3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \qquad y = \frac{5}{t+2} \qquad t \geqslant 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \qquad e < x \leqslant f$$

and a, b, c, d, e and f are constants to be found.

(5)

(b) State the range of g.

(2)

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6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = 8\cos t + 2 \qquad y = 6\sin t - 3 \qquad 0 < t \leqslant 2\pi$$

The point P lies on C where $t = \frac{\pi}{3}$

(a) Use parametric differentiation to find the exact gradient of the **tangent** to C at P.

(3)

The **normal** to C at P intersects the x-axis at the point R.

(b) Find, using algebra, the exact coordinates of R.

(6)

(c) Find a Cartesian equation of C. The equation must not involve trigonometric expressions.

(2)

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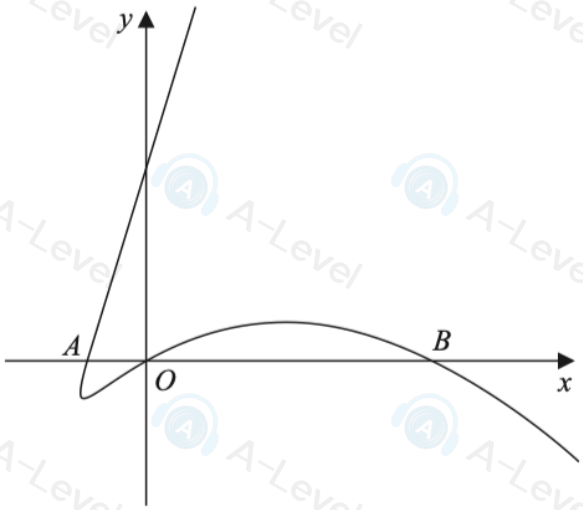

Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

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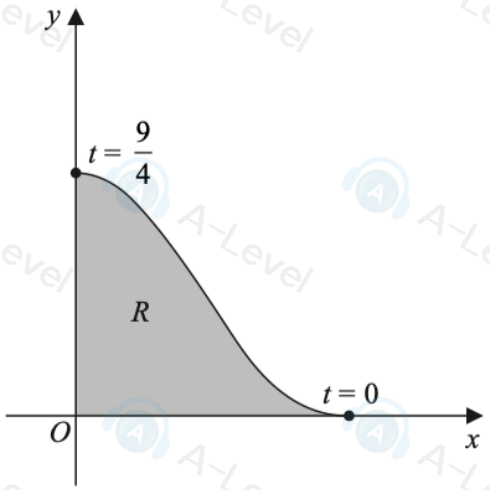

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K\int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}}\,\mathrm{d}t$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

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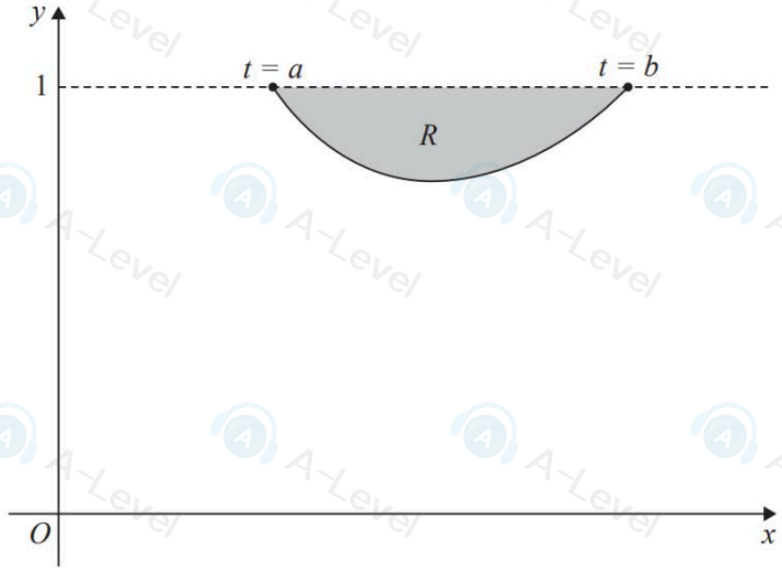

Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where M and k are constants to be found.

(5)

(c) (i) Write $\dfrac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R, giving your answer in simplest form.

(6)

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8. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

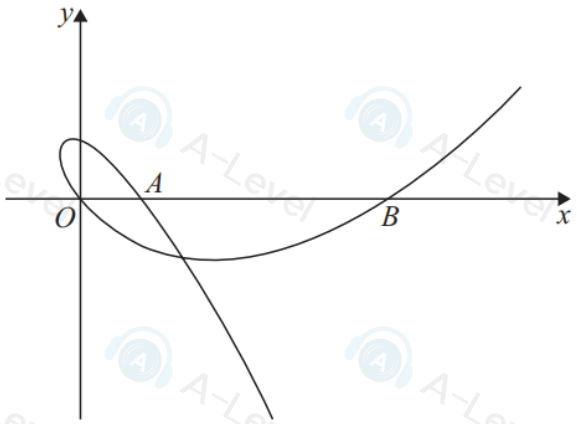

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = t^2 + 2t \qquad y = t^3 - 9t \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B as shown in Figure 3.

(a) Find the coordinates of point A and show that point B has coordinates (15, 0).

(3)

(b) Find the equation of the tangent to the curve at B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The tangent to the curve at B cuts the curve again at point X.

(c) Use algebra to find the coordinates of X, showing each stage of your working.

(5)

(Total for Question 8 is 12 marks)