3. A curve has parametric equations

$$x = \frac{t+15}{t+4} \qquad y = \frac{5}{t+2} \qquad t \geqslant 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax+b}{cx+d} \qquad e < x \leqslant f$$

and a, b, c, d, e and f are constants to be found.

(5)

(b) State the range of g.

(2)

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

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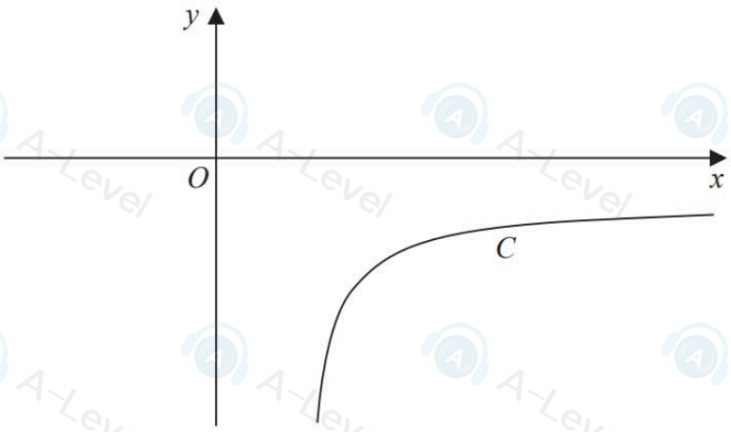

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \qquad y = \sqrt{3}\tan\left(t + \frac{\pi}{3}\right) \qquad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t

(3)

(b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are constants.

(4)

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(c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found.

(5)

Leave blank

2. The curve C has parametric equations

$$x = \frac{t^4}{2t + 1} \qquad y = \frac{t^3}{2t + 1} \qquad t > 0$$

(a) Write down $\frac{x}{y}$ in terms of t, giving your answer in simplest form.

(1)

(b) Hence show that all points on C satisfy the equation

$$x^3 - 2xy^3 - y^4 = 0$$

(3)

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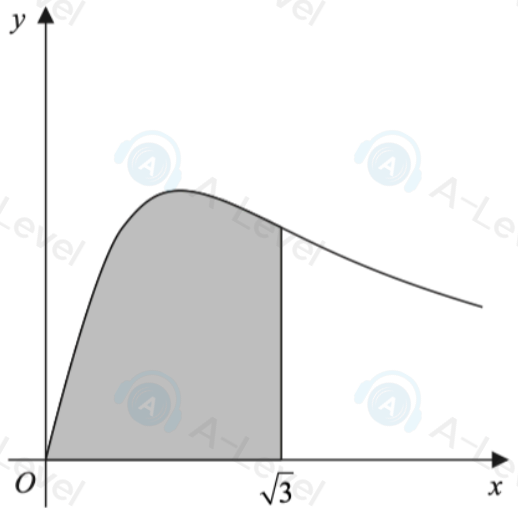

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan\theta \qquad y = 2\sin 2\theta \qquad \theta \geqslant 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta)\, \mathrm{d}\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

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3. The curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that C has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where p and q are constants to be found.

(6)

(b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where a, b, c and d are constants.

(3)

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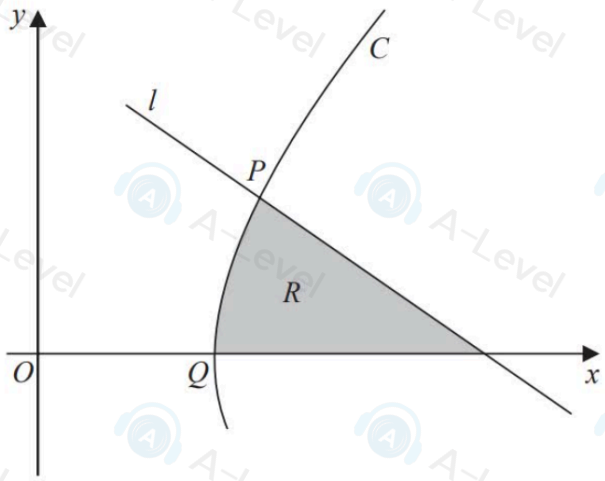

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R, shown shaded in Figure 2 is bounded by the curve C, the line l and the x-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x-axis.

(7)

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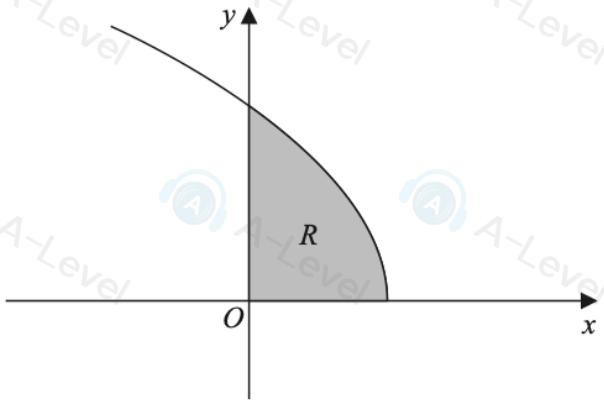

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t \, \mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

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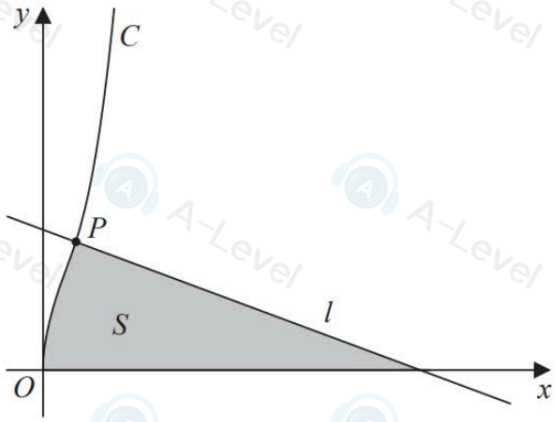

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

(6)

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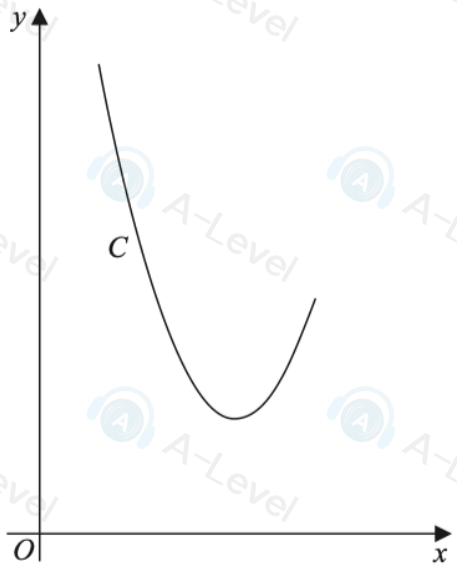

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

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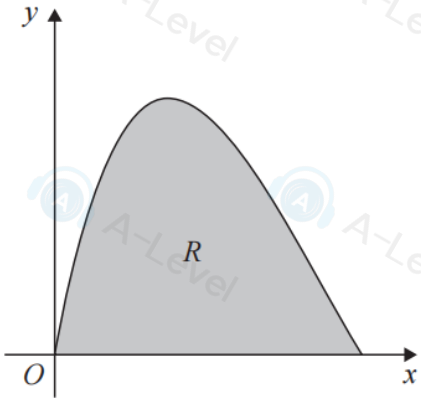

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, \mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

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