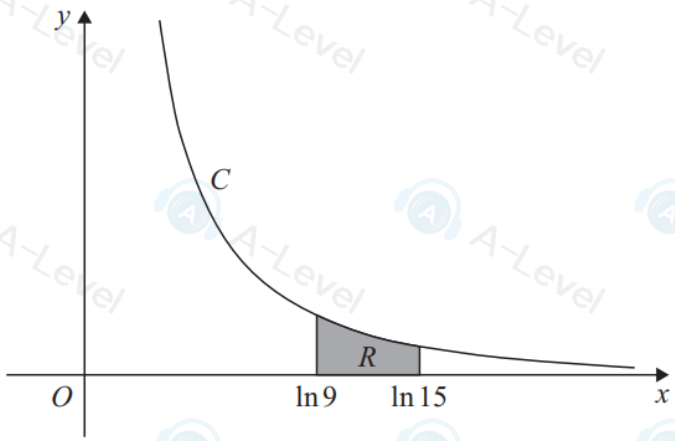


Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t+1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

3. The curve C is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to C at the point $(2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x\cos 3x \, dx$$

giving your answer in simplest form.

(5)

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2.

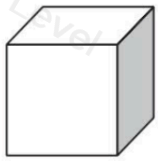


Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the surface area of the cube is S cm^2
- the volume of the cube is V cm^3

Given that the surface area of the cube is increasing at a constant rate of $4\,\text{cm}^2\,\text{s}^{-1}$

(a) show that $\frac{dx}{dt} = \frac{k}{x}$ where k is a constant to be found,

(4)

(b) show that $\frac{dV}{dt} = V^p$ where p is a constant to be found.

(3)

7. The volume V cm^3 of a spherical balloon with radius r cm is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

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4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{5 + 17x - 10x^2}{x(1 - x)(2x + 1)} \qquad x > 1$$

(a) Write $f(x)$ in the form

$$\frac{A}{x} + \frac{B}{1 - x} + \frac{C}{2x + 1}$$

where A, B and C are constants to be found.

(3)

(b) Hence, use algebraic integration to find the exact value of

$$\int_2^4 f(x)\,dx$$

Write your answer in the form $p \ln 2 + q \ln\left(\frac{3}{5}\right)$, where p and q are integers to be found.

(5)

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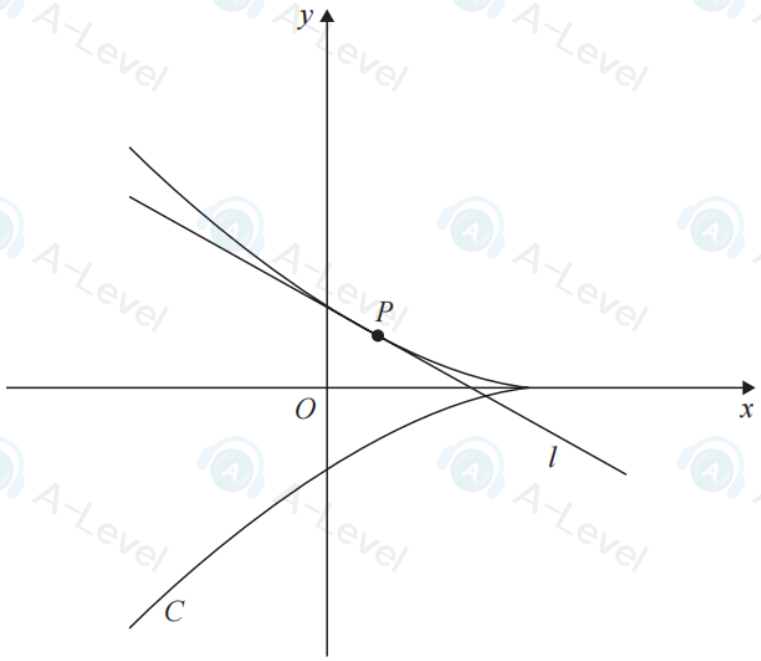


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\frac{dy}{dx} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

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3. $$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

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