7. The volume $V\,\text{cm}^3$ of a spherical balloon with radius $r\,\text{cm}$ is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

3. The curve C is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to C at the point $(2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

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1. The binomial expansion of

$$(3 + kx)^{-2} \qquad |kx| < 3$$

where k is a non-zero constant, may be written in the form

$$A + Bx + Cx^2 + Dx^3 + \ldots$$

where A, B, C and D are constants.

(a) Find the value of A

(1)

Given that $C = 3B$

(b) show that

$$k^2 + 6k = 0$$

(3)

(c) Hence (i) find the value of k

(ii) find the value of D

(3)

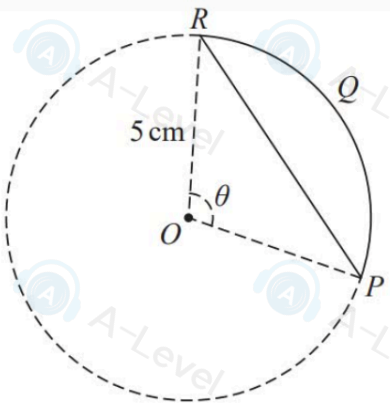

Figure 1

Figure 1 shows a sketch of a segment $PQRP$ of a circle with centre O and radius 5 cm.

Given that

- angle POR is θ radians
- θ is increasing, from 0 to π, at a constant rate of 0.1 radians per second
- the area of the segment $PQRP$ is A cm^2

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K(1 - \cos\theta)$$

where K is a constant to be found.

(2)

(b) Find, in cm^2 s^{-1}, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

(4)

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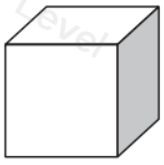

x cm

Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is V cm^3

Given that the volume of the cube is increasing at a constant rate of $5\,\text{cm}^3\text{s}^{-1}$

(a) find $\frac{\mathrm{d}x}{\mathrm{d}t}$ giving your answer in terms of x.

(3)

(b) Hence find the value of $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $V = 64$

(2)

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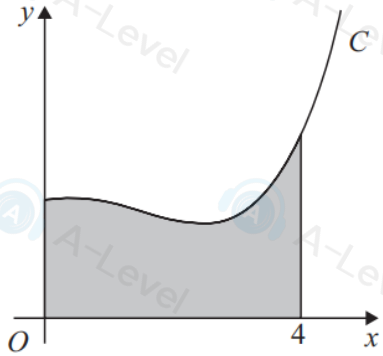

Figure 1

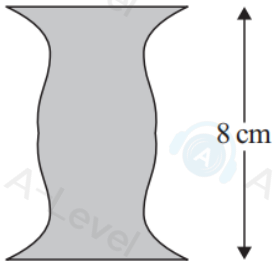

Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^x$$

The finite region, shown shaded in Figure 1, is bounded by C, the y-axis, the x-axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x-axis to form a solid S.

(a) Show that the exact volume of S is given by

$$\pi\int_0^4 \left(A + B\cos 2x + Ce^x \cos x + De^{2x}\right)\mathrm{d}x$$

where A, B, C and D are constants to be found.

(4)

(b) Find $\int e^x \cos x \, \mathrm{d}x$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together. The paperweight is 8 cm high.

(c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

3. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

(Total for Question 3 is 7 marks)

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4. $f(x) = \sqrt{1 - 4x^2} \qquad |x| < \frac{1}{2}$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of $f(x)$. Give each coefficient in simplest form.

(4)

(b) By substituting $x = \frac{1}{4}$ into the binomial expansion of $f(x)$, obtain an approximation for $\sqrt{3}$

Give your answer to 4 decimal places.

(2)

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1. The curve C has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C.

Find the equation of the normal to C at the point P, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(7)

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6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} dx = 2\left(x^3 + 1\right)^k \left(x^3 - A\right) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

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