**9.**

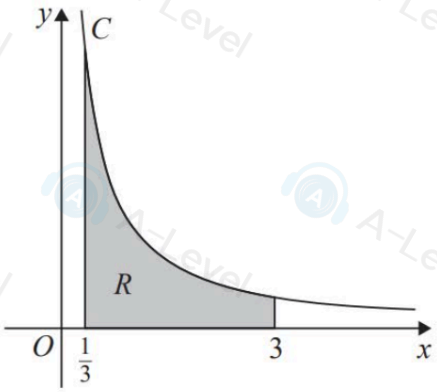


**Figure 3**

The curve $C$, shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\left(\arctan\sqrt{x}\right)}$$

The region $R$, shown shaded in Figure 3, is bounded by $C$, the line with equation $x = 3$, the $x$-axis and the line with equation $x = \frac{1}{3}$

The region $R$ is rotated through 360° about the $x$-axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume $V$ of the solid formed is given by

$$k\int_a^b \frac{1}{u^2}\,\mathrm{d}u$$

where $k$, $a$ and $b$ are constants to be found.

**(6)**

(b) Hence, using algebraic integration, find the value of $V$ in simplest form.

**(3)**

Leave blank

**4.** Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,\mathrm{d}x$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(8)**

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**6.** Use integration by parts to show that

$$\int e^{2x}\cos 3x\,dx = pe^{2x}\sin 3x + qe^{2x}\cos 3x + k$$

where $p$ and $q$ are rational numbers to be found and $k$ is an arbitrary constant.

**(6)**

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**2.** The curve $C$ has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \qquad y > 0$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$

**(6)**

Curve $C$ cuts the $y$-axis at the point $P$

(b) Find the exact value of the gradient of the tangent to $C$ at $P$

**(2)**

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**4.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

(i) The volume, $V$, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the radius of the balloon is 5 cm.

[The volume $V$ of a sphere of radius $r$ is given by the formula $V = \frac{4}{3}\pi r^3$]

**(4)**

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, $h$ cm, at a particular point in the cave is modelled by the differential equation

$$\frac{dh}{dt} = \frac{k}{h^3}$$

where $k$ is a constant and $t$ hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- $T$ hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of $T$.

Give your answer to one decimal place.

**(6)**

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**6.** Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3+1}} \mathrm{d}x = 2\left(x^3+1\right)^k \left(x^3 - A\right) + c$$

where $k$ and $A$ are constants to be found and $c$ is an arbitrary constant.

**(5)**

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**9.** Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, $A\,\text{cm}^2$, covered by the bacteria, $t$ days after the bacteria start to grow, is modelled by the differential equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r}\right)^2$$

where $p$, $q$ and $r$ are integers to be found.

**(7)**

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

**(2)**

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**7.** The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, $x$, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where $k$ is a constant and $t$ is the number of years after monitoring began.

(a) Show that $k = 1.8$

**(2)**

(b) Hence calculate the long-term population of goats predicted by this model.

**(1)**

In a **second** model, the number of goats, $x$, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

**(3)**

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

**(5)**

(e) Find the long-term population of goats predicted by this **second** model.

**(1)**

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**6.**

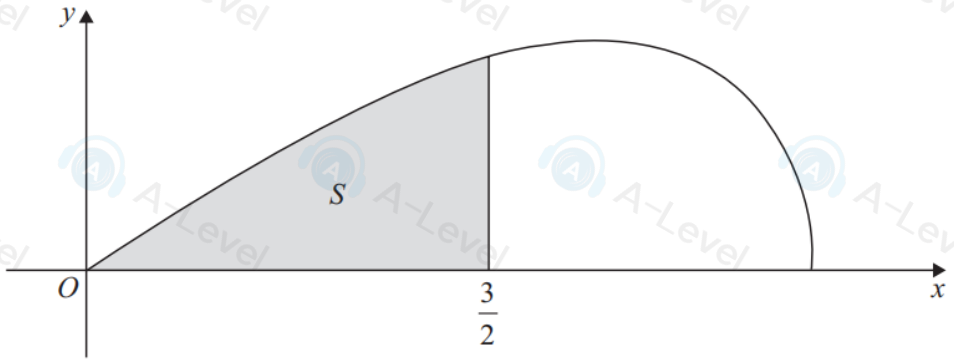


**Figure 2**

Figure 2 shows a sketch of the curve with parametric equations

$$x = 3\sin t \qquad y = 2\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The finite region $S$, shown shaded in Figure 2, is bounded by the curve, the $x$-axis and the line with the equation $x = \frac{3}{2}$

The shaded region $S$ is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution is given by

$$k\int_0^a \sin^2 t \cos^3 t \, \mathrm{d}t$$

where $k$ and $a$ are constants to be given in terms of $\pi$.

**(4)**

(b) Use the substitution $u = \sin t$, or otherwise, to find the exact value of this volume, giving your answer in the form $\frac{p\pi}{q}$ where $p$ and $q$ are integers.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(5)**

**(Total for Question 6 is 9 marks)**

**7.**

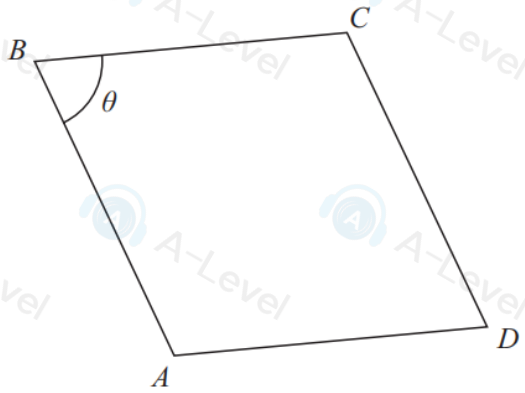


**Figure 2**

**In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\overrightarrow{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90°$

(a) find the exact value of $\cos\theta$, giving your answer in simplest form.

**(3)**

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where $k$ is an integer to be found.

**(3)**

**3:** Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y\frac{dy}{dx} = xe^{-y^2} \qquad y \geqslant 0 \quad x \geqslant 5$$

giving your answer in the form $y^2 = f(x)$

**(5)**

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