8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

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6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

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6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is $45°$

(b) find the exact possible values of α

(6)

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7. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to l,

(a) find $\overrightarrow{OA}$

(2)

The point X lies on l.

Given that X is the point on l that is closest to the origin,

(b) find the coordinates of X.

(5)

The points O, X and A form the triangle OXA.

(c) Find the exact area of triangle OXA.

(3)

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9. Relative to a fixed origin O, the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l, where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b.

(3)

(b) Find the vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, -3, 2)$

(c) Find the size of angle CAB.

(3)

(d) Find the exact area of the triangle CAB.

(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB.

(e) Find the coordinates of the two possible positions of D.

(4)

(Total for Question 9 is 14 marks)

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

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2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

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