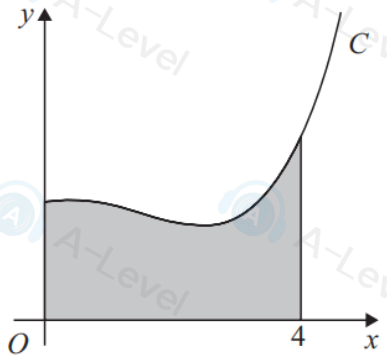

Figure 1

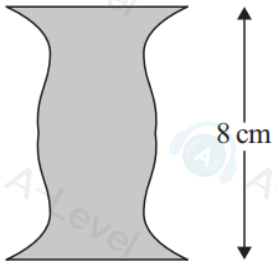

Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^{x}$$

The finite region, shown shaded in Figure 1, is bounded by C, the y-axis, the x-axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x-axis to form a solid S.

(a) Show that the exact volume of S is given by

$$\pi\int_{0}^{4}\left(A + B\cos 2x + Ce^{x}\cos x + De^{2x}\right)\mathrm{d}x$$

where A, B, C and D are constants to be found.

(4)

(b) Find $\int e^{x}\cos x \, \mathrm{d}x$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together. The paperweight is 8 cm high.

(c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_{0}^{\frac{\pi}{6}} x\cos 3x \, \mathrm{d}x$$

giving your answer in simplest form.

(5)

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10.

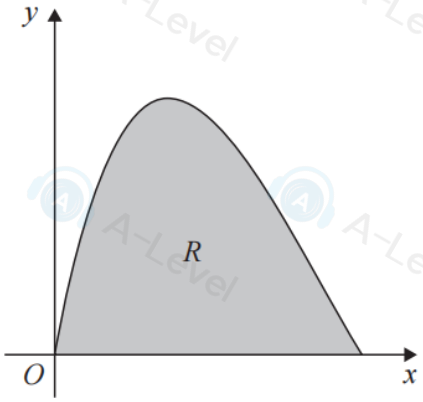

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t \sin^2 t \cos t \, \mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4}\, \mathrm{e}^{\sqrt{2x+1}} \, \mathrm{d}x$$

may be expressed in the form

$$\int_a^b ku^2 \mathrm{e}^u \, \mathrm{d}u$$

where a, b and k are constants to be found.

(4)

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4}\, \mathrm{e}^{\sqrt{2x+1}} \, \mathrm{d}x$$

giving your answer in simplest form.

(5)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of $\int_1^e \frac{\ln x}{x^2} dx$

Write your answer in the form $a + \frac{b}{e}$, where a and b are integers.

(6)

(Total for Question 2 is 6 marks)

---

7. (a) Express $\frac{1}{(4-x)(2-x)}$ in partial fractions.

(2)

The mass, x grams, of a substance at time t seconds after a chemical reaction starts is modelled by the differential equation

$$\frac{dx}{dt} = k(4-x)(2-x) \qquad t \geqslant 0 \qquad 0 \leqslant x < 2$$

where k is a constant.

Given that when $t = 0$, $x = 0$

(b) solve the differential equation and show that the solution can be written as

$$x = \frac{4 - 4e^{2kt}}{1 - 2e^{2kt}}$$

(*Solutions relying on calculator technology are not acceptable.*)

(6)

Given that $k = 0.1$

(c) find the value of t when $x = 1$, giving your answer, in seconds, to 3 significant figures.

(2)

(Total for Question 7 is 10 marks)

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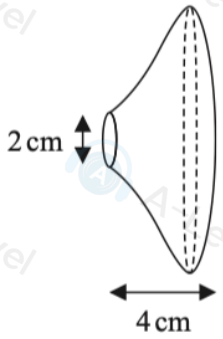

Figure 3

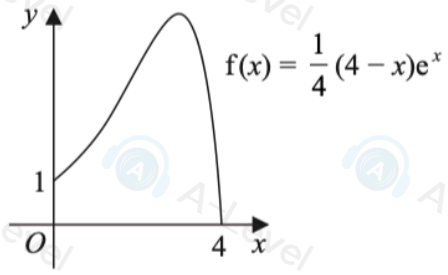

Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x-axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4 - x)e^x \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, $V\,\text{cm}^3$, of the doorknob is given by

$$V = K\int_0^4 (x^2 - 8x + 16)e^{2x}\,dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^q + r)\,\text{cm}^3$ where p, q and r are simplified rational numbers to be found.

(5)

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7. (a) Show that the substitution $u = \tan x$ transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3}\,dx$$

to

$$\int \frac{u}{(k + u^2)^n}\,du$$

where k and n are integers to be found.

(4)

(b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3}\,dx$$

(2)

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5. (i) Find

$$\int x^2 e^{4x} \, dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} \, dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

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7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Use the substitution $x = 4\sin\theta$ to find the exact value of

$$\int_2^{2\sqrt{3}} \frac{1}{(16-x^2)^{\frac{3}{2}}} \, dx$$

(6)

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7. The current, x amps, at time t seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where k is a constant.

Initially there is zero current in the circuit.

(a) Solve the differential equation to find an equation, in terms of k, for the current in the circuit at time t seconds.
Give your answer in the form $x = f(t)$.

(6)

Given that in the long term the current in the circuit approaches 7 amps,

(b) find the value of k.

(2)

(c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

(3)

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