blank

5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} 13 \\ -1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to one decimal place.

(3)

A circle with centre A and radius 35 cuts the line l_1 at the points P and Q.

Given that the x coordinate of P is greater than the x coordinate of Q,

(c) find the coordinates of P and the coordinates of Q.

(4)

DO NOT WRITE IN THIS AREA

DO

6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is $45°$

(b) find the exact possible values of α

(6)

DO NOT WRITE IN THIS AREA

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

3.

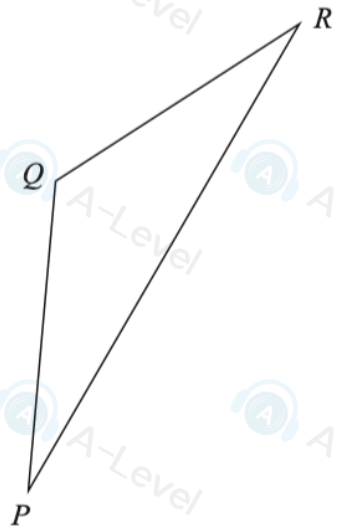


Figure 1

Figure 1 shows a sketch of triangle PQR.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$

(2)

(b) Find the size of angle PQR, in degrees, to three significant figures.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

8: Relative to a fixed origin O,

- the point A has position vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
- the point B has position vector $3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

Given that the line l passes through A and B,

(a) (i) find $\overrightarrow{AB}$

(ii) find a vector equation for l.

(3)

The point C has position vector $p\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, where p is a constant.

Given that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$,

(b) find the possible values of p.

(4)

The points A, B and C form a triangle ABC.

Given also that $p > 0$

(c) find the area of triangle ABC, giving your answer to 3 significant figures.

(4)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN TH
DO NOT WRITE IN THIS AREA DO NOT WI

Leave blank

7. With respect to a fixed origin O,

- the line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where λ is a scalar constant
- the point A has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that X is the point on l nearest to A,

(a) find

(i) the coordinates of X

(ii) the shortest distance from A to l.
Give your answer in the form $\sqrt{d}$, where d is an integer.

(7)

The point B is the image of A after reflection in l.

(b) Find the position vector of B.

(2)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

---

4. Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $|\overrightarrow{PP'}|$, giving your answer as a simplified surd.

(2)

DO NOT WRITE IN THIS AREA