4. A discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(2-x) & x = 0, 1 \\ k(3-x) & x = 2, 3 \\ k(x+1) & x = 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$ **(2)**

Find the exact value of

(b) $P(1 \leqslant X < 4)$ **(1)**

(c) $E(X)$ **(2)**

(d) $E(X^2)$ **(2)**

(e) $\text{Var}(3X + 1)$ **(3)**

2. The discrete random variable X has the following probability distribution.

x	-2	-1	0	1	3
$P(X = x)$	0.15	a	b	a	0.4

(a) Find $E(X)$. **(2)**

Given that $E(X^2) = 4.54$

(b) find the value of a and the value of b. **(5)**

The random variable $Y = 3 - 2X$

(c) Find $\text{Var}(Y)$. **(3)**

5. Franca is the manager of an accountancy firm. She is investigating the relationship between the salary, £x, and the length of commute, y minutes, for employees at the firm. She collected this information from 9 randomly selected employees.

The salary of each employee was then coded using $w = \dfrac{x - 20\,000}{1000}$

The table shows the values of w and y for the 9 employees.

w	6	8	8	–1	25	15	3	–2	19
y	45	50	35	65	25	40	50	75	20

(You may use $\sum w = 81 \quad \sum y = 405 \quad \sum wy = 2490 \quad S_{ww} = 660 \quad S_{yy} = 2500$)

(a) Calculate the salary of the employee with $w = -2$

(1)

(b) Show that, to 3 significant figures, the value of the product moment correlation coefficient between w and y is –0.899

(3)

(c) State, giving a reason, the value of the product moment correlation coefficient between x and y

(1)

The least squares regression line of y on w is $y = 60.75 - 1.75w$

(d) Find the equation of the least squares regression line of y on x giving your answer in the form $y = a + bx$

(3)

(e) Estimate the length of commute for an employee with a salary of £21 000

(2)

Franca uses the regression line to estimate the length of commute for employees with salaries between £25 000 and £40 000

(f) State, giving a reason, whether or not these estimates are reliable.

(2)

3. The discrete random variable X has probability distribution

$$P(X = x) = \frac{1}{5} \qquad x = 1, 2, 3, 4, 5$$

(a) Write down the name given to this distribution.

(1)

Find

(b) $P(X = 4)$

(1)

(c) $F(3)$

(1)

(d) $P(3X - 3 > X + 4)$

(2)

(e) Write down $E(X)$

(1)

(f) Find $E(X^2)$

(2)

(g) Hence find $Var(X)$

(2)

Given that $E(aX - 3) = 11.4$

(h) find $Var(aX - 3)$

(4)

5. The discrete random variable X has the following probability distribution

x	-2	0	2	4
$P(X = x)$	a	b	a	c

where a, b and c are probabilities.

Given that $E(X) = 0.8$

(a) find the value of c.

(2)

Given also that $E(X^2) = 5$ find

(b) the value of a and the value of b,

(4)

(c) $\text{Var}(X)$

(2)

The random variable $Y = 5 - 3X$

Find

(d) $E(Y)$

(1)

(e) $\text{Var}(Y)$

(2)

(f) $P(Y \geqslant 0)$

(4)

Leave blank

6. The random variable A represents the score when a spinner is spun. The probability distribution for A is given in the following table.

a	1	4	5	7
$P(A = a)$	0.40	0.20	0.25	0.15

(a) Show that $E(A) = 3.5$
(2)

(b) Find $\text{Var}(A)$
(3)

The random variable B represents the score on a 4-sided die. The probability distribution for B is given in the following table where k is a positive integer.

b	1	3	4	k
$P(B = b)$	0.25	0.25	0.25	0.25

(c) Write down the name of the probability distribution of B.
(1)

(d) Given that $E(B) = E(A)$ state, giving a reason, the value of k.
(1)

The random variable $X \sim N(\mu, \sigma^2)$

Sam and Tim are playing a game with the spinner and the die.

They each spin the spinner once to obtain their value of A and each roll the die once to obtain their value of B.
Their value of A is taken as their value of μ and their value of B is taken as their value of σ.
The person with the larger value of $P(X > 3.5)$ is the winner.

(e) Given that Sam obtained values of $a = 4$ and $b = 3$ and Tim obtained $b = 4$ find, giving a reason, the probability that Tim wins.
(2)

(f) Find the largest value of $P(X > 3.5)$ achievable in this game.
(4)

(g) Find the probability of achieving this value.
(2)

6. A biased coin has probability 0.4 of showing a head. In an experiment, the coin is spun until a head appears. If a head has not appeared after 4 spins, the coin is not spun again. The random variable X represents the number of times the coin is spun.

For example, $X = 3$ if the first two spins do not show a head but the third spin does show a head. The coin would not then be spun a fourth time since the coin has already shown a head.

(a) Show that $P(X = 3) = 0.144$

(1)

The table gives some values for the probability distribution of X

x	1	2	3	4
$P(X = x)$		0.24	0.144	

(b) (i) Write down the value of $P(X = 1)$

(ii) Find $P(X = 4)$

(3)

(c) Find $E(X)$

(2)

(d) Find $\text{Var}(X)$

(3)

The random variable H represents the number of heads obtained when the coin is spun in the experiment.

(e) Explain why H can only take the values 0 and 1 and find the probability distribution of H.

(2)

(f) Write down the value of

(i) $P(\{X = 3\} \cap \{H = 0\})$

(ii) $P(\{X = 4\} \cap \{H = 0\})$

(2)

The random variable $S = X + H$

(g) Find the probability distribution of S

(4)

7. The cumulative distribution of a discrete random variable X is given by

x	1	2	3	4
$\mathrm{F}(x)$	$\frac{1}{13}$	$\frac{2k-1}{26}$	$\frac{3(k+1)}{26}$	$\frac{k+4}{8}$

where k is a positive constant.

(a) Show that $k = 4$

(1)

(b) Find the probability distribution of the discrete random variable X

(3)

(c) Using your answer to part (b), write down the mode of X

(1)

(d) Calculate $\mathrm{Var}(13X - 6)$

(5)

Leave blank

4. A spinner can land on the numbers 10, 12, 14 and 16 only and the probability of the spinner landing on each number is the same.
The random variable X represents the number that the spinner lands on when it is spun once.

(a) State the name of the probability distribution of X.

(1)

(b) (i) Write down the value of $E(X)$

(1)

(ii) Find $\text{Var}(X)$

(2)

A second spinner can land on the numbers 1, 2, 3, 4 and 5 only.
The random variable Y represents the number that this spinner lands on when it is spun once. The probability distribution of Y is given in the table below

y	1	2	3	4	5
$P(Y = y)$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$

(c) Find (i) $E(Y)$

(2)

(ii) $\text{Var}(Y)$

(3)

The random variable $W = aX + b$, where a and b are constants and $a > 0$
Given that $E(W) = E(Y)$ and $\text{Var}(W) = \text{Var}(Y)$

(d) find the value of a and the value of b.

(5)

Each of the two spinners is spun once.

(e) Find $P(W = Y)$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO N